THE LOCAL HARMONIC DECOMPOSITION: A TOOL FOR EXTRACTING ANGLE INFORMATION FROM WAVEFIELDS

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ABSTRACT

This paper describes the local harmonic decomposition: it is an efficient algorithm which can be used to extract angle information from a wavefield. It is then shown that this algorithm can be used in the context of geophysical processing to produce reflection angle gathers during a shot-record migration. Shot-record wave-equation migration is a very accurate method for imaging geophysical data in complex propagation velocity media. However, producing one output image of the acoustic reflectors is not enough, multiple images where each image corresponds to a certain reflection angle with respect with the local dip of the reflector allows more information to be gained about this reflector. This angle information can also be used to check the validity of the velocity model used during the propagation. Tests performed on synthetic geophysical data proves the ability of the local harmonic decomposition to sort the reflected energy according to local reflection angle.

1. INTRODUCTION

In geophysical exploration, an acoustic wavefield is emitted through the earth, reflected by the earth interfaces and scattered back to the surface where it is measured. In order to obtain a precise image of these earth interfaces causing the reflected energy, a process called migration must be used. The shot-record migration consists in propagating through an acoustic medium of variable but known velocity, a synthetic wavefield representing the incident emitted wavefield, and back-propagating through the same medium the recorded data representing the scattered reflected wavefield. A cross-correlation of these two wavefields gives the reflectivity of the subsurface for each depth point. However this reflectivity has no angle information. Rickett and Sava [1] have proposed a method for angle-gather computation that seems to produce some artefacts in case of large shot intervals, which can be the case in shot-record migrations.

Our motivation is to review the theory behind angle gathers, derive an exact expression for the angle gathers, then derive an efficient algorithm that matches this expression within the usual limits of a wave-equation migration, defined by a maximum propagation angle and a maximum spatial wavenumber.

2. LOCAL ANGLE TRANSFORM

Let's $w(x)$ be a wavefield at a given frequency ω and a given depth z. We want to decompose this wavefield in the sum of angle components. The general case is that the wavefield $w(x)$ is the sum of its downgoing component $w_+(x)$ and its upgoing component $w_-(x)$. We define the angle θ such as $\theta = 0$ corresponds to vertical downgoing propagation. θ is defined on $[-\frac{\pi}{2}, \frac{\pi}{2}]$ for the downgoing wavefield ($\cos\theta \geq 0$), and on $[\frac{\pi}{2}, \frac{3\pi}{2}]$ for the upgoing wavefield ($\cos\theta \leq 0$). Let's consider the downgoing component first. We can take the Fourier transform:

$$w_+(x) = \frac{1}{2\pi} \int_{-\frac{\omega}{c}}^{\frac{\omega}{c}} e^{jkx} \hat{W}_+(k) dk, \tag{1}$$

where c is the local velocity and where we have neglected the evanescent energy. We now define the angle by the change of variable:

$$k = \frac{\omega}{c} \sin\theta, \tag{2}$$

$$w_+(x) = \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{j\frac{\omega}{c}\sin\theta x} \frac{\omega}{c} \cos\theta \;\; \hat{W}_+(\frac{\omega}{c}\sin\theta) d\theta. \tag{3}$$

We define an angle transform on $\theta = [-\frac{\pi}{2}, \frac{\pi}{2}]$ by:

$$\begin{aligned} W_+(\theta) &= \tfrac{\omega}{c}\cos\theta \;\; \hat{W}_+(\tfrac{\omega}{c}\sin\theta) \\ &= \tfrac{\omega}{c}\cos\theta \int_{-\infty}^{\infty} e^{-j\frac{\omega}{c}\sin\theta x} w_+(x) dx. \end{aligned} \tag{4}$$

With that definition, $w_+(x)$ can be decomposed in:

$$w_+(x) = \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{j\frac{\omega}{c}\sin\theta x} W_+(\theta) d\theta. \tag{5}$$

We can define a local angle transform by changing equation (4) to:

$$W_+(x,\theta) = \frac{\omega}{c}\cos\theta \int_{-\infty}^{\infty} e^{j\frac{\omega}{c}\sin\theta u} w_+(x-u) du. \tag{6}$$

A similar equation can be found in Xie and Wu [2] without the cosine term. This term ensures that averaging the local angle components gives the local field. The computation of such local angle transform is done by a small window local Fourier Transform. As this computation must be done at each output point, this can be computer-intensive.

3. LOCAL HARMONIC DECOMPOSITION

I will now describe an efficient algorithm which can be used to compute the local angle transform, or better, to bypass it. Starting from equation (6) we can, for each x, take the Fourier transform on the angle variable. As the angle variable is periodic with period 2π, this corresponds to an harmonic decomposition.

$$W_+(x,\theta) = \sum_{n=-\infty}^{\infty} \hat{w}_n^+(x) e^{jn\theta}. \tag{7}$$

The important result is that $\hat{w}_n^+(x)$ can be computed directly from $w_+(x)$. As $W_+(x,\theta)$ is zero on $[\frac{\pi}{2}, \frac{3\pi}{2}]$:

$$\hat{w}_n^+(x) = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-jn\theta} W_+(x,\theta) d\theta. \quad (8)$$

Inserting equation (6):

$$\hat{w}_n^+(x) = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-jn\theta} \frac{\omega}{c}\cos\theta \int_{-\infty}^{\infty} e^{j\frac{\omega}{c}\sin\theta u} w_+(x-u) du d\theta, \quad (9)$$

interchanging the order of the integrals and making the change of variable $k = \frac{\omega}{c}\sin\theta$, we obtain:

$$\hat{w}_n^+(x) = \int_{-\infty}^{\infty} w_+(x-u) du \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-jn\theta}\frac{\omega}{c}\cos\theta e^{j\frac{\omega}{c}\sin\theta u} d\theta = \int_{-\infty}^{\infty} w_+(x-u) du \frac{1}{2\pi}\int_{-\frac{\omega}{c}}^{\frac{\omega}{c}} e^{-jn\arcsin\frac{ck}{\omega}} e^{jku} dk. \quad (10)$$

Defining in the k domain the filters:

$$F_n^+(k) = e^{-jn\arcsin\frac{ck}{\omega}} = \left(\sqrt{1-(ck/\omega)^2} - jck/\omega\right)^n, \quad (11)$$

we obtain, $f_n^+(x)$ being the filters in the x domain:

$$\hat{w}_n^+(x) = \int_{-\infty}^{\infty} f_n^+(u) w_+(x-u) du. \quad (12)$$

We see that we can compute $\hat{w}_n^+(x)$ by filtering $w_+(x)$ by the filters $f_n^+(x)$, which, with the aid of equation (2), may be defined symbolically as $F_n^+(k) = e^{-jn\theta}$. The filters can be defined recursively by:

$$\begin{array}{rcll} F_0^+(k) & = & 1 & \\ F_{n+1}^+(k) & = & H_{k_0}^+(k) F_n^+(k), & n \geq 0 \\ F_{n-1}^+(k) & = & \overline{H_{k_0}^+(k)} F_n^+(k), & n \leq 0, \end{array} \quad (13)$$

where the filter $H_{k_0}^+(k) = e^{-j\theta}$, which depends on $k_0 = \omega/c$ is:

$$H_{k_0}^+(k) = \sqrt{1-(k/k_0)^2} - jk/k_0. \quad (14)$$

The local harmonic decomposition can therefore be computed by:

$$\begin{array}{rcll} w_0^+(x) & = & w_+(x) & \\ w_{n+1}^+(x) & = & h_{k_0}^+(x) * w_n^+(x), & n \geq 0 \\ w_{n-1}^+(x) & = & h_{k_0}^+(-x) * w_n^+(x), & n \leq 0, \end{array} \quad (15)$$

When the upgoing part is considered, $w(x) = w_+(x) + w_-(x)$, the local harmonic decomposition becomes:

$$\hat{w}_n(x) = \int_{-\infty}^{\infty} f_n^+(u) w_+(x-u) du + \int_{-\infty}^{\infty} f_n^-(u) w_-(x-u) du, \quad (16)$$

where the $f_n^-(x)$ are defined as before except that the sign of the square root, which is the sign of $\cos\theta$, is negative instead of positive.

4. COMPUTATION OF THE LOCAL HARMONIC DECOMPOSITION

The above derivation shows that the local harmonic decomposition of an angle gather can be computed directly by recursive filtering of the data by the filter $H_{k_0}^+(k)$ given by equation (14).

4.1 Theoretical impulse response

It is interesting to derive the theoretical impulse response $h_{k_0}^+(x)$ of the filter $H_{k_0}^+(k)$. Setting $k_0 = 1$ and working with Laplace transform (setting $p = jk$), we obtain from equation (14):

$$H_1^+(p) = \sqrt{1+p^2} - p. \quad (17)$$

Starting with the Laplace transform of the Bessel function of order 0, $J_0(x)$:

$$u(x)J_0(x) \rightarrow \frac{1}{\sqrt{1+p^2}}, \quad (18)$$

where $u(x)$ is Heaviside function, we take the derivative in x of the l.h.s. while multiplying the r.h.s.by p:

$$\delta(x) + u(x)J_0'(x) = \delta(x) - u(x)J_1(x) \rightarrow \frac{p}{\sqrt{1+p^2}}, \quad (19)$$

$\delta(x)$ being Dirac's function. The Laplace transform of $\delta(x)$ being 1, we obtain

$$-u(x)J_1(x) \rightarrow -1 + \frac{p}{\sqrt{1+p^2}}. \quad (20)$$

We divide the l.h.s. by $-x$ while integrating the r.h.s. in p:

$$u(x)\frac{J_1(x)}{x} \rightarrow \sqrt{1+p^2} - p + C. \quad (21)$$

The integration constant C is zero because $\sqrt{1+p^2} - p = \frac{1}{\sqrt{1+p^2}+p}$ goes to zero when p goes to infinity, and $u(x)\frac{J_1(x)}{x}$ is a step function at $x = 0$ with no terms in $\delta(x)$. We can therefore write:

$$h_{k_0}^+(x) = u(x)\frac{J_1(k_0 x)}{x}. \quad (22)$$

Figure 1 and 2 shows the spectrum $H_{k_0}^+(k)$ and the impulse response $h_{k_0}^+(x)$.

4.2 Filter synthesis

The next step is to synthesize the filters $H_{k_0}^+(k)$ for every useful k_0, that is for:

$$\frac{\omega_{min}}{c_{max}} \leq k_0 \leq \frac{\omega_{max}}{c_{min}}. \quad (23)$$

The synthesized filters have to be accurate for $|k| \leq k_0 \sin\theta_{max}$, where θ_{max} is the maximum propagation angle and $|k| \leq \alpha k_{Nyq}$, where α is the fraction of the spatial Nyquist bandwidth to be preserved, k_{Nyq} being the spatial Nyquist wavenumber. They also have to be less than one in modulus everywhere for stability reasons. This last condition makes IIR filters a good choice.

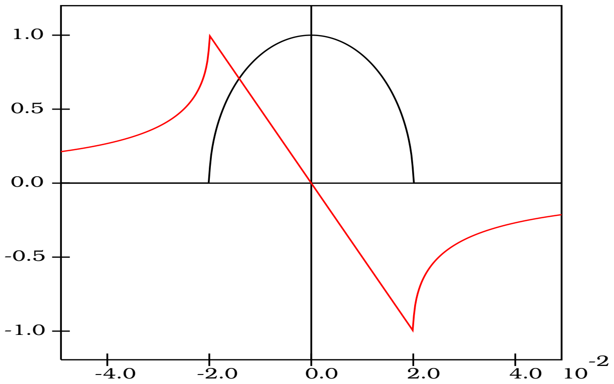

Figure 1: Spectrum $H^+_{k_0}(k)$: real and imaginary parts

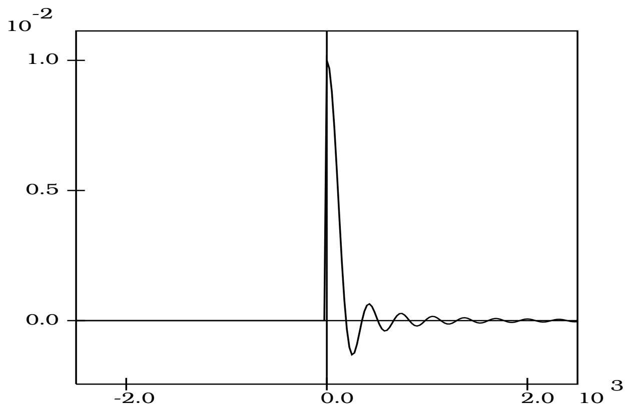

Figure 2: Impulse response $h^+_{k_0}(x)$

4.3 Laterally variable velocity

Once the filters $H^+_{k_0}(k)$ have been tabulated for the useful k_0 values, we use a so-called "explicit" filtering structure, in which the local harmonic decomposition is computed through recursions of the type $w^+_{n+1}(x) = h^+_{k_0(x)}(x) * w^+_n(x)$, with $w^+_0(x) = w_+(x)$, where the filter coefficients are space-varying by being taken from the tabulated value for $\frac{\omega}{c(x)}$, taking into account the local velocity $c(x)$ which can be laterally varying.

5. HARMONIC IMAGING

The proposed local harmonic decomposition can be used to compute various angle gathers during a shot-record wave-equation migration. For a given shot s, frequency f and depth z the incident wavefield is $i(x)$ and the scattered wavefield is $s(x)$. We consider the local angle transform $I(x,\theta)$ and $S(x,\theta)$ of $i(x)$ and $s(x)$. Figure 3 shows the incident and scattered wavefields (which are plane waves, not rays). The angle-dependent reflectivity $R(x,\theta)$ relates $I(x,\theta)$ to $S(x,\theta)$

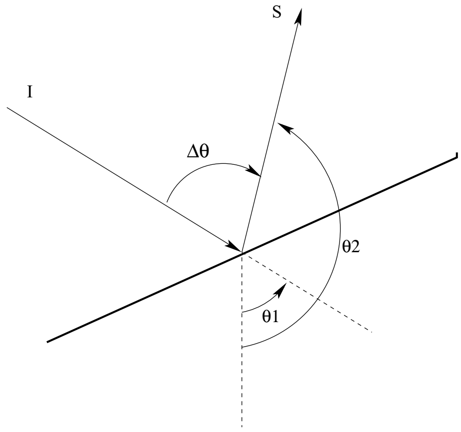

Figure 3: Angle dependent reflectivity: $S(\theta_2) = R(\Delta\theta)I(\theta_1)$

by:

$$\begin{aligned} S(x,\theta_2) &= \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} R(x,\pi-\theta_1+\theta_2)I(x,\theta_1)d\theta_1 \\ &= \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} G(x,\theta_2-\theta_1)I(x,\theta_1)d\theta_1, \end{aligned} \tag{24}$$

by defining $G(x,\Delta\theta) = R(x,\pi+\Delta\theta)$. Equation (24) is, for each x, an angle convolution. Taking the Fourier transform in angle therefore simplifies it to:

$$\hat{s}_n(x) = \hat{g}_n(x)\hat{i}_n(x). \tag{25}$$

A kinematic angle gather can be computed by:

$$\hat{g}_n(x) = \overline{\hat{i}_n(x)}\hat{s}_n(x). \tag{26}$$

Therefore the local harmonic decomposition of the reflectivity can be computed by the relation:

$$\hat{r}_n(x) = (-1)^n\overline{\hat{i}_n(x)}\hat{s}_n(x), \tag{27}$$

where $\hat{i}_n(x)$ and $\hat{s}_n(x)$ are the local harmonic decompositions of $i(x)$ and $s(x)$. After computing a certain number of harmonics and stacking them in frequency and shot, the angle gather $R(x,\theta)$ can be computed by

$$R(x,\theta) = \sum_{n=-N}^{N} \hat{r}_n(x)e^{jn\theta}. \tag{28}$$

Equation (27) can be considered as an harmonic imaging generalizing the conventional scalar imaging:

$$r(x) = \overline{i(x)}s(x). \tag{29}$$

The angle gather $R(x,\theta)$ obtained by this method has the property that its average in θ is the conventional imaging $\overline{i(x)}s(x)$. Indeed, the average is the harmonic zero and $\hat{r}_0(x) = \overline{\hat{i}_0(x)}\hat{s}_0(x) = \overline{i(x)}s(x)$.

5.1 Other imaging condition

We can use other imaging conditions than equation (27) to compute various other angle gathers at no extra computational cost:

$$\begin{aligned} \hat{r}_n(x) &= \overline{\hat{i}_n(x)}s(x) \\ \hat{r}_n(x) &= \overline{i(x)}\hat{s}_n(x) \\ \hat{r}_n(x) &= (-1)^n\overline{\hat{i}_n(x)}\hat{s}_{-n}(x), \end{aligned} \tag{30}$$

will compute a gather indexed in incident angle, scattered angle and twice the local dip respectively. The third imaging condition comes from the fact that the local dip of the reflector can be seen from Figure 3 as being $\theta_{dip} = \frac{\pi}{2} - \frac{\theta_1+\theta_2}{2}$.

6. SYNTHETIC DATA EXAMPLE

Figure 4 shows the migration of the 2D SEG/EAGE Salt model and the angle gather corresponding to the last displayed location. The gather clearly displays the sub-salt reflections.
In order to work with a dataset with greater angle coverage, an other dataset was used: it is a 2D dataset with a distance between shots of 100m and consisting in a set of linear reflectors of various dips. It was generated with the Picrocol velocity model, with a lateral velocity gradient added. Figure 5 shows the velocity model, Figure 6 the migration with the exact velocity model. Figure 7 shows the angle gather for the location 5km with the correct velocity. The velocity can be checked to be correct for all events. Figure 8 shows the same angle gather when the migration is done with a velocity 4% too slow, which is confirmed by the curvature of the events. All events on the angle gathers are centered on 0 degree despite their various dips and don't have artefacts.

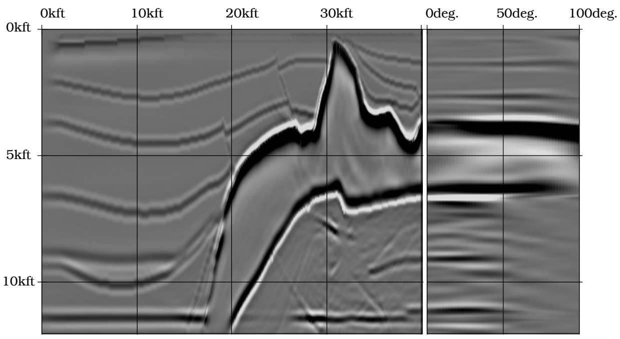

Figure 4: 2D SEG/EAGE Salt model and angle gather

7. CONCLUSION

It has be shown that local angle information can be extracted from a wavefield by direct computing of a local harmonic decomposition. In the case of shot-record migration, using an harmonic imaging, similar to conventional imaging but applied to the local harmonic decomposition of the incident and scattered fields, allows the efficient computation of artefact-free angle gathers, as proven by synthetic data tests.

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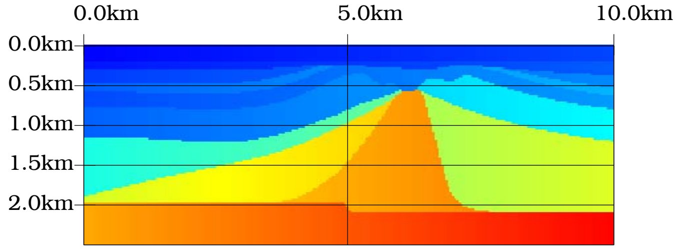

Figure 5: Velocity model

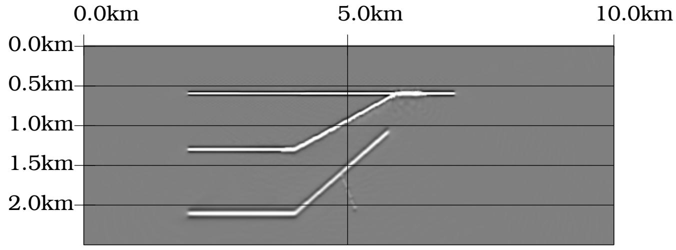

Figure 6: Migration

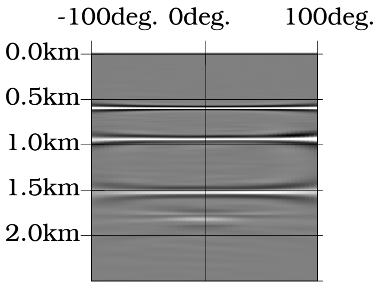

Figure 7: Angle gather (100% velocity)

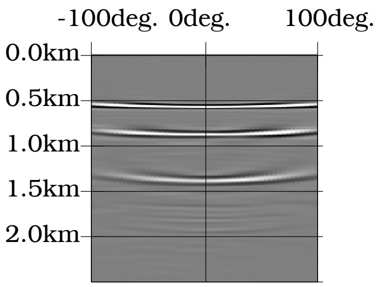

Figure 8: Angle gather (96% velocity)