BEARING AND RANGE ESTIMATION USING WIDE-BAND MUSIC METHOD

Zineb Saidi[(1)], *Salah Bourennane*[(2)], *Laurent Guillon*[(1)], *and Patrick Sanchez*[(3)]

[(1)]IRENav (EA 3634), Ecole Navale
Lanvéoc Poulmic, BP600, 29240 Brest−Armées, France.
saidi,guillon@ecole-navale.fr
[(2)] Institut Fresnel, UMR CNRS 6133-EGIM, D.U. de saint Jérome
13 397 Marseille Cedex 20,France.
salah.bourennane@fresnel.fr
[(3)]Laboratoire de Mécanique et d'Acoustique, Unité Propre du CNRS - UPR 7051
31 chemin Joseph-Aiguier 13402 Marseille cedex 20, France.
sanchez@lma.cnrs-mrs.fr

ABSTRACT

This study deals with the bearing and the range estimation for buried objects problem. We propose a new method that combines the array processing approaches with an accurate acoustical modeling for the buried objects localization problem in the underwater acoustics environment. This method incorporates the exact solution for the scattered field (instead of using the plane wave model) in the MUSIC method, uses the focusing operator to decorrelate the signals and estimates both the range and the bearing objects. Finally, the performances of the proposed method are validated on experimental data recorded during an underwater acoustics experiments.

1. INTRODUCTION

MUSIC method is widely used in underwater acoustics for objects (sources) bearing estimation. It is based on the assumption that the wave front model in known and usually the considered objects are far from the array. Consequently, plane wavefront approximation is possible [6]. In many applications, where this assumption is not valid, the objects localization needs both, bearing and range estimation.
Many techniques addressed to estimate both the range and the bearing of objects using Matched Field Processing applied to the narrow-band signals [7], high-order subspace [5], genetic algorithm [4], geoacoustic inversion methods [1], and so on. Some of these techniques do not take into account the interactions between objects, or they are applied only for narrow band signals, and the others are validated only on simulated data. In our study, we have developed a method that solves the objects localization problem for wide-band correlated signals in the underwater acoustics domain and experimental data are used to validate its performances. This method is based on the incorporation of the exact solution of the scattered field by an elastic cylindrical shell, in the MUSIC method and uses the focusing operator to decorrelate the signals. The objects structures are assumed known.
We begin this study with a summary of the MUSIC method, presented in *section* 2. A modified MUSIC method, for narrow-band and wide-band signals, is developed, respectively, in *section* 3 and 4. The developed method is validated on experimental data. The experimental set up is described in *section* 5 and the obtained results are shown and discussed in *section* 6. The conclusion is outlined in *section* 7.

2. LOCALIZATION WITH THE MUSIC METHOD

Consider K objects observed by a linear array of N sensors. The output signal of the nth sensor can be described by

$$r_n(t) = \sum_{k=1}^{K} s_k(t - \tau_{n,k}) + b_n(t), \quad n = 1,2,...,N, \tag{1}$$

where $s_k(t)$ is the signal associated with the kth wavefront $(k = 1,2,...,K)$, $\tau_{n,k}$ is the delay associated with the signal propagation time from the kth object to the nth sensor and $b_n(t)$ is the additive noise at the nth sensor. The noise is assumed to be uncorrelated with the signals, uncorrelated from sensor to sensor, and to have variance σ^2. We assume that each sensor have a unit gain.
When the object is in the far-field region of the array, the wavefront can be assumed to be plane and MUSIC method allows us to estimate the bearing of this plane wave associated to this object. The spatial spectrum of the MUSIC method is given by

$$\mathbf{p}_{\mathbf{MUSIC}}(\theta) = \frac{\mathbf{1}}{\mathbf{a}(\theta)^{+}\mathbf{V_b}\mathbf{V_b^{+}}\mathbf{a}(\theta)}, \tag{2}$$

where the superscript "$(.)^+$" represents the Hermitian transpose and $\mathbf{a}(\theta) = \left[1,\, e^{-2j\pi f \frac{d\sin\theta}{c}}\,,\, ...\,,\, e^{-2j\pi f (N-1)\frac{d\sin\theta}{c}}\right]^T$ is the plane wave direction vector, the superscript "$[.]^T$" represents the transpose, $\mathbf{V_b}$ is the eigenvectors associated to the noise subspace, c is the sound speed in the medium, d is the interspacing of the sensors, f is the signal frequency and j is the complex operator.

3. BEARING AND RANGE ESTIMATION FOR NARROW-BAND SOURCES

In this section, the bearing and range estimation problem is solved for narrow-band signals with the assumption that the objects are in a free space. These objects have all a cylindrical shell.

3.1 Exact solution

Consider an infinite elastic cylinder of outer radius a, inner radius b, in a free space, located at (r_1, θ_1) the range and the bearing object respectively, associated to the first sensor of

the array (figure 1). The two fluids outside and inside the shell are labeled by 1 and 3, respectively, sound velocities $c_{1,3}$, the wavenumbers $k_{1,3}$. Suppose that the incident field is a plane wave with an angle θ_{inc}. In order to calculate the exact solution for the scattered field $p_s(r_1,\theta_1)$ a decomposition of the different fields is used, according to the Bessel (J_m, N_m) and Hankel (H_m) functions [2]. We adopt cylindrical coordinates.
In medium 1, the pressure is taken as $p = p_i + p_s$, with a given incident plane-wave pressure

$$p_i = p_0 \sum_{m=0}^{\infty} j^m \varepsilon_m J_m(k_1 r_1)\cos(m\theta_{r1}), \qquad (3)$$

where $\theta_{r1} = \theta_1 - \theta_{inc}$, p_0 a constant
and a scattered pressure

$$p_s = p_0 \sum_{m=0}^{\infty} i^m \varepsilon_m b_m H_m^{(1)}(k_1 r_1)\cos(m\theta_{r1}), \qquad (4)$$

we use $\varepsilon_0 = 1, \varepsilon_1 = \varepsilon_2 = ... = 2$, $b_m, ..., g_m$ are coefficients and m is the number of modes. The time dependent term $e^{(-j2\pi ft)}$ has been assumed for each field variable.
In medium 2, the displacement vector $\mathbf{u}$ is written as

$$\mathbf{u} = -\nabla\psi + \nabla \times \mathbf{A}, \qquad (5)$$

where ψ is the scalar potential and $\mathbf{A}$ is the vector potential. The solutions are written

$$\psi = p_0 \sum_{m=0}^{\infty} j^m \varepsilon_m [c_m J_m(k_l r_1) + d_m N_m(k_l r_1)]\cos(m\theta_{r1}), \quad (6)$$

$$\mathbf{A} = p_0 \sum_{m=0}^{\infty} j^m \varepsilon_m [e_m J_m(k_t r_1) + f_m N_m(k_t r_1)]\cos(m\theta_{r1}), \quad (7)$$

where k_t and k_l are respectively the transversal and the longitudinal wavenumber. .
In medium 3, one has again a compressional wave, which must be regular at the origin:

$$p_z = p_0 \sum_{m=0}^{\infty} j^m \varepsilon_m g_m J_m(k_3 r_1)\cos(m\theta_{r1}). \qquad (8)$$

Both at $r_1 = a$ and $r_1 = b$, the following boundary conditions have to be satisfied:

- the pressure in the fluid equals the normal component of stress in the solid,
- the normal component of displacement is continuous,
- the tangential component of shearing stress are zero.

Using these boundary conditions, coefficients $b_m, ..., g_m$ can be calculated [2].

3.2 The modified MUSIC for narrow-band sources

The scattered field model, developed in the previous subsection, is incorporated in the MUSIC method instead of the plane wave model. The spatial spectrum of the modified MUSIC method is given by

$$\mathbf{p_{MUSICnb}}(r,\theta) = \frac{1}{\mathbf{p_s}(r,\theta)^{+}\mathbf{V_b}\mathbf{V_b}^{+}\mathbf{p_s}(r,\theta)}, \qquad (9)$$

where $\mathbf{p_s}(r,\theta) = [p_s(r_1,\theta_1),\ p_s(r_2,\theta_2),\ ...\ ,\ p_s(r_N,\theta_N)]^T$, represents the direction vector and $p_s(r_n,\theta_n)$ is the scattered field associated to the nth sensor. We have used the general pythagore theorem to calculate the couple (r_n,θ_n) associated to the nth sensor as shown in figure 1. The obtained r_n, θ_n are given by

$$r_n = \sqrt{r_{n-1}^2 - d^2 - 2r_{n-1}d\cos(\frac{\pi}{2} + \theta_{n-1})} \qquad (10)$$

$$\theta_n = \cos^{-1}[\frac{d^2 + r_n^2 - r_{n-1}^2}{2r_{n-1}d}], \quad n = 2, ..., N. \qquad (11)$$

Equations (10) and (11) are incorporated in equation (4) to obtain $p_s(r_n,\theta_n)$.

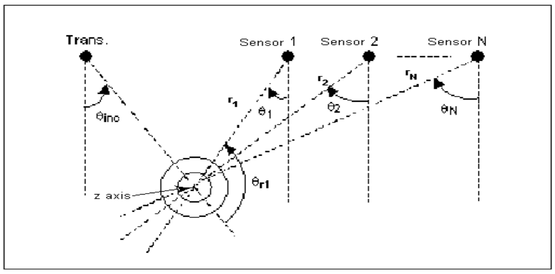


Figure 1: Range and bearing object associated to each sensor.

4. BEARING AND RANGE ESTIMATION FOR WIDE-BAND SOURCES

In the previous section, the modified MUSIC method has been developed for a narrow-band signals. In this section, wide-band signals are considered. We propose to apply the bilinear focusing operator [9] technique which divides the frequency band into L narrow-bands. This technique transforms the received signals in the L bands into the focusing frequency f_0 and consequently decorrelates the signals [10]. Here, f_0 is the center frequency of the spectrum of the received signal and it is chosen as the focusing frequency.
The following is the step-by-step description of the technique:

1. using an ordinary beamformer to find an initial estimate of r, θ and the number of objects K,
2. filling the directional matrix $\hat{\mathbf{P}}_s(f_l) = [\mathbf{p_{s1}}(r,\theta,f_l),\ \mathbf{p_{s2}}(r,\theta,f_l),\ ...\ ,\ \mathbf{p_{sK}}(r,\theta,f_l)]$, where each component of the directional vector $\mathbf{p_{sk}}(\mathbf{r},\theta)$, associated to the kth objects, is filled using equation (4),
3. estimating the spectral matrix output sensors data $\mathbf{\Gamma}(f_l)$ at frequency f_l,
4. calculating sources spectral matrix at frequency f_l using:

$$\begin{aligned}\mathbf{\Gamma}_s(f_l) &= (\hat{\mathbf{P}}_\mathbf{s}^{+}(f_l)\hat{\mathbf{P}}_\mathbf{s}(f_l))^{-1}\hat{\mathbf{P}}_\mathbf{s}^{+}(f_l) \\ &\quad [\mathbf{\Gamma}(f_l) - \hat{\sigma}^2(f_l)\mathbf{I}]\hat{\mathbf{P}}_\mathbf{s}(f_l) \\ &\quad (\hat{\mathbf{P}}_\mathbf{s}^{+}(f_l)\hat{\mathbf{P}}_\mathbf{s}(f_l))^{-1}, \end{aligned} \qquad (12)$$

where, $\mathbf{I}$ is the identity matrix and $\hat{\sigma}^2$ the estimated noise variance.

5. calculating the average of the spectral matrices associated to the objects:

$$\mathbf{\Gamma}_s(f_0) = \frac{1}{L}\sum_{l=1}^{L}\mathbf{\Gamma}_s(f_l), \quad 1 \le l \le L \tag{13}$$

6. calculating $\hat{\mathbf{\Gamma}}(f_0) = \hat{\mathbf{P}}_\mathbf{s}(f_0)\mathbf{\Gamma}_s(f_0)\hat{\mathbf{P}}_\mathbf{s}^{+}(f_0)$ and $\hat{\mathbf{\Gamma}}(f_l) = \mathbf{\Gamma}(f_l) - \hat{\sigma}^2(f_l)\mathbf{I}$,
   then, the noise variance is estimated by :

$$\hat{\sigma}^2(f_l) = \frac{1}{N-K}\sum_{i=K+1}^{N}\lambda_i(f_l), \tag{14}$$

   where $\lambda_i(f_l)$ is the ith eigenvalue of $\mathbf{\Gamma}(f_l)$,
7. estimating the bilinear focusing operator:

$$\mathbf{T}(f_0, f_l) = \mathbf{V}(f_0)\mathbf{V}^{+}(f_l), \tag{15}$$

   where $\mathbf{V}(f_0)$ and $\mathbf{V}(f_l)$ are the eigenvector matrices of $\mathbf{\Gamma}(f_0)$ and $\mathbf{\Gamma}(f_l)$, respectively,
8. calculating the focused spectral matrix:

$$\hat{\mathbf{\Gamma}}(f_0) = \frac{1}{L}\sum_{l=1}^{L}\mathbf{T}(f_0, f_l)\mathbf{\Gamma}(f_l)\mathbf{T}^{+}(f_0, f_l), \tag{16}$$

9. using a detection method (AIC or MDL) to find the true number of sources [8].

The modified spatial spectrum of MUSIC method for wide-band correlated signals is given by

$$\mathbf{p}_{MUSICwb}(r,\theta) = \frac{1}{\mathbf{p_s}(r,\theta,f_0)^{+}\mathbf{V_{b0}}\mathbf{V_{b0}}^{+}\mathbf{p_s}(r,\theta,f_0)}, \tag{17}$$

where $\mathbf{V_{b0}}$ is the eigenvector matrices of $\hat{\mathbf{\Gamma}}(f_0)$ associated to the noise subspace.

5. EXPERIMENTAL SET UP

5.1 Experimental methods

Time domain measurements in an experimental water tank have been used in order to evaluate the performances of the developed method. The experimental set up is shown in figure 2 where all the dimensions are given in meter. The bottom of the tank is full of fine and homogeneous sand where are buried six cylindrical shells, between 0 and 0.005 m, of different dimensions (table 1). We have done six experiments. The transmitter is fixed at an incident angle 60 ° and the receiver moves horizontally from the initial to the final position with a step size $d = 0.002$ m. The distance, between the transmitter, the RR' axis and the receiver, remains the same for all the experiments. For the three first experiments, we have fixed the receiver axis at 0.2 m from the bottom of the tank and the RR' axis is positioned on the 1^{st}, the 2^{nd} and the 3^{rd} cylinders couple. For the three last experiments, the receiver axis is fixed at 0.4 m and we have repeated the same experiments as in the first time.

	1^{st} couple	2^{nd} couple	3^{rd} couple
inner radius a (m)	0.01	0.018	0.02
Filled of	air	water	air
Separated by (m)	0.13	0.16	0.06

Table 1: characteristics of the various cylinders (the inner radius $b = a - 0.001$ m)

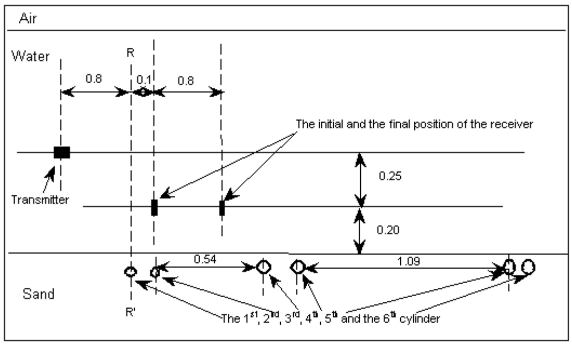


Figure 2: Experimental set up

5.2 Experimental data

Typical sensor output signals corresponding to one experiment are shown in figure 3. Sensor output signal in time and frequency domain are presented in figure 4. The frequency Band is $[fmin = 150, fmax = 250]$ kHz, the center frequency is $f_0 = 200$ kHz and the sampling rate is 2 MHz.

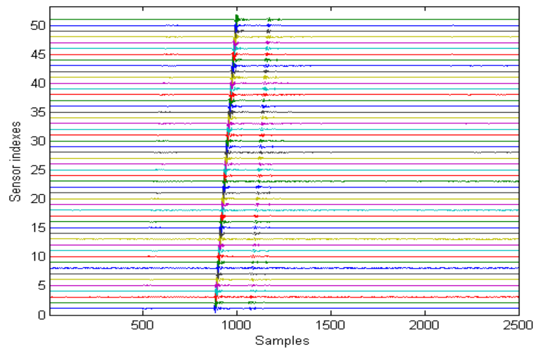
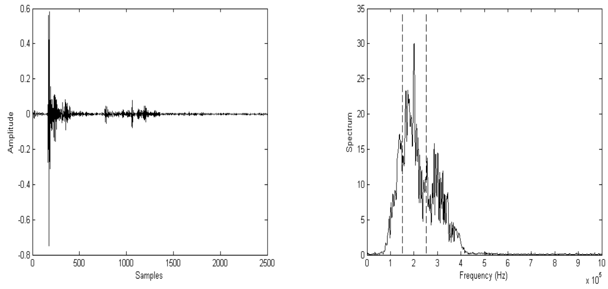


Figure 3: Observed sensor output signals

5.3 Results and discussion

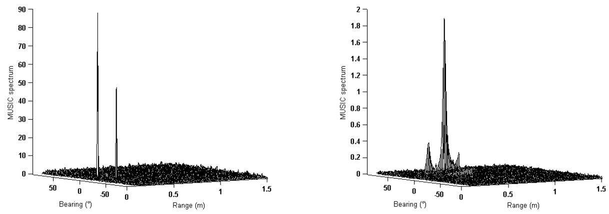
The considered cylinders are buried under the sand which has geoacoustic characteristics near to those of water. Then, we can make the assumption that the cylinders are in a free space. The seven steps, listed above, are applied on each experimental data. A sweeping on r and θ have been applied ($[0.2, 1.5]$ m for r and $[-90°, 90°]$ for θ). The obtained spatial spectrum of the modified MUSIC method are shown in figures 5, 6 and 7. For each experiment, only one cylinders couple is radiated by the transmitter.

Table 2 gives the real and the estimated range and bearing values obtained with the modified MUSIC method. The indexes 1 and 2 in table 2are the 1^{st} and the 2^{nd} cylinder of each couple of cylinders. Note that, the difference between the estimated value $(r_{1,2est}, \theta_{1,2est})$ and the real value $(r_{1,2re}, \theta_{1,2re})$ is very small and only two cylinders that have not been detected in Exp.4 and Exp.6, because, the received echo, associated to these cylinders, is rather weak.



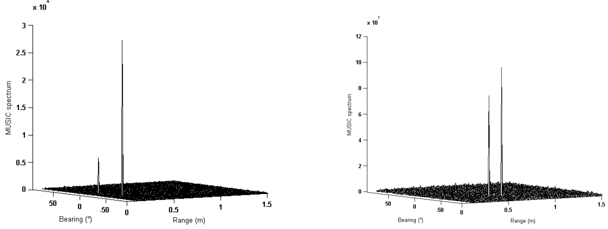
(a) (b)

Figure 4: Typical sensor output signal from the experiment. (a) Time signal. (b) Frequency signal



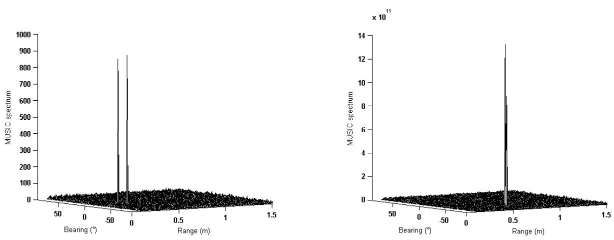
(a) (b)

Figure 5: Spectrum of the modified MUSIC method associated to the first cylinders couple. (a) The receiver is at 0.2 m from the bottom (Exp.1). (b) The receiver is at 0.4 m from the bottom (Exp.4)



(a) (b)

Figure 6: Spectrum of the modified MUSIC method associated to the second cylinders couple. (a) The receiver is at 0.2 m from the bottom (Exp.2). (b) The receiver is at 0.4 m from the bottom (Exp.5)



(a) (b)

Figure 7: Spectrum of the modified MUSIC method associated to the third cylinders couple. (a) The receiver is at 0.2 m from the bottom (Exp.3). (b) The receiver is at 0.4 m from the bottom (Exp.6)

6. SUMMARY AND CONCLUSION

The array processing approaches, as the MUSIC method and the focusing operator, are combined with the exact

	Exp.1	Exp.4	Exp.2	Exp.5	Exp.3	Exp.6
r_{1re}(m)	0.24	0.65	0.26	1.24	0.26	0.65
$\theta_{1re}(^\circ)$	-25	-50	-34	-70	-34	-50
r_{1est}(m)	0.25	0.63	0.29	1.21	0.28	0.63
$\theta_{1est}(^\circ)$	-23	-52	-33	-70	-32	-52
r_{2re}(m)	0.22	0.56	0.24	1.17	0.22	0.64
$\theta_{2re}(^\circ)$	8	-41	-22	-69	4	-49
r_{2est}(m)	0.25	–	0.25	1.2	0.23	–
$\theta_{2est}(^\circ)$	9	–	-20	-65	6	–

Table 2: (r, θ) real (re) and estimated (est) values with the modified MUSIC method (negative θ is clockwise from the vertical)

solution of the scattered field in order to estimate both the range and the bearing objects. The performances of this method are investigated through real data associated to many cylinders buried under the sand. The proposed method is superior in terms of performance to the conventional method when the objects are in the far field and even in the near field. The range and the bearing objects are estimated with a significantly good accuracy due to the free space assumption.

ACKNOWLEDGMENTS

We would like to thank Dr J. P. Sessarego, from the LMA (Laboratory of Mechanic and Acoustics), Marseille, France, for helpful discussions and technical assistance.

REFERENCES

[1] N. Chapman and A. Tolstoy, *Benchmarking geoacoustic inversion methods*, Special Issue, J. Conf. Acous., 6 (1 and 2), 289 pages, 1998.

[2] R. D. Doolittle and H. Uberall, "Sound scattering by elastic cylindrical shells", *J. Acoust. Soc. Am.*, 39 (2), 1966

[3] M. A. Doron and J. Weiss, "On focusing for wideband array processing", *IEEE Transaction On signal Processing*, 40 (6),1992

[4] Edwards, D.J. and Keane, A.J., "Numerical techniques for efficient sonar bearing and range searching in the near field using genetic algorithms", *in Evolutionary Algorithms in Engineering Applications*, ISBN 3-540-62021-4, 1996.

[5] M. Haardt, R. N. Challa, and S. Shamsunder, "Improved bearing and range estimation via high-order subspace based Unitary ESPRIT", *IEEE Computer Society Press*, Nov. 1996

[6] P. S. Naidu, *Sensor array signal processing*, CRC Press, 2001

[7] A. Sahin and E. L. Miller, "Object-based localization of buried objects using high resolution array processing techniques", *in Proc. of SPIE- AeroSense*, vol. 2765, May 1996

[8] M. Wax and T. Kailath, "Detection of signals information therotic criteria", *IEEE Trans. Acoust., Speech, Signal processing*, 33 (2), April 1985

[9] S. Valaee and P. Kabal, "Wideband array processing using a two-sided correlation transformation", *IEEE Transaction On signal Processing*,43 (1),1995

[10] H. Wang and M. Kaveh, "Coherent signal-subspace processing for the detection and estimation of angles of arrival of multiple wideband sources", *IEEE Trans. Acoust., Speech, Signal Processing*,33 (4),1985