PILOT-SYMBOL-AIDED ITERATIVE CHANNEL ESTIMATION FOR OFDM-BASED SYSTEMS

Berna Özbek [*†], *Didier Le Ruyet* [*] *and Cristiano Panazio* [*]

[*] Electronics and Communications Laboratory, Conservatoire National des Arts et Métiers (CNAM)
292 rue Saint Martin, 75141 Paris, France
phone: + (33) 1 40272081, fax: + (33) 1 40272779, email: {ozbek,leruyet,cpanazio}@cnam.fr
[†] Electrical and Electronics Engineering Department, Izmir Institute of Technology
Urla, 35430, Izmir, Turkey
phone: + (90) 232 7506582, fax: + (90) 232 7506505, email: bernaozbek@iyte.edu.tr

ABSTRACT

In this paper, we propose a pilot-symbol-aided iterative channel estimation for coded OFDM-based systems. We use the symbol APP provided by the channel decoder to form groups of virtual pilots. According to their reliabilities, we combine these groups to improve the channel estimation. We also compare the proposed algorithm with the EM algorithm.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) based systems are strong candidates for an air interface of future fourth-generation mobile wireless systems which provide high data rates and high mobility. In order to achieve the potential advantages of OFDM-based systems, the channel coefficients should be estimated with minimum error. The channel estimation can be improved using more pilot symbols [1]. However, it causes data rate reduction or bandwidth expansion. Therefore, spectrally efficient channel estimation techniques should be considered. In this case, the iterative techniques provide an improvement on the channel estimator performance without requiring additional pilots.

For single carrier systems in flat fading channels, iterative channel estimation methods based on adaptive filtering algorithms using all feedback information provided by channel decoder has been proposed in [2]. For least-squares (LS) and interpolation based channel estimation techniques, it is possible to increase the number of pilots using all the hard decision symbols obtained by the channel decoder [3]. It has been shown that depending on the quality of the feedback information, iterative channel estimation techniques do not always improve the channel estimation accuracy [2] [4].

OFDM-based systems are generally used in time varying frequency selective fading channels. In this case, the channel transfer function changes across the subcarriers and OFDM symbols. Therefore, the pilots are distributed uniformly in both time and frequency axes [5]. For pilot-symbol-aided channel estimation techniques, first the channel coefficients that belong to the pilot subcarriers are estimated using the LS. Then, these estimates are interpolated over the entire frequency-time grid using two dimensional or two one dimensional Wiener filtering [6]. These methods have been enhanced using all feedback information in [7] [8]. However, these estimation methods are performed assuming the statistical properties of the channel are perfectly known at the receiver side. Alternatively, the interpolation part can be performed using discrete Fourier transform (DFT)-based techniques which provide an efficient tradeoff between complexity and performance [9]. Moreover, the EM algorithm can be also applied to enhance the channel estimation by using all feedback information [12].

In this paper, we consider pilot-symbol-aided iterative channel estimation methods for coded OFDM-based systems using the LS technique with the DFT-based interpolation in frequency axes and the linear interpolation in time axes. While increasing the number of virtual pilots, we propose to use the reliability of feedback information instead of using all hard decision symbols. Thus, we avoid the channel estimation errors caused by unreliable symbols as derived in [4] by reducing the impact of unreliable data in the channel estimation. Since the mean square error (MSE) of the LS channel estimator is inversely proportional to the number of pilots, we observe an improvement on the channel estimator by adding the reliable virtual pilots for OFDM-based systems.

This paper is organized as follows. First, we describe the coded OFDM system model over time varying frequency selective channels in section 2. Then, we give the proposed iterative channel estimation method in section 3. In section 4, we extend the application of the proposed method to orthogonal space-time block coded (STBC) OFDM systems. Finally, we give performance results.

2. SYSTEM MODEL

In this section, we will examine the transmitter and receiver structure of coded OFDM systems including the pilot distribution. Then, we will describe the pilot-symbol-aided channel estimation techniques with the DFT-based interpolation in frequency axes and the linear interpolation in time axes.

2.1 Transmitter Structure

As illustrated in Figure 1, the vector of data bits $\mathbf{b}$ is first encoded with an error correcting code such as a convolutional or a turbo code and interleaved to obtain the vector of coded data bits. This vector is mapped accordingly to the constellation and the vector of data symbols is obtained. For pilot-symbol-aided channel estimation, N_f and N_t pilot symbols are inserted periodically with the distance D_f and D_t in frequency and time grid respectively, as shown in Figure 2.

The serial symbol vector $\mathbf{s}$ is converted to the transmitted vector $\mathbf{S}$ with length $K \times N$ by a serial-to-parallel converter where K is the number of total subcarriers in one OFDM symbol and N is the total number of OFDM

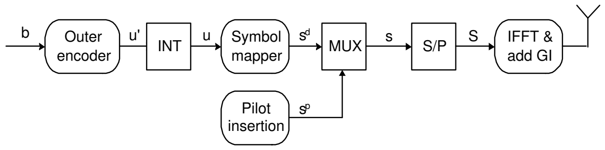

Figure 1: The transmitter structure of coded OFDM system

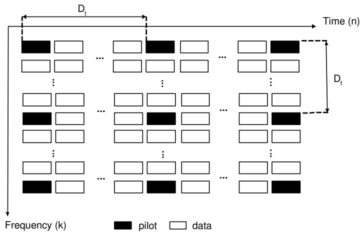

Figure 2: The pilot arrangement for OFDM systems in the frequency-time grid

symbols in the frame. Each OFDM symbol vector in the frame $\mathbf{S}(n) = [\ s_1(n) \quad s_2(n) \quad \ldots \quad s_K(n)\ ]$ is modulated onto K subcarriers by an inverse fast Fourier transform (IFFT) and the guard interval (GI) is added. Then, the OFDM frame is transmitted through the time varying frequency selective channels. This channel is described using baseband equivalent impulse response as $\mathbf{h}(n) = [\ h_1(n) \quad h_2(n) \quad \ldots \quad h_{L_f}(n)\ ]^T$ where L_f is the length of channel. We assume that the channel is constant over one OFDM symbol and the channel length is equal or smaller than the length of the GI in order to avoid intercarrier and intersymbol interference at the receiver.

In the pilot arrangement, N_f can be chosen as greater or equal to the channel length, $N_f \geq L_f$, and N_t can be selected by satisfying the channel coherence time as $D_t \leq \frac{1}{2 f_d T_{\text{OFDM-GI}}}$ where f_d is the maximum Doppler frequency and $T_{\text{OFDM-GI}}$ is the duration of one OFDM symbols including the GI.

2.2 Receiver Structure

As illustrated in Figure 3, after removing the GI and applying FFT, the received signal is obtained as

$$r_k(n) = H_k(n) s_k(n) + n_k(n) \quad k = 1,2,...,K \quad n = 1,2,...,N \tag{1}$$

where $H_k(n)$ is the complex channel coefficient in frequency domain for the kth subcarrier and the nth OFDM symbol, $n_k(n)$ is the sample of the FFT transformed additive white Gaussian noise with variance $\ ^2_N$ per real dimension and $s_k(n)$ can be a data symbol or a pilot symbol according to its place in the frame.

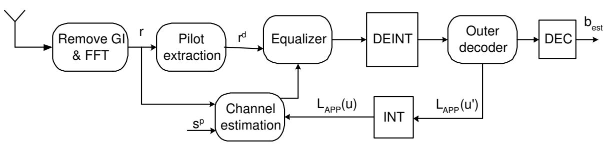

Figure 3: The receiver structure of coded OFDM system

2.3 Pilot-symbol-aided channel estimation

Using the received signals at the position of pilots, LS channel estimate can be performed in the frequency domain as

$$\hat{H}_{k'}(n') = \frac{r_{k'}(n')}{s_{k'}(n')} = H_{k'}(n') + \frac{n_{k'}(n')}{s_{k'}(n')} \tag{2}$$

where k' and n' represent the place of pilot symbols in frequency and time axes respectively and $\hat{H}_{k'}(n')$ is the estimated channel coefficient belonging to the pilot subcarriers. When the pilot and data symbols have normalized power, the estimation error is calculated as $\ ^2_{ch} = 2\ ^2_N$.

Thus, the estimated channel vector $\hat{\mathbf{H}}^p(n') = [\ \hat{H}_1(n') \quad \hat{H}_{D_f+1}(n') \quad \hat{H}_{2D_f+1}(n') \quad \ldots \quad \hat{H}_{(N_f-1)D_f+1}(n')]^T$ can be reconstructed for each pilot OFDM symbol. In order to obtain estimated channel coefficients for all subcarriers, we use the DFT-based interpolation in frequency domain. First, we transform the frequency channel estimate $\hat{\mathbf{H}}^p(n')$ into time domain as

$$\hat{\mathbf{h}}^p(n') = \mathbf{F}^{-1} \hat{\mathbf{H}}^p(n') \tag{3}$$

where $\mathbf{F}$ is the $N_f \times N_f$ DFT matrix. Then, we apply a filtering matrix to $\hat{\mathbf{h}}^p(n')$ assuming that the channel response is limited to L_f to obtain the filtered channel response $\hat{\mathbf{h}}^{(0)}(n') = \mathbf{W}\hat{\mathbf{h}}^p(n')$ where $\mathbf{W}$ is the $L_f \times N_f$ filtering matrix. Then, applying DFT, we get the initial channel estimates for the n'th OFDM symbol as

$$\hat{\mathbf{H}}^{(0)}(n') = \mathbf{V}\hat{\mathbf{h}}^{(0)}(n') \tag{4}$$

where $\mathbf{V}$ is the $K \times L_f$ matrix obtained from the first L_f columns of the $K \times K$ DFT matrix.

In order to estimate the other channel coefficients in the frame, we simply apply linear interpolation in the time domain using the estimated channel coefficients for each subcarrier. As a result, we obtain all the estimated channel coefficients in the frame to reconstruct the data symbols using simple one-tap frequency domain equalizer as

$$\tilde{s}_{k''}(n'') = \frac{r_{k''}(n'')}{\hat{H}^{(0)}_{k''}(n'')} \tag{5}$$

where k'' and n'' represent the place of data symbols in frequency and time domain respectively.

2.4 Outer Decoder

Since we use a soft-input soft-output outer decoder, we need to calculate the channel log-likelihood ratio (LLR) from $\tilde{s}_{k''}(n'')$. The channel LLR of each bit $u_{mk''}(n'')$ for $m = 1,2,...,M_c$ with M_c is the number of bits per symbol is expressed as

$$L(u_{mk''}(n'')) = \ln \frac{\displaystyle\sum_{\mathbf{u}\in\mathbb{U}_{mk'',+1}} e^{-\frac{|\tilde{s}_{k''}(n'') - \ _i(u)|^2}{\ ^2_{k''}(n'')}}}{\displaystyle\sum_{\mathbf{u}\in\mathbb{U}_{mk'',-1}} e^{-\frac{|\tilde{s}_{k''}(n'') - \ _i(u)|^2}{\ ^2_{k''}(n'')}}} \tag{6}$$

where $\ _i$ is the symbol in the constellation where $i = 1,2,...,2^{M_c}$, $\mathbb{U}_{mk'',+1}$ is the set of 2^{M_c-1} vectors $\mathbf{u}$ with

$u_{mk''} = +1$ and $^{2}_{k''}(n'') = \frac{2 \; ^{2}_{N}}{|\hat{H}^{(0)}_{k''}(n'')|^2}$ is the effective noise variance.

The channel LLR information given in (6) is first deinterleaved and then is sent to the soft-input soft-output decoder which produces a posteriori probability (APP), $L_{\text{APP}}(\mathbf{u}')$, about the transmitted bits as well as the parity bits. This soft information is fed back to the channel estimator after interleaving. These soft estimates are also used to produce hard decision symbols, $\hat{s}_{k''}(n'')$.

3. PROPOSED ITERATIVE CHANNEL ESTIMATION

In order to improve the channel estimation accuracy, it is possible to add virtual pilots using an iterative channel estimation and data detection. The reduction of channel estimation error variance can be defined by a factor of $_{v} = \frac{N_f N_t}{N_f N_t + N_v}$ where N_v is the number new virtual pilots that are added in the OFDM frame [1].

One method is to use the hard decision symbols $\hat{s}_{k''}(n'')$ as new virtual pilots, calculate the LS channel estimation as given in (2) and then apply interpolation methods such as DFT interpolation [10].

In this paper, before applying interpolation, we propose to divide these LS channel estimates into groups that contains N_f equally spaced estimates. Thus, we obtain $K/N_f - 1$ virtual pilot groups. For each group, we perform a DFT-based interpolation as given in (3) considering their corresponding delays to obtain $\hat{\mathbf{h}}^{(1)}_{g}(n')$ for $g = 2, 3, ..., K/N_f$. Instead of averaging the group estimates, we propose to combine them by taking into account their reliability which is calculated using the APP information. In order to obtain the reliability of symbol, we use the bit error probability from APP information [11] as

$$P_{g,mi} = \frac{1}{1 + e^{|L_{\text{APP}}(u_{mi})|}} \quad (7)$$

where $i = g, g + D_f, g + 2D_f, ..., (N_f - 1)D_f + g$.

From this predicted probability, we estimate the probability of correctness of the associated group and can define the reliability factor of each group as

$$P_{rel}(g) = \prod_{n=1}^{N_f} \prod_{m=1}^{M_c} (1 - P_{g,mn}) \quad (8)$$

As a result, the combining stage is performed by

$$\hat{\mathbf{h}}^{(1)}(n') = \frac{\hat{\mathbf{h}}^{(0)}(n') + \sum_{g=2}^{K/N_f} P_{rel}(g)\hat{\mathbf{h}}^{(1)}_{g}(n')}{1 + \sum_{g=2}^{K/N_f} P_{rel}(g)} \quad (9)$$

Then, we calculate $\hat{\mathbf{H}}^{(1)}(n')$ as

$$\hat{\mathbf{H}}^{(1)}(n') = \mathbf{V}\hat{\mathbf{h}}^{(1)}(n') \quad (10)$$

After applying the linear interpolation in the time domain, the symbols are estimated at the second iteration as in (5).

We compare the proposed channel estimation with the unbiased EM algorithm that uses APP information [12]. This algorithm can be applied as

$$\hat{\mathbf{h}}^{(1)}(n) = \mathbf{R}_n^{-1}\mathbf{P}_n\mathbf{r}(n) \quad (11)$$

where $\mathbf{R}_n = \mathbf{V}^H \text{diag}(\text{E}(\mathbf{X}^*(n)))\text{diag}(\text{E}(\mathbf{X}(n)))\mathbf{V}$, $\mathbf{P}_n = \mathbf{V}^H \text{diag}(\text{E}(\mathbf{X}^*(n)))$, $\text{E}(X_k(n)) = \sum_{i=1}^{2^{M_c}} {}_{i}\text{APP}_{k,n}({}_{i})$ and $\text{APP}_{k,n}({}_{i})$ which is the symbol APP probability.

Then, we obtain the frequency domain channel vector as given in (10).

4. APPLICATION TO ORTHOGONAL STBC-OFDM

The proposed channel estimation method can be easily extended to orthogonal STBC-OFDM systems with two transmit antennas [13]. For these systems, the number of pilots in time domain should be increased proportionally to the number of transmit antennas. At the receiver side, the LS channel estimation is performed as

$$\hat{\mathbf{H}}_{k'}(n') = (\mathbf{S}_{k'}(n'))^H \mathbf{r}_{k'}(n') = \mathbf{H}_{k'}(n') + (\mathbf{S}_{k'}(n'))^H \mathbf{n}_{k'}(n') \quad (12)$$

where $\mathbf{S}_{k'}(n') = \begin{bmatrix} S_{k'}(n') & S_{k'}(n'+1) \\ -S_{k'}(n'+1)^* & S_{k'}(n')^* \end{bmatrix}$ and $\mathbf{H}_{k'}(n') = [H_{1,k'}(n') \quad H_{2,k'}(n')]$ with $H_{i,k'}(n')$ that is the channel coefficient belongs to ith transmit antenna.

Then, the DFT-based interpolation can be applied for the channel coefficients of each transmit antenna individually. After obtaining the feedback information from the channel decoder, the virtual groups are reconstructed and the reliability of each group is calculated for each transmit antenna.

5. PERFORMANCE EVALUATION

In this section, we evaluate the proposed iterative channel estimation method for coded OFDM systems. The time varying frequency fading channel has 8 taps and normalized Doppler frequency is chosen as $f_D T_{\text{OFDM-GI}} = 0.01$. We use a $[7,5]$ convolutional code with rate $1/2$ and a random interleaver. One OFDM frame consists of 33 OFDM symbols with 256 subcarriers. We use uniform spacing of pilot symbols with $D_t = 16$ ($N_t = 3$) in the time domain and with $D_f = 32$ ($N_f = 8$) in the frequency domain. The total number of pilot and data symbols in one frame are 24 and 8424 respectively. In this system, the data-to-pilot power ratio is about 25dB. Since the OFDM symbol which includes 248 data and 8 pilot symbols, we can reconstruct 31 virtual pilot groups besides 1 actual pilot group at the second iteration.

In order to evaluate the proposed iterative channel estimation method, first we examine the MSE$= E\|H_k(n) - \hat{H}_k(n)\|^2$ versus the quality of feedback information. Therefore, we assume the APP as a $N(^{2}_{\text{APP}}/2, ^{2}_{\text{APP}})$ Gaussian random variable. Choosing the noise variance as $^{2}_{N} = 0.1581$, we illustrate the MSE for different values of $^{2}_{\text{APP}}$ in Figure 4. According to the results, the proposed iterative channel estimation avoids the estimation error when the quality of feedback information is poor. Since 248 new virtual pilots has added to the OFDM frame, the $_{v}$ factor is calculated as 0.03 which is also the MSE ratio between the first and second iteration at $^{2}_{\text{APP}} = 20$.

In Figure 5, we give the bit-error-rate (BER) performance of coded OFDM system. According to the results, the use of the reliability factor in the combining process, the BER performance improves significantly compared to first iteration. Moreover, in the region of interest, the proposed algorithm gives the same BER performance with the EM algorithm while avoiding the $L_f \times L_f$ channel inversion.

6. CONCLUSION

In this paper, we have proposed an algorithm which combines virtual pilot groups by taking into account their reliabilities provided from feedback information. We have enhanced the channel estimation compared to the pilot only method. We have also shown that the proposed algorithm gives the same performance as the unbiased EM algorithm. Similar to the EM algorithm, the proposed method can be also applied to OFDM blind channel estimation.

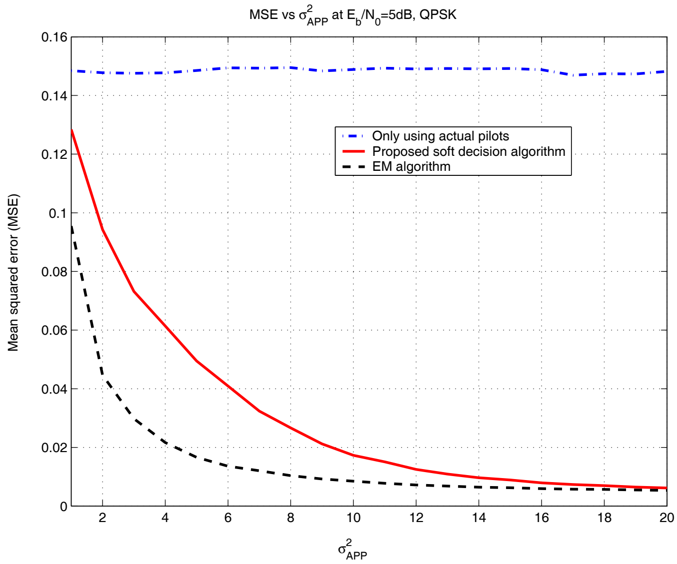

Figure 4: The MSE versus the variance of feedback information

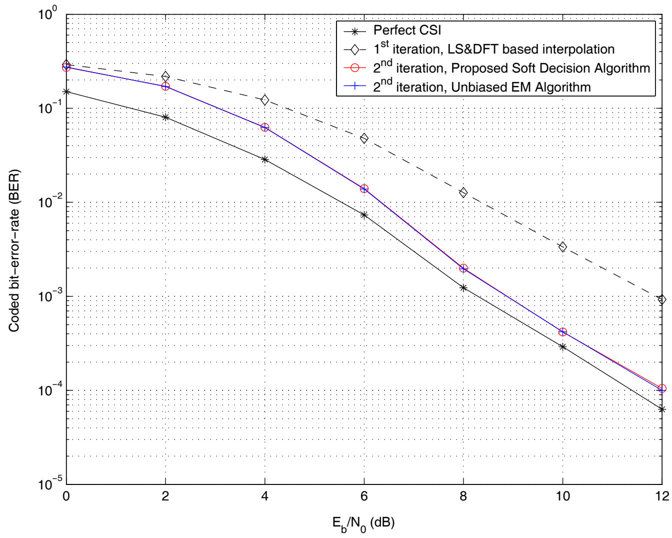

Figure 5: The BER performance of coded OFDM system

REFERENCES

[1] D. Slock, "Signal Processing challenges for wireless Communication," in *Proc. 1st Int. Sym. on Control, Communications and Signal Processing*, Tunisia, March 21-24 2004, pp. 881–892.

[2] R. Otnes and M. Tuchler, "Soft iterative channel estimation for Turbo equalization: Comparison of channel estimation Algorithms," in *Proc. IEEE ICCS'02*, Singapore, Nov. 2002, pp. 72–76.

[3] N. Nefedov and M. Pukkila, "Iterative channel estimation for GPRS," *Proc. IEEE Personnal and Mobile Radio Communication*, pp. 999–1003, 2003.

[4] S. Lasaulce and N. Sellami, "On the impact of using unreliable data on the bootstrap channel estimation performance," in *Proc. IEEE SPAWC'03*, Italy, June 2003, pp. 348–352.

[5] R. Nilsson, O. Edfors, M. Sandell and P. O. Brjesson, "An analysis of two-dimensional pilot-symbol assisted modulation for OFDM," in *Proc. IEEE ICPWC'97*, pp. 71–74, 1997.

[6] P. Hoeher, S. Kaiser and I. Robertson, "Two dimensional pilot-symbol-aided channel estimation by Wiener Filtering," in *Proc. IEEE ICASSP'97*, pp. 1845–1848, April 1997.

[7] M. Flament, B. Mielczarek and A. Svensson, "Joint channel estimation and turbo decoding for OFDM-based systems," in *Proc. Int. Symp. on Wireless Personal Multimedia Communication*, pp. 1299 – 1303, Oct. 2002.

[8] F. Sanzi, S. Jelting, and J. Speidel, "A comparative study of iterative channel estimators for mobile OFDM systems," *IEEE Tran. on Wireless Communication*, vol. 2, pp. 849–859, Sep. 2003.

[9] M. J. F. Garcia, J. M. Paez-Borrallo and S. Zazo, "DFT-based channel estimation in 2D-pilot-symbol-aided OFDM wireless systems," in *Proc. IEEE VTC'01 Spring*, pp. 810–814, 2001.

[10] B. Han, X. Gao, X. You, J. Wang and E. Costa, "An Iterative Joint Channel Estimation and Symbol Detection Algorithm Applied in OFDM System with high data to pilot power ratio," in *Proc. IEEE ICC'03*, pp. 810-814, 2003.

[11] P. Hoeher, I. Land and U. Sorger, "Log-likelihood values and Monte Carlo Simultion-Some Fundamental Results," in *Proc. 2nd Int/ Sym. on Turbo Codes and Related Topics*, pp. 43-46, 2000.

[12] G. K. Kaleh, R. Vallet,"Joint Parameter Estimation and Sybol detection for linear or nonlinear unknown channels," in *Proc. IEEE Trans. on Communication*, pp. 2406-2413, 1994.

[13] K. F. Lee and D. B. Williams, "A space-time coded transmitter diversity technique for frequency selective fading channels," in *Proc. Sensor Array and Multichannel Signal Processing Workshop*, pp. 149-152, 2000.