A NEW STABLE ADAPTIVE IIR FILTER FOR ACTIVE NOISE CONTROL SYSTEMS

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ABSTRACT

The multimodal error surface and instability problem of adaptive IIR filters invokes more research. In this paper, an algorithm with the required conditions for stability is proposed based on hyperstabilty theory and evaluated in active noise control systems. The performance of the algorithm is compared to that of the FuRLMS and SHARF algorithms using computer simulations. It is shown that, in general, the proposed algorithm has a faster convergence rate. To check the stability, the algorithm is evaluated in the presence of impulsive noise. Simulation results show that while the proposed algorithm is converging in a stable manner, the other two algorithms diverge.

1. INTRODUCTION

Active Noise Control (ANC) systems have widely been investigated for canceling unwanted acoustic noise in various applications. An ANC system works based on producing an anti-noise similar to the primary (unwanted) noise, but with an opposite phase and equal amplitude [1, 2]. The sum of these two signals reduces the acoustic noise. An active noise control system makes use of an adaptive FIR or IIR filter to generate the anti-noise signal. The use of IIR filters in ANC systems is of interest when there is a feedback in the system from the secondary-path towards the reference signal or the noise is broadband for which the effect of poles and zeros of the secondary path transfer function are considerable. It is obvious that in such cases an FIR filter with a large order is required to model the system. This, however, leads to a slow convergence of the incorporated adaptive algorithm; hence, the ANC system may become unstable.

To adapt IIR active noise or vibration control systems, the FuRLMS algorithm has been addressed [3, 4]. The convergence properties of this algorithm are not well understood. The theory of hyperstability for stable adaptation of IIR filters has also been reported in ANCs and other applications [5-6]. It will be shown that the adaptation process of an IIR filter can be considered as a linear system with a nonlinear time-varying feedback path. The output-error HARF algorithm has been successfully applied in a wide range of situations for stable adaptation of IIR filters. The modified version of the latter algorithm has been referred to as the SHARF algorithm in which the error signal drives a smoothing filter to enhance stability of the algorithm [5]. However, as it will be shown the convergence rate and stability of the SHARF algorithm will be improved by the proposed algorithm.

Although, the above algorithms have been applied to active noise and vibration control systems [7-9], there is still a large number of challenging points to be further investigated.

This is effectively due to the existence of the secondary-path transfer function in ANC systems in comparison with the conventional output error identification problems.

In this paper, an adaptive algorithm is used in an ANC system with IIR structure. Then, the stability conditions of the system are derived based on the hyperstability theory of popov when a secondary-path transfer function exists in the system. In [8] a similar approach has been considered for a constant step-size parameter. Here, we use an adaptive step-size parameter to improve the convergence rate and to reduce the minimum mean square error. Using simulation results, the convergence and stability of the proposed algorithm with those of the FuRLMS and SHARF algorithms is compared. The input signal is assumed to be embedded in either white Gaussian or impulsive α -stable noise.

2. ADAPTATION SYNTHESIS OF IIR FILTER

Fig. 1 shows a typical ANC system with an IIR control filter. The Recursive Least-Squares criterion (1) is used to adjust the IIR filter coefficients where the error signal $\varepsilon(i)$ is computed by (2). It must be noted that in (1) n is the current time while the indices i *'s* denotes the previous times up to *n*.

$$J(n) = \sum_{i=1}^{n} \varepsilon^2(i) \qquad (1)$$

$$\varepsilon(i) = d(i) - \left[\hat{\boldsymbol{\theta}}^T(n)\boldsymbol{\varphi}(i-1)\right] * s(n) \qquad (2)$$

where *d(i)* is the disturbance to be cancelled at time *i*, *s(n)* is the secondary-path impulse response at time *n*, * is linear convolution operator, $\boldsymbol{\varphi}(i-1)$ denote the regression vector and $\hat{\boldsymbol{\theta}}(n)$ is vector of the filter parameters as:

$$\hat{\boldsymbol{\theta}}(n) = \left[\hat{a}_1(n), \hat{a}_2(n), \ldots, \hat{a}_{n_A}(n), \hat{b}_1(n), \hat{b}_2(n), \ldots, \hat{b}_{n_B}(n)\right]^T$$

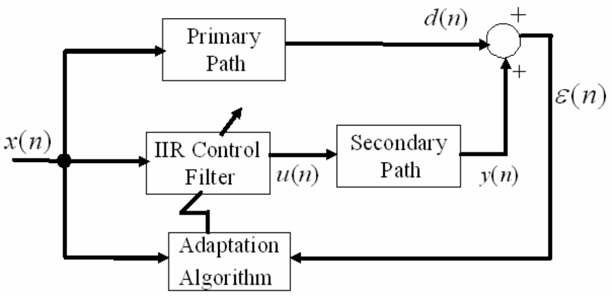

Figure 1. A typical active noise control system with IIR control filter

n_A, n_B are the orders of denominator and numerator of the control filter respectively. By combining the convolution of secondary path to the regression vector in (2) the control objective (1) may be formulized as:

$$\min_{\hat{\theta}(n)} J(n) = \sum_{i=1}^{n} \left[d(i) - \hat{\boldsymbol{\theta}}^T(n)\boldsymbol{\varphi}_f(i-1)\right]^2 \quad (3)$$

where $\boldsymbol{\varphi}_f(i-1)$ is the filtered regression vector whose elements are the filtered version of control and reference signals as follows:

$$u_f(i) = s(n) * u(i) \quad x_f(i) = s(n) * x(i)$$

$$\boldsymbol{\varphi}_f(i-1) = \left\lfloor -u_f(i-1), \ldots, -u_f(i-n_A), \ldots, x_f(i-n_B) \right\rfloor$$

Using the same method presented in [9], we derive this algorithm for the case that an estimation of secondary-path transfer function is available. So, the filtered version of the reference signal is included in the derivations. The proposed algorithm updates the coefficients of the adaptive IIR filter (shown in Fig. 1) at time *n* using equations (4-11):

$$\hat{\boldsymbol{\theta}}(n+1) = \hat{\boldsymbol{\theta}}(n) + F(n)\boldsymbol{\varphi}_f(n)v(n+1) \quad (4)$$

$$v(n+1) = \frac{v^0(n+1)}{1 + \boldsymbol{\varphi}_f^T(n)F(n)\boldsymbol{\varphi}_f(n)} \quad (5)$$

$$F(n+1) = \frac{1}{\lambda_1(n)}\left[F(n) - \frac{F(n)\boldsymbol{\varphi}_f(n)\boldsymbol{\varphi}_f^T(n)F(n)}{\frac{\lambda_1(n)}{\lambda_2(n)} + \boldsymbol{\varphi}_f^T(n)F(n)\boldsymbol{\varphi}_f(n)}\right] \quad (6)$$

$$v^0(n+1) = \varepsilon^0(n+1) + \sum_{j=1}^{n_C} c_j \varepsilon(n+1-j) \quad (7)$$

$$\varepsilon^0(n+1) = d(n+1) - \hat{\boldsymbol{\theta}}^T(n)\boldsymbol{\varphi}_f(n) \quad (8)$$

$$\varepsilon(n-i) = d(n-i) - \hat{\boldsymbol{\theta}}^T(n-i)\boldsymbol{\varphi}_f(n-i-1) \quad (9)$$

$$\boldsymbol{\varphi}_f(n) = \left\lfloor -u_f(n), \ldots, -u_f(n-n_A), \ldots, x_f(n-n_B) \right\rfloor \quad (10)$$

$$u_f(i) = \hat{s}(n) * u(i), \quad x_f(i) = \hat{s}(n) * x(i) \quad (11)$$

Here $F(n)$ is the adaptation gain matrix updated iteratively by (6), $\lambda_1(n)$ and $\lambda_2(n)$ are two weighting sequences which determine how adaptation gain change in time, $\varepsilon^0(n)$ is the priori error and $\varepsilon(n)$ is the posteriori error. In (11) $\hat{s}(n)$ is an estimation of secondary path impulse response and c_j 's are the coefficients of an FIR filters used to filter the posteriori error which is described in more details in next section.

3. INVESTIGATION OF STABILITY USING HYPERSTABILITY THEOREM

Assuming slow adaptation, the control filter may be replaced before the secondary path transfer function [1] in which $x_f(n)$ is the filtered version of the reference signal $x(n)$ (Fig 2). In order for the ANC system to be defined as an output-error identification problem, Fig. 2 may equivalently be depicted as Fig. 3.

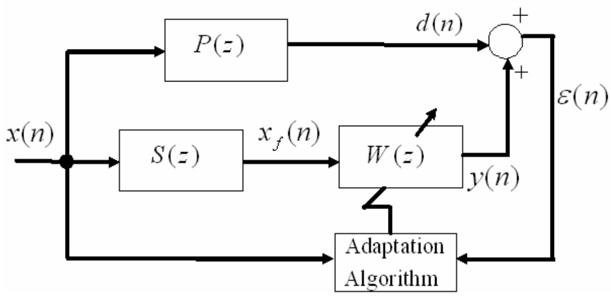

Figure 2. Block diagram of Fig. 1 under slow adaptation.

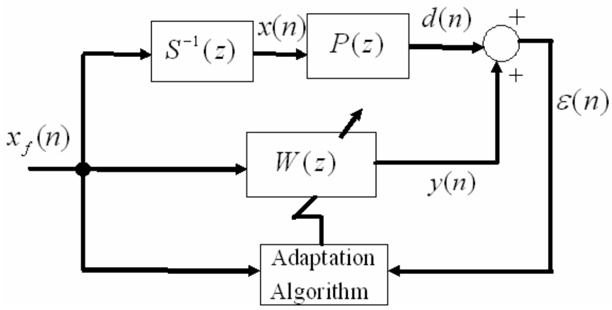

Figure 3. Block diagram of Fig. 2 whit $x_f(n)$ as input signal.

It is clear that the secondary-path transfer function should be minimum phase to be able to model its inverse by $W(z)$. The stability condition of the algorithm proposed in the previous section is proved by the following theorem.

Theorem: The algorithm (4-11) used to update the coefficients of adaptive IIR filter is stable and will converge to the desired filter weights if and only if $\frac{\hat{S}(z^{-1})D(z^{-1})}{A_{sz}(z^{-1})A_{pp}(z^{-1})}$ be strictly positive real where it is assumed $S(z^{-1}) = \frac{A_{sz}(z^{-1})}{A_{sp}(z^{-1})}$ and $P(z^{-1}) = \frac{A_{pz}(z^{-1})}{A_{pp}(z^{-1})}$, besides under the perfect modeling of cancellation path transfer function by an FIR filter this condition will be simplified to the positive realness of $\frac{C(z^{-1})}{A_{pp}(z^{-1})}$.

Proof: To show that the filter weights of adaptive IIR filter in figure (1) converges to the desired values in a stable manner and minimizes the performance index (3), it is equivalent to prove that the estimated parameters vector $\hat{\boldsymbol{\theta}}(n)$ in figure (3) leads to the values for which the posteriori estimation error converges asymptotically to zero, that is:

$$\lim_{n\to\infty} \varepsilon(n+1) = 0 \qquad (12)$$

For this purpose, it is required to calculate the posteriori estimation error based on variations of the vector $\hat{\boldsymbol{\theta}}(n)$. By defining the posteriori estimation error as $\varepsilon(n+1) = d(n+1) - y(n+1)$ and considering the numerator and denominator of $P(z)S^{-1}(z)$ as $B(z^{-1})$ and $A(z^{-1})$ respectively, the posteriori estimation error may be rewritten as:

$$\varepsilon(n+1) = -A^*(z^{-1})d(n) + B(z^{-1})x_f(n) - y(n+1) \qquad (13)$$

$$A^*(z^{-1}) = a_1 + a_2 z^{-1} + \ldots + a_{n_A} z^{-n_A+1}$$

By adding and subtracting the term $A^*(z^{-1})y(n)$ to the right-hand side of (13) we obtain

$$\varepsilon(n+1) = -A^*(z^{-1})\varepsilon(n) + \boldsymbol{\varphi}_f^T(n)\left[\theta - \hat{\theta}(n+1)\right] \qquad (14)$$

where θ is the vector of optimal values of filter weights. Then, using the definition of $A(z^{-1})$, we may more compactly present (14) as:

$$\varepsilon(n+1) = \frac{1}{A(z^{-1})}\left[-\boldsymbol{\varphi}_f^T(n)\tilde{\theta}(n+1)\right] \qquad (15)$$

where $\tilde{\theta}(n+1) = \theta - \hat{\theta}(n+1)$. Subtracting both sides of (4) from θ and then multiplying by $\boldsymbol{\varphi}_f^T(n)$, we may have:

$$\phi_f^T\tilde{\boldsymbol{\theta}}(n+1) = \phi_f^T\tilde{\boldsymbol{\theta}}(n) + \phi_f^T F(n)\boldsymbol{\varphi}_f(n)v(n+1) \qquad (16)$$

Based on (15) and (16) Fig. 4 is used to show the adaptation algorithm with a linear time-invariant part in the feedforward path and a nonlinear time-varying part in the feedback path. According to the *popov hyperstability theory,* this system is stable when the feedback system is passive and thefeedforward system is strictly passive. The passivity ofthe feedback has been proved in [9]. Here, we prove the strict passivity of the feedforward path. Since $A(z^{-1})$ is the denominator of the system, its strict passivity may not be realizable in some situations. To overcome this problem, the posteriori estimation error can be filtered as:

$$v(n+1) = C(z^{-1})\varepsilon(n+1) \qquad (17)$$

where $C(z^{-1}) = 1 + \sum_{i=1}^{n_C} c_i z^{-i}$ is a stable polynomial with an order less than or equal to n_A. Since $A(z^{-1})$ is the denominator of $P(z)S^{-1}(z)$, the condition for thestability of the proposed algorithm will be the strictly positive realness of $\frac{\hat{S}(z^{-1})C(z^{-1})}{A_{sz}(z^{-1})A_{pp}(z^{-1})}$. If we assume that $S(z^{-1})$ is an FIR filter which can perfectly be modeled by another FIR filter so that $S(z^{-1}) = \hat{S}(z^{-1})$, the above condition is simplified to the strictly positive realness of this transfer function $\frac{C(z^{-1})}{A_{pp}(z^{-1})}$.

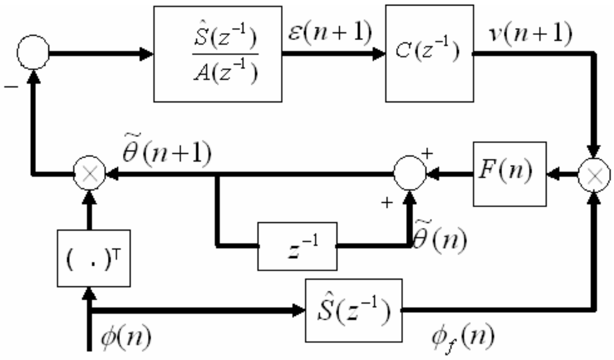

Figure 4. Equivalent feedback system and adaptive algorithm used in output error modeling.

4. SIMULATION RESULTS

Using computer simulations, the stability and performance of the proposed algorithm is evaluated based on (4-11) and compared to the other well-known IIR ANC algorithms. Note that the performance of an adaptive algorithm is normally justified based on its convergence rate and steady state error.

It is assumed that the primary-path transfer function is an IIR filter defined as $P(z) = \frac{\beta_0 + \beta_1 z^{-1}}{1 + \alpha_1 z^{-1} + \alpha_2 z^{-2}}$ and the secondary-path transfer function is an FIR filter defined by $S(z) = c_0 + c_1 z^{-1} + c_2 z^{-2}$. The filter coefficients c_0, c_1, and c_2 are selected 1, 0.7, and 0.6 respectively [5]. It is assumed the estimated secondary-path is equal to the actual one. The filter $C(z^{-1})$ with the coefficients [1 -.9] is used for filtering the error and establishing the positive real condition. The input signal is white uniform noise with zero mean and variance of unity. The results are shown by averaging 100 independent trials of the experiments.

In the first experiment, the parameters of the primary-path transfer function are chosen as β_0=0.05, β_1=-0.4, α_1=-1.1314, and α_2=0.25. Then, the transfer function has two poles at 0.83 and 0.3 with an error surface containing two minima. In order for the control filter to model the primary and inverse of the secondary paths, $W(z)$ is chosen as an IIR filter (1st-order numerator and 3rd-order denominator). Initial values of coefficients in all the cases are set to zero. The initial value of the gain matrix in (6) is $F(1) = 4$ and the weightings are selected as $\lambda_1 = 1$, $\lambda_2 = 0$. In Fig. 5 the convergence rates of the FuRLMS and SHARF algorithms are compared. To obtain the highest convergence rates, maximum values of step-size parameters are selected in all the cases such that any further increase of them will unstable the system. As seen in Fig. 5, due to perfect modeling of the control filter and converging to the global minimum in all of cases, the steady state error is zero while the proposed algorithm has a higher convergence rate with respect to those of the other algorithms.

Accordingly, the convergence behavior of the first and fourth coefficients of the denominator of $W(z)$ is seen in Fig. 5 which is in agreement with the above result.

In the second experiment the parameters of $P(z)$ are chosen as $\beta_0 = 0.05, \beta_1 = 0, \alpha_1 = -1.75, \alpha_2 = 0.81$. In this case, the primary-path transfer function (plant) has two complex conjugate poles with the amplitude of 0.9 and the local minimum is placed in the error surface such that it is hard for the algorithms to reach. The controller $W(z)$ is an IIR filter with a zero-order numerator and a third-order denominator. Initial value of the gain matrix is $F(1) = 200I$ and $\lambda_1 = 1$, $\lambda_2 = 1$. The conditions applied to the other algorithms are the same as the previous experiment. As clearly seen in Fig. 6, in the FuRLMS algorithm does not converge. Also, the proposed algorithm has a higher convergence rate than the SHARF algorithm. The convergence behavior of the coefficients also show the superiority of the proposed algorithm compared to the other ones.

5. CONCLUSION

In this paper, we investigated the stability of adaptive IIR controllers in ANC applications. This was carried out by synthesizing an RLS-type adaptive algorithm and deriving the required conditions for guaranteeing the stability based on hyperstabilty theory. Using computer simulations, the performance of this algorithm was compared to that of the FuRLMS and SHARF algorithms in IIR ANC systems. The results show that the proposed algorithm has a faster convergence speed than the SHARF algorithm and the FuRLMS is unstable.

6. ACKNOWLEDGEMENT

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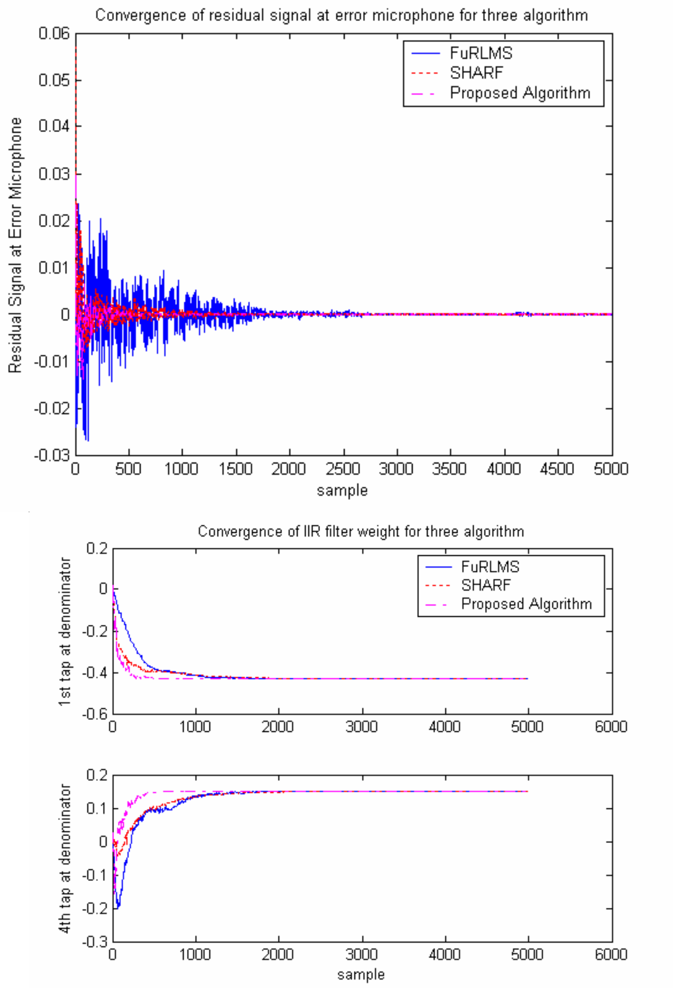

Figure 5. Error signals (up), convergence behavior of two coefficients of IIR controller (down).

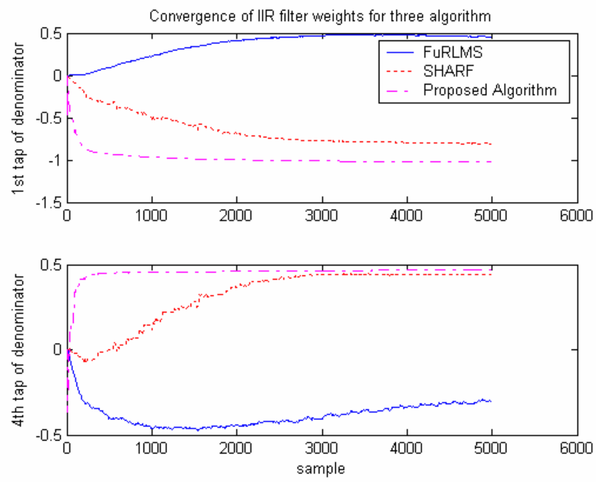

Figure 6. Error signals (up), convergence behavior of two coefficients of IIR controller(down).