THREE-DIMENSIONAL AUTOREGRESSIVE PARAMETER ESTIMATION FROM NOISY DATA

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ABSTRACT

This paper deals with the three-dimensional Autoregressive (3-D AR) model parameter estimation from noisy data. We develop an algorithm to estimate the transversal AR parameters corresponding to the Quarter-Space (QS) region of support without a priori knowledge of additive noise power. The transversal parameters and the noise variance are both obtained as a solution of a quadratic eigenvalue problem. The performance of the proposed algorithm is evaluated by numerical examples.

1. INTRODUCTION

Recently, three-dimensional autoregressive (3-D AR) models have been applied in 3-D data processing [1]-[4]. For example, the 3-D AR model is used for modelling, analysis, synthesis of a set of homogenous 3-D textures [1][2]. It is well known that any model-based approach for data representation and processing involves two important stages, viz., i) the determination of the model order, and ii) the estimation of the transversal model's parameters. This problem has been extensively treated for the 1-D AR model but , in the 3-D case, a few papers available and they often deal with one or the other estimation problem [5]-[7]. For example, in [5] the Yule-Walker (YW) equations of noiseless 3-D AR model are solved efficiently via a recursive algorithm, which takes advantage of the cubic Toeplitz structure of the autocorrelation matrix. Dealing with the model order, the authors in [7] proposed an algebraic method for Gaussian 3-D quarter space AR model and presented a comparative study with the informational criterion methods. Good results are obtained in the absence of measurements noise. However, in the presence of noise; these methods provide seriously degraded results. So in this paper we present an approach which take into account the noisy data case to reduce the bias of the estimated parameters.

The aim of this paper is to develop a numerical algorithm to estimate the 3-D AR transversal parameters, and the noise variance from a finite number of noisy measurements. The AR model order is assumed to be known a priori. The 3-D AR model considered here is assumed to be Gaussian, stable and spatial shift invariant. The proposed algorithm is inspired by the algorithm proposed for one-dimensional AR process in [8]. The AR parameters are generally obtained via the resolution of the YW equations. In the 1-D case, the YW equations are easily represented in matrix format. However, the 3-D AR processes leads to a complicate set of YW equations caused by the region of support. Thus, we first address in section 2 the problem of expressing the autocorrelation function (ACF) matrix of 3-D QS AR random fields in terms of the model parameters. In section 3, we introduce the noisy 3-D AR model and develop our method to estimate the AR parameters and the noise variance. In Section 5, we present numerical examples.

2. NOISELESS 3-D AR MODEL

Let $\{x(m,n,t)\}$ be a second-order zero-mean stationary 3-D ergodic process satisfying a causal QS 3-D AR model of order $p=(p_1,p_2,p_3)$

$$\sum_{k_1=0}^{p_1}\sum_{k_2=0}^{p_2}\sum_{k_3=0}^{p_3} a_{k_1,k_2,k_3}\, x(m-k_1,n-k_2,t-k_3)=e(m,n,t)\,, \qquad (1)$$

where $\{a_{k_1,k_2,k_3}\}$ are the transversal AR coefficients such that $a_{0,0,0}=1$, the input generator process $\{e(m,n,t)\}$ is assumed to be zero-mean, white noise, 3-D Gaussian process with variance σ_e^2.

We recall that the autocorrelation function (ACF) of the 3-D homogenous process is defined as follows:

$$r_y(h_1,h_2,h_3)=E[\,y(m,n,t)y(m-h_1,n-h_2,t-h_3)\,]\,, \qquad (2)$$

where $E[.]$ denotes the mathematical expectation operator. According to (1), the ACF satisfies the following autoregressive 3-D Yule-Walker (YW) equations

$$\sum_{k_1=0}^{p_1}\sum_{k_2=0}^{p_2}\sum_{k_3=0}^{p_3} a_{k_1,k_2,k_3}\, r_x(h_1-k_1,h_2-k_2,h_3-k_3)= \sigma_e^2\,\delta(h_1,h_2,h_3) \qquad (3)$$

where $\delta(h_1,h_2,h_3)$ is the 3-D Kronecker delta function.

To represent the 3-D YW equations in the matrix format, we propose the following construction :

For a fixed $h=(h_1,h_2,h_3)$, we concatenate the 3-D AR parameter coefficients $\{a_{k_1,k_2,k_3}\}$ $k_i=0,\cdots,p_i$; $i=1,2,3$, and the corresponding ACF samples into two $(p_1+1)(p_2+1)(p_3+1)\times 1$ vectors $\boldsymbol{\theta}$ and $\boldsymbol{r}_{h_1}^{h_2,h_3}$ as follows:

$$\boldsymbol{\theta}=[\,\underline{\underline{\boldsymbol{\theta}}}_0;\underline{\underline{\boldsymbol{\theta}}}_1;\cdots;\underline{\underline{\boldsymbol{\theta}}}_{p_1}\,]^T\ ;\ \boldsymbol{r}_{h_1}^{h_2,h_3}=[\,\underline{\underline{\boldsymbol{r}}}_0;\underline{\underline{\boldsymbol{r}}}_1;\cdots;\underline{\underline{\boldsymbol{r}}}_{p1}\,]^T \qquad (4)$$

where, T is the transpose operator, and

$$\underline{\underline{\theta}}_{k_1} = [\ \underline{\theta}_{k_1,0}; \underline{\theta}_{k_1,1}; \cdots ; \underline{\theta}_{k_1,p_2}\ ]^T ;$$

$$\underline{\underline{r}}_{k_1} = [\ \underline{r}_{k_1,0}; \underline{r}_{k_1,1}, \ldots, \underline{r}_{k_1,p_2}\ ]^T \quad (5)$$

and

$$\underline{\theta}_{k_1,k_2} = [\ a_{k_1,k_2,0}; a_{k_1,k_2,1}; \cdots ; a_{k_1,k_2,p_3}\ ]^T ,$$

$$\underline{r}_{k_1,k_2} = [\ r_{k_1,k_2,0}, r_{k_1,k_2,0}, \ldots, r_{k_1,k_2,p_3}\ ]^T \quad (6)$$

with $r_{k_1,k_2,k_3} = r_x(h_1 - k_1, h_2 - k_2, h_3 - k_3)$,

Thus, equation (3) can be rewritten:

$$\theta^T r_{h_1}^{h_2,h_3} = \sigma_e^2 \delta(h_1, h_2, h_3) \quad (7)$$

which leads to the 3-D YW equations in a matrix form:

$$\boldsymbol{R}_x \boldsymbol{\theta} = \sigma_e^2 \underline{\boldsymbol{h}} \quad (8)$$

where $\underline{h}$ has dimensions $(p_1+1)(p_2+1)(p_3+1)\times 1$ such as $\underline{h} = [1, 0, \cdots, 0]^T$. The matrix $\boldsymbol{R}_x$ is a block-block Toeplitz matrix (cubic Toeplitz). It is structured as block Toeplitz matrix of size $(p_1+1)\times(p_1+1)$

$$\boldsymbol{R}_x = \begin{bmatrix} \boldsymbol{R}_0 & \boldsymbol{R}_{-1} & \cdots & \boldsymbol{R}_{-p_1} \\ \boldsymbol{R}_1 & \boldsymbol{R}_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \boldsymbol{R}_{-1} \\ \boldsymbol{R}_{p_1} & \cdots & \boldsymbol{R}_1 & \boldsymbol{R}_0 \end{bmatrix} \quad (9)$$

where each block entry $\boldsymbol{R}_k$ is a $(p_2+1)\times(p_2+1)$ Toeplitz-block-Toeplitz matrix of the form

$$\boldsymbol{R}_k = \begin{bmatrix} \boldsymbol{R}_k^0 & \boldsymbol{R}_k^{-1} & \cdots & \boldsymbol{R}_k^{-p_2} \\ \boldsymbol{R}_k^1 & \boldsymbol{R}_k^0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \boldsymbol{R}_k^{-1} \\ \boldsymbol{R}_k^{p_2} & \cdots & \boldsymbol{R}_k^1 & \boldsymbol{R}_k^0 \end{bmatrix} \quad (10)$$

each submatrix $\boldsymbol{R}_k^l$ is a $(p_3+1)\times(p_3+1)$ Toeplitz one as

$$\boldsymbol{R}_k^l = \begin{bmatrix} r_x(k,l,0) & r_x(k,l,-1) & \cdots & r_x(k,l,-p_3) \\ r_x(k,l,1) & r_x(k,l,0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & r_x(k,l,-1) \\ r_x(k,l,p_3) & \cdots & r_x(k,l,1) & r_x(k,l,0) \end{bmatrix} \quad (11)$$

Since the input variance σ_e^2 is unknown, the system in (8) cannot be used directly to estimate the AR transversal parameters. Thus, a constrained system is needed. Indeed, since the first element of θ is assumed to be one i.e. $a_{0,0,0} = 1$, we eliminate the first equation in (8) and move the first column of the remaining matrix to the right-hand side of the system to obtain the following modified system:

$$\tilde{\boldsymbol{R}}_x \tilde{\theta} = -\tilde{r} \quad (12)$$

Both the two vectors $\tilde{\theta}$ and $\tilde{r}$ have dimensions of $((p_1+1)(p_2+1)(p_3+1)-1)\times 1$ where $\tilde{\theta}$ contains the unknown transversal parameters and $\tilde{r}$ contains the corresponding ACF sample. Therefore, the AR parameters can be determined by solving (12). The input variance σ_e^2 can be computed using (3) for $(h_1, h_2, h_3) = (0,0,0)$

3. NOISY 3-D AR MODEL

Assuming that, the process $x(m,n,t)$ is corrupted by zero mean white noise $\{v(m,n,t)\}$, to yield the 3-D observable noisy observation

$$y(m,n,t) = x(m,n,t) + v(m,n,t) \quad (13)$$

where the additive noise $\{v(m,n,t)\}$ and the input generator process $\{e(m,n,t)\}$ are assumed mutually independent. The signal-to-noise ratio (SNR) of the system is defined by

$$SNR = 10 \log_{10} \frac{\sigma_x^2}{\sigma_v^2} \ dB \quad (14)$$

where σ_x^2 and σ_v^2 are respectively the variance of the signal and the variance of the additive noise.

In the presence of noisy data, the AR parameters estimated from the YW equation described in the previous section will be biased since

$$r_y(h_1, h_2, h_3) = r_x(h_1, h_2, h_3) + \sigma_v^2 \delta(h_1, h_2, h_3) \quad (15)$$

However, this method is able to estimate the AR model parameters for large SNR. When the SNR is small, the estimation results are influenced by the large Gaussian noise.

More explicitly, the 3-D YW equations which relate the AR parameters with the ACF of the noisy process have the following form:

$$\sum_{k_1=0}^{p_1} \sum_{k_2=0}^{p_2} \sum_{k_3=0}^{p_3} a_{k_1,k_2,k_3} r_y(h_1 - k_1, h_2 - k_2, h_3 - k_3) =$$

$$\begin{cases} \sigma_e^2 + \sigma_v^2 & (h_1, h_2, h_3) = (0,0,0) \\ a_{h_1,h_2,h_3} \sigma_v^2 & (h_1, h_2, h_3) \in [0,p] - \{(h_1, h_2, h_3) = (0,0,0)\} \\ 0 & elsewhere \end{cases} \quad (16)$$

where $[0,p] = [0,p_1]\times[0,p_2]\times[0,p_3]$.

Considering the coupled equation (15) and (8), it is clear that the 3-D YW equations can be represented in the matrix format as follows:

$$[\ \boldsymbol{R}_y - \sigma_v^2 \boldsymbol{I}\ ]\theta = \sigma_e^2 \underline{\boldsymbol{h}} \quad (17)$$

where $\boldsymbol{R}_y$ is block-block Toeplitz autocorrelation matrix of the noisy process structured as $\boldsymbol{R}_x$, and $\boldsymbol{I}$ is the identity matrix of dimensions :

$(p_1+1)(p_2+1)(p_3+1)\times(p_1+1)(p_2+1)(p_3+1)$.

The system of equations (17) cannot be solved directly since it contains $(p_1+1)(p_2+1)(p_3+1)$ nonlinear equations in the AR parameters and the noise variance. However, by choosing $p_i \le h_i \le 2p_i$, such as $(h_1, h_2, h_3) \ne (p_1, p_2, p_3)$, for $i = 1,2,3$, in (16) the resulting equations are linear and will not involve $r_y(0,0,0)$

$$\sum_{k_1=0}^{p_1} \sum_{k_2=0}^{p_2} \sum_{k_3=0}^{p_3} a_{k_1,k_2,k_3} r_x(h_1 - k_1, h_2 - k_2, h_3 - k_3) = 0 \quad (18)$$

This system of equations is the 3-D version of the extended YW equations (EYW). It makes it possible to estimate the AR parameters using a block-block Toepltiz system obtained by an extend technique as the one used in the noiseless case (12). Unfortunately, the EYW method provides poor estimation accuracy due to the use of high lags autocorrelation estimates, which tends to be inaccurate and have a larger estimation variance [8]-[10]. To compensate for errors in the estimated ACF lags, the overdetermined extended YW equations (OEYW) have been used in 1-D case [8] [11]. Their extension in the 3-D case can be defined in the matrix form as follows:

$$\widetilde{\boldsymbol{R}}_y \boldsymbol{\theta} = \boldsymbol{0} \tag{19}$$

where

$$\widetilde{\boldsymbol{R}}_y = \begin{bmatrix} \widetilde{\boldsymbol{R}}_{p_1+1} & \widetilde{\boldsymbol{R}}_{p_1} & \cdots & \widetilde{\boldsymbol{R}}_1 \\ \widetilde{\boldsymbol{R}}_{p_1+2} & \widetilde{\boldsymbol{R}}_{p_1+1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \widetilde{\boldsymbol{R}}_{p_1} \\ \widetilde{\boldsymbol{R}}_{p_1+q_1} & \cdots & \widetilde{\boldsymbol{R}}_{p_1+2} & \widetilde{\boldsymbol{R}}_{p_1+1} \end{bmatrix} \tag{20}$$

$$\widetilde{\boldsymbol{R}}_k = \begin{bmatrix} \widetilde{\boldsymbol{R}}_k^{p_2+1} & \widetilde{\boldsymbol{R}}_k^{p_2} & \cdots & \widetilde{\boldsymbol{R}}_k^{1} \\ \widetilde{\boldsymbol{R}}_k^{p_2+2} & \widetilde{\boldsymbol{R}}_k^{p_2+1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \widetilde{\boldsymbol{R}}_k^{p_2} \\ \widetilde{\boldsymbol{R}}_k^{p_2+q_2} & \cdots & \widetilde{\boldsymbol{R}}_k^{p_2+2} & \widetilde{\boldsymbol{R}}_k^{p_2+1} \end{bmatrix} \tag{21}$$

and

$$\widetilde{\boldsymbol{R}}_k^{l} = \begin{bmatrix} r_y(k,l,p_3+1) & r_y(k,l,p_3) & \cdots & r_y(k,l,1) \\ r_y(k,l,p_3+2) & r_y(k,l,p_3+1) & & \vdots \\ \vdots & & \ddots & r_y(k,l,p_3) \\ r_y(k,l,p_3+q_3) & \cdots & r_y(k,l,p_3+2) & r_y(k,l,p_3+1) \end{bmatrix} \tag{22}$$

where $q_i > p_i$ for $i = 1,2,3$. The linear system of equations in (18) also evades $r_y(0,0,0)$ and uses more equations than unknowns parameters. The AR parameters can be obtained solving (19). A problem with the EYW and OEYW methods is that they use only a set of linear equations corresponding to large-lag autocorrelation sample. In fact, the ACF of the noisy process always includes significant errors at all lags for $h_1 > p_1$, or $h_2 > p_2$, or $h_3 > p_3$ mainly resulting from the additive noise and the use of a small number of data. In addition, these methods don't take into account the first $(p_1+1)(p_2+1)(p_3+1)$ nonlinear equations based on relatively lower ACF lag estimates. To alleviate this problem we propose in the following parts a method based on joint linear and nonlinear equations.

3.1 The proposed estimation method

According to the previous section, the AR parameters and the variance noise satisfies two systems of equations. The first one given in (17) is nonlinear; the second described in (19) is linear. The nonlinear system can be rewritten as follows:

$$[\ \overline{\boldsymbol{R}}_y - \sigma_v^2 \boldsymbol{B}_1\ ]\boldsymbol{\theta} = \boldsymbol{0} \tag{23}$$

where $\overline{\boldsymbol{R}}_y$ is obtained by removing the first line in the matrix $\boldsymbol{R}_y$. The matrix $\boldsymbol{B}_1$ is such as:

$$\boldsymbol{B}_1 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & \vdots \\ \vdots & \vdots & \cdots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{24}$$

The coupled equations (19) and (23) can be combined and written as a generalized eigenvalue problem:

$$[\ \boldsymbol{R} - \sigma_v^2 \boldsymbol{B}\ ]\theta = 0 \tag{25}$$

where the two matrices $\boldsymbol{R}$, and $\boldsymbol{B}$ are defined by:

$$\boldsymbol{R} = \begin{bmatrix} \overline{\boldsymbol{R}}_y \\ \widetilde{\boldsymbol{R}}_y \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_1 \\ \boldsymbol{0} \end{bmatrix} \tag{26}$$

Consequently, the AR parameters and the noise variance satisfy the following quadratic eigenvalue problem:

$$[\ \boldsymbol{R} - \lambda \boldsymbol{B}\ ]^T [\ \boldsymbol{R} - \lambda \boldsymbol{B}\ ] v = \boldsymbol{0} \tag{27}$$

which can be written as follows:

$$[\ \boldsymbol{H}_0 + \lambda \boldsymbol{H}_1 + \lambda^2 \boldsymbol{H}_2\ ] v = \boldsymbol{0} \tag{28}$$

where $\boldsymbol{H}_0$, $\boldsymbol{H}_1$, and $\boldsymbol{H}_3$ are three symmetric matrices given by $\boldsymbol{H}_0 = \boldsymbol{R}^T\boldsymbol{R}$, $\boldsymbol{H}_1 = -(\boldsymbol{R}^T\boldsymbol{B} + \boldsymbol{B}^T\boldsymbol{R})$, and $\boldsymbol{H}_2 = \boldsymbol{B}^T\boldsymbol{B}$.

The classical approach to solve the quadratic eigenvalue problem is to turn them into linear eigenvalue problems by introducing a new vector $w = \lambda v$ [12]. In our case this lead to the double size generalized linear eigenvalue problem:

$$[\boldsymbol{H} - \lambda \boldsymbol{G}]\begin{bmatrix} v \\ w \end{bmatrix} = \boldsymbol{0} \tag{29}$$

where $\boldsymbol{H}$, and $\boldsymbol{G}$ are defined by

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_0 & 0 \\ 0 & \boldsymbol{I} \end{bmatrix},\ \text{and}\ \boldsymbol{G} = \begin{bmatrix} -\boldsymbol{H}_1 & -\boldsymbol{H}_2 \\ \boldsymbol{I} & 0 \end{bmatrix} \tag{30}$$

where $\boldsymbol{I}$ is the identity matrix of dimensions $n \times n$ where $n = ((p_1+1)(p_2+1)(p_3+1)-1)$. This approach allows determining eigenvalues and eigenvectors numerically, since for the generalized linear eigenvalue problem like (29) the mathematical methods are well established [13]. The solutions of (29) are a set of complexes numbers and their conjugates. If $(\lambda, [w\ \ v]^T)$ is one of these solutions, then v is an eigenvector of (28) associated to the eigenvalue λ. Furthermore, the eigenvalues solving (29) are the estimates of the additive noise variance σ_v^2. Only one eigenvector solving (28) corresponds to the exact transversal AR coefficients. For the noiseless case, there is one solution to (19) and (23) associated to the eigenvalue $\lambda = 0$. Then, for the noisy case, the correct solution i.e., the transversal AR coefficients, is the eigenvector corresponding to the only real eigenvalue obtained as a solution for (28). This single real eigenvalue is the estimate of the variance noise. Finally, the variance of the input generator process can be estimated using (16) for $(h_1,h_2,h_3) = (0,0,0)$.

The proposed method can be summarized as follows:

1. Compute the ACF estimate $\hat{r}_y(.,.,.)$.
2. Form the matrices $\boldsymbol{R}$, and $\boldsymbol{B}$ defined in (26).
3. Construct the matrices $\boldsymbol{H}$, and $\boldsymbol{G}$ defined in (30).
4. Solve the generalized eigenvalue problem $[\boldsymbol{H} - \lambda \boldsymbol{G}]\underline{b} = \boldsymbol{0}$ in λ and $\underline{b}$.
5. The estimate of the noise variance is the real generalized eigenvalue λ solving 4.
6. The transversal parameters are the first n elements of the eigenvector b_0 associated to λ obtained in 4.

4. EXPRIMENTS RESULTS

In this section, we present a numerical example to provide a verification of the theoretical results. We generate a noisy observations $y(m,n,t) = x(m,n,t) + v(m,n,t)$, of size $M \times N \times T = 128 \times 128 \times 128$, where $x(m,n,t)$ is a QS 3-D AR $(1,1,1)$ process given by

$$x(m,n,t) = 0.75x(m,n-1,t) + 0.82x(m-1,n,t) - 0.615x(m-1,n-1,t) \\ + 0.9x(m,n,t-1) - 0.675x(m,n-1,t-1) - 0.738x(m-1,n,t-1) \\ + 0.5535x(m-1,n-1,t-1) + e(m,n,t)$$

The additive noise $v(m,n,t)$ is zero mean white Gaussain noise with a variance $\sigma_v^2 = 36.24$ to produce signal-to-noise ratio of 10 dB $(SNR = 10\,dB)$.

To build the matrix $\tilde{\boldsymbol{R}}_y$ described in (17) we used the parameters $q_1 = p_1$, $q_2 = p_2$, and $q_3 = p_3$. The ACF sample was computed using the unbiased estimate given by

$$\hat{r}_y(h_1,h_2,h_3) = \frac{1}{(M-h_1)(N-h_2)(T-h_3)} \sum_{m=1}^{M-h_1} \sum_{n=1}^{N-h_2} \sum_{t=1}^{T-h_3} y(m,n,t) y(m+h_1, n+h_2, t+h_3)$$

The estimated AR parameters and the noise variance using the developed method, and the OEYW method are presented in the table I. We remark that, for a moderate value of SNR, the results of the proposed method outperforms those obtained with the OEYE method. However, in practice, some technical modifications have been introduced in the proposed algorithm. Indeed, due to the errors in estimating the autocorrelation block matrix, all the eigenvectors and eigenvalues solving quadratic eigenvalue problem appear in complex conjugate. Thus, the eigenvalue corresponding to the noise variance σ_v^2 is also complex. As an alternative, in this example we have selected the eigenvalue having smallest modulus as the noise variance estimate, and the transversal parameters as the real part of its corresponding eigenvector.

	True coefficients	Proposed Method	OEYW Method
$a_{0,1,0}$	-0.750	-0.7136	-0.8156
$a_{1,0,0}$	-0.820	-0.8223	-0.8534
$a_{1,1,0}$	0.615	0.5913	0.6731
$a_{0,0,1}$	-0.900	-0.9055	-0.9338
$a_{0,1,1}$	0.675	0.6456	0.7458
$a_{1,0,1}$	0.738	0.7478	0.7787
$a_{1,1,1}$	-0.5535	-0.5375	-0.5970
σ_e^2	10	10.505	15.3677
σ_v^2	36.24	34.8530	43.6763

TABLE I: RESULTS OF ESTIMATED PARAMETERS

5. CONCLUSION

In this paper, we have addressed the problem of estimating the parameters of tree-dimensional autoregressive random fields corrupted by additive white noise. The 3-D AR model considered here is assumed to be Gaussian, stable and spatial shift invariant with quarter-space region of support. We first addressed the problem of expressing the 3-D Yule Walker equations of noiseless 3-D AR field in matrix form. This expression was then employed in the noisy case to show that the model parameters are a solution of a quadratic eigenvalue problem. The performance of the proposed method is evaluated using numerical examples.

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