NEW ORTHOGONAL BINARY USER CODES FOR MULTIUSER SPREAD SPECTRUM COMMUNICATIONS

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ABSTRACT

Walsh codes are perfectly orthogonal binary (antipodal) block codes that found many popular applications over many decades including synchronous multi-user communications. It is well known that they perform poorly for asynchronous multi-user communications. Therefore, the Gold codes are the preferred user codes in asynchronous CDMA communications. In this paper, new sets of binary orthogonal user codes are introduced for asynchronous spread spectrum multi-user communications. It is shown in this paper that the proposed binary user code family outperforms the Walsh codes significantly and they match in performance with the popular, nearly orthogonal Gold codes closely for asynchronous multi-user communications. We present that there are a good number of such independent code sets of different sizes available in the binary space. They might help us to increase service/multi-service capabilities of future communications systems.

1. INTRODUCTION

This paper introduces complementary orthogonal sets of binary space to the popular Walsh codes for spread-spectrum service communications applications. Note that Walsh family is a small subset of binary space with well-regulated functions in the bases of transforms [1-2]. In addition to the linear-phase property of the basis functions, Walsh sets do not include any two sequences in the set with the same number of zero-crossings. If one relaxes the latter, complementary orthogonal sets in the binary space with still linear phase are obtained.

The computational power available today allowed us to search for and obtain those new binary transform sets reported in this paper. We compared their performance with widely used codes like Gold codes and Walsh codes. Our comparisons include their time and frequency domain properties along with their BER performance in AWGN channels. It is shown that these new orthogonal binary codes outperform Walsh codes significantly and provide a comparable performance with Gold codes in all the measures and service scenarios considered in this paper. A sample list of these new codes is also given in the paper for further studies.

2. MATHEMATICAL REMARKS AND SEARCH

Earlier studies in the literature on orthogonal binary transforms suggest that linear phase, zero mean codes outperform non-linear and non-zero mean codes in their BER performance [3-4]. Accordingly, sample space for the design of the proposed new orthogonal codes is only limited to zero mean and linear phase codes in this study. Therefore, for 8-bit (length 8) codes, binary sample space consists of 22 unique codes, for 16-bit codes there are 326 candidate codes, and for 32-bit codes there are about 38,000 codes.

Walsh codes perform poorly in AWGN channels for asynchronous communications when the circular shift of the second code (following code in a stream of codes) matches with the first code or the complement of the first code for any particular chip delay. Note that the decimal values of all n-bit (size n) Walsh codes in a set are multiples of $2^{(n/2)}+1$ or $2^{(n/2)}-1$. For example, 16-bit codes of the size 16 Walsh set are multiples of 255 or 257. In our orthogonal code design and search, such strict code conditions are avoided and the number of codes in the set that are multiples of $2^{(n/2)}+1$ or $2^{(n/2)}-1$ is minimized. Moreover, we excluded the condition of having only one code in the set for a given number of zero-crossings that was a requirement in the Walsh family.

Orthogonal code sets with linear phase are iteratively selected from the binary sample space for the given dimension. Our studies have shown that in any n-bit sample space (n-dimensional space), n-1 orthogonal code sets can be formed. DC code is added to the set to make it a complete binary code set. Using this method, a number of independent binary code sets can be generated from the sample space.

Note that the degrees of freedom is very limited when the dimension of the space is low, i.e. short codes or small values of n. As an example, for the 8-bits and 16-bits cases, there are not any other binary orthogonal sets available that do not share some of their basis functions with the Walsh family. These orthogonal binary code sets can be formed by carefully including certain Walsh codes common in both sets. In contrast, for the cases of 32 bits or higher, a number of unique orthogonal binary code sets can be generated. Table I displays the functions of a typical new 16-bit code family in decimals for convenience where the 0's replaced by -1 value of the binary codes along with the Walsh set. Note that 8 of these functions are common with size 16 Walsh set.

Table I: A typical new 16-bit code family where common functions with Walsh set are bolded.

Function Index	New Code Set	Walsh Set
1	**65535**	**65535**
2	383	43690
3	3727	52428
4	**39321**	**39321**
5	12979	61680
6	**42405**	**42405**
7	**50115**	**50115**
8	15683	38550
9	21717	65280
10	**43605**	**43605**
11	**52275**	**52275**
12	23333	39270
13	**61455**	**61455**
14	26393	42330
15	26857	49980
16	**38505**	**38505**

Similarly, a typical set of size 32 binary sequences for the proposed family along with the Walsh set is displayed in Table II. Note that except the first function, which is DC, none of the basis functions of the new orthogonal binary set is a common function with the Walsh set for 32-bit codes.

There are several other 32 dimensional unique binary sets available in the binary space.

Table II: A typical new 32-bit code family along with the Walsh set.

Function Index	New Code Set	Walsh Set
1	**4294967295**	**4294967295**
2	5973503	2863311530
3	629325403	3435973836
4	1193068317	2576980377
5	1915216974	4042322160
6	1634031993	2779096485
7	1011666371	3284386755
8	116291424	2526451350
9	1059591420	4278255360
10	1455520917	2857740885
11	1850886774	3425946675
12	1539580890	2573637990
13	429217383	4027576335
14	902022060	2774181210
15	1956395310	3275539260
16	1891164913	2523502185
17	774992779	4294901760
18	1792447657	2863289685
19	199813167	3435934515
20	304520119	2576967270
21	446421336	4042264335
22	477474360	2779077210
23	593345220	3284352060
24	699821460	2526439785
25	939458579	4278190335
26	1202818530	2857719210
27	1286539981	3425907660
28	1292890290	2573624985
29	1366974090	4027518960
30	1567797573	2774162085
31	2064330529	3275504835
32	1760139030	2523490710

3. PERFORMANCE COMPARISONS

Typical 32-bit orthogonal Walsh and proposed codes along with a 31-bit, nearly orthogonal, Gold codes are displayed in Figure 1. Magnitude response functions of these codes are shown in Figure 2. Note that the sample sequence of the proposed orthogonal codes has more evenly spread frequency spectrum compared to sample Walsh code of the same length.

Cross-correlation sequences between a typical pair of codes (2-user case) for the three families under consideration are displayed in Figure 3. It is observed that Gold and proposed codes have similar cross-correlation (inter-code correlation) while sample Walsh pair has worse correlation properties.

For 32 bit codes, comparisons of sums and variances of the maximum, and sum of aperiodic cross correlations between all the pairs of codes for Walsh, Gold, and the proposed codes (three distinct orthogonal codes; Ortho1, Ortho2, Ortho3) are given in Table III. It is observed from the table that the cross-correlation properties of the proposed codes and Gold codes are comparable while the cross-correlations of Walsh code pairs are inferior to others. These inter-code and intra-code correlation properties dictate the performance of a service communications system. Therefore, choosing the best possible user codes with minimum intra-code and inter-code correlation properties will improve the system performance.

We considered an asynchronous communications scenario with 2 users in the system. The goal here is to investigate the BER performance of the communications system with AWGN noise assumption and employing different user code families. This helps us to understand better the variations of the inter-code and inter-code correlations of the codes whenever the noise is at presence. The randomness of channel noise will perturb the noise-free correlation properties of the user codes presented in Table III. Communications performance is computed by taking the average of BER performances over all the possible pairs of codes. Figure 4 displays BER performances of 16-bit Walsh and proposed code families. It is clearly seen from this figure that the latter significantly outperforms the first. Similarly, Figure 5 displays the BER curves for the case of 32-bits length orthogonal and 31-bits Gold codes. It is observed from these BER curves that the performance of the proposed orthogonal code (Ortho3; set3) outperforms Walsh codes significantly and it closely matches with that of Gold code. Similarly, the other sets of the proposed orthogonal binary code family, namely Ortho1 (set1) and Ortho2 (set2) perform comparable to the popular Gold codes.

4. CONCLUSIONS

The growing demand for orthogonal, fixed power (binary/antipodal) user codes require additional codes to be available for service spread spectrum communications applications. We presented in this paper that the Walsh codes utilize only a small portion of the orthogonal binary space due to their restrictions that are not necessarily important for service communications. We proposed a design methodology and derived a number of orthogonal code sets that outperform Walsh codes and closely match with Gold codes for asynchronous CDMA applications. Service capabilities of existing communications systems might be improved by simultaneously using these independent code sets for different users in software radios. Formulating the problem with more elegance and extending the code design for any length using mathematical analysis tools are currently under study.

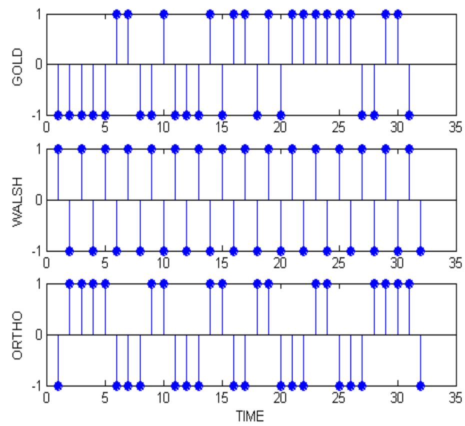


Fig. 1: Time domain representations of typical 32-length codes for Walsh and proposed (Ortho) families along with a 31-length Gold code.

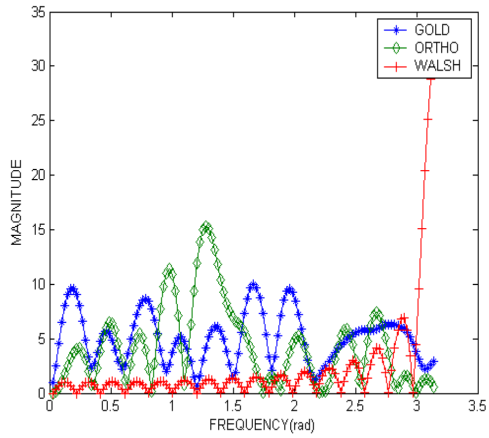


Fig. 2: Magnitude response functions of Walsh, Gold, and proposed orthogonal (ortho) binary codes plotted in Figure 1.

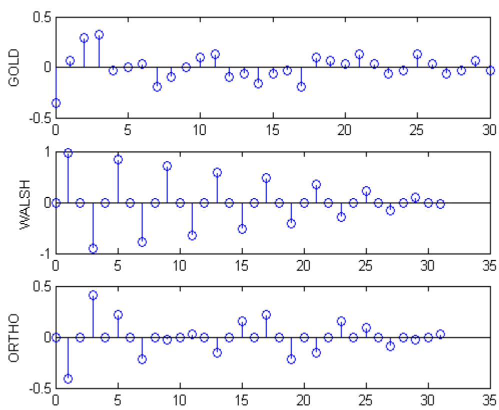
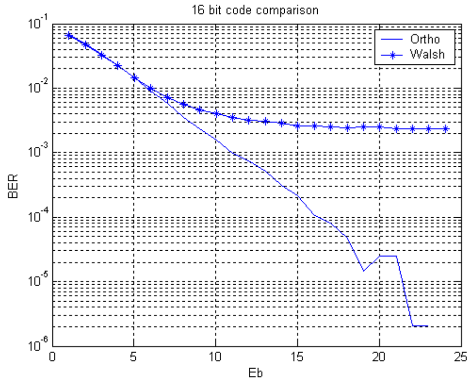


Fig. 3: Cross-correlation (inter-code correlation) sequences between typical pairs of codes.

Fig. 4: BER performance of length 16 Walsh and proposed codes for asynchronous communications channel in 2-users scenario.



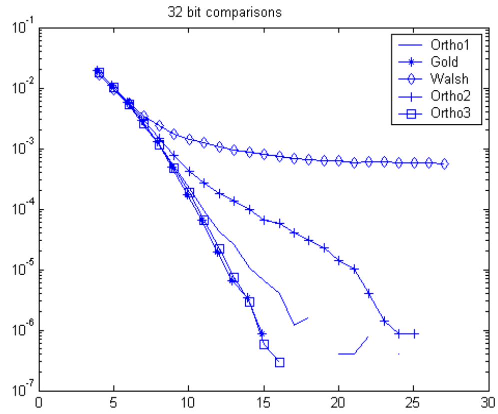


Fig. 5: BER performance of various codes for asynchronous communications channel in 2-users scenario.

TABLE III: Cross - Correlation (inter-code correlations) Comparisons of 31-bit Gold code and 32-bit Orthogonal Walsh and proposed codes.

	Walsh		**Gold**		**Ortho1**		**Ortho2**		**Ortho3**	
	Max	**Total**	**Max**	**Total**	**Max**	**Total**	**Max**	**Total**	**Max**	**Total**
Sum:	103	935	138	1343	140	1336	133	1225	144	1361
Variance:	.041	2.21	.0018	.1844	.0076	.4086	.0132	.8240	.0054	.2435

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