BLIND MULTIUSER DETECTION BY ACCELERATED SUBSPACE TRACKING

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ABSTRACT

We propose a new blind multiuser detection scheme based on an accelerated subspace tracking in rapidly time-varying channel scenarios. The proposed subspace tracker is derived by combining the projection approximation subspace tracking (PAST) with the internal model principle, approximately expressing the changing parameters with an expansion of polynomial time functions. The proposed subspace tracker can still maintain the linear computational complexity to the number of users, similar to the PAST. Furthermore, the effectiveness of the proposed multiuser detector is validated in synchronous DS-CDMA systems with Rayleigh fading through some numerical simulations.

1. INTRODUCTION

Multiuser detection techniques are necessary for future wireless communication systems to substantially increase the capacity of code-division multiple-access (CDMA) systems [1], and a lot of schemes have been proposed (e.g. [2] [3] and the references within). Especially, some suboptimal linear detectors such as the decorrelating detector (DD) and minimum mean-square error detector (MMSED) have attracted much attention due to their efficiency and simplicity [4]. Another advantage of these detectors is to realize blind algorithms by combining subspace trackers. In this case, the performance of the multiuser detectors highly depends on the tracking ability of the subspace methods.

The projection approximation subspace tracking (PAST) [5] is one of the most efficient subspace estimators because of not only the tracking ability but also the computational simplicity. Some multiuser detectors have been proposed based on the PAST or PASTd algorithms which is modified by introducing the deflation steps [6] [7]. Actually, however, fast time-varying channel scenarios such that users move with high or accelerating speed make subspace tracking difficult. Furthermore, the simultaneous estimation of an appropriate forgetting factor and subspace is undesirable from the standpoint of the computational complexity.

In this work, we propose an accelerated subspace tracking algorithm by introducing the internal model principle to the PASTd, in which an external input expressed by a finite order of polynomial time function can be cancelled if a corresponding model with a same or higher order is included in the closed loop. The proposed subspace tracker can still maintain the linear computational complexity to the number of users similar to the PASTd. And a new blind multiuser detection scheme is described with the linear detectors such as the DD and MMSED in synchronous direct-sequence CDMA (DS-CDMA) scenarios employing antenna arrays. Furthermore, the effectiveness of the proposed method is shown in some numerical simulations under the situation that the spatial characteristics of the users change rapidly.

This paper is organized as follows. Section 2 briefly illustrates the data model of a DS-CDMA system and two blind linear multiuser detectors in terms of the signal subspace. The proposed subspace tracking algorithm and its convergence analysis are described in Section 3. Section 4 contains numerical simulation results, and the concluding remarks are given in Section 5.

2. DATA MODEL AND BLIND LINEAR MULTIUSER DETECTORS

2.1 DS-CDMA Data Model

Consider a synchronous DS-CDMA system of K simultaneous users. The discrete baseband signal at the mth antenna element within a symbol interval T is expressed as [6]

$$\mathbf{x}_m(n) = \sum_{k=1}^{K} A_k(n) b_k(n) \mathbf{s}_k a_m(\theta_k(n)) + \xi_m(n), \tag{1}$$

where $A_k(n)$, $b_k(n)$ and $\mathbf{s}_k$ denote, respectively, the received amplitude, transmitted symbols of ± 1, and the normalized signal waveform vector of the kth user. $\mathbf{s}_k$ is given as $\mathbf{s}_k = (1/\sqrt{L})[c_k(0), \cdots, c_k(L-1)]^T$, where L is the processing gain and $c_k(n) \in \{\pm 1\}$ is a signature sequence. $\xi_m(n) \in \mathbf{C}^{L\times 1}$ and $a_m(\theta_k(n))$ denote additive white Gaussian noise (AWGN) and the array steering vector, consisting of the direction-of-arrival (DOA) $\theta_k(n)$ of the kth user, respectively. The receiver is assumed M-element array and the structure of the steering vector $\mathbf{a}(\theta_k) = [a_1(\theta_k), \cdots, a_M(\theta_k)]^T$ depends on array geometry. From (1), the output of M-element antenna array can be vectorized as

$$\mathbf{x}(n) = \sum_{k=1}^{K} A_k(n) b_k(n) \tilde{\mathbf{s}}_k(n) + \xi(n), \tag{2}$$

where $\mathbf{x}(n) = [\mathbf{x}_1^T(n), \cdots, \mathbf{x}_M^T(n)]^T \in \mathbf{C}^{ML\times 1}$, the so-called spatio-temporal signature $\tilde{\mathbf{s}}_k(n) \in \mathbf{C}^{ML\times 1}$ is given by Kronecker product such as $\tilde{\mathbf{s}}_k(n) = \mathbf{a}(\theta_k(n)) \otimes \mathbf{s}_k$ [7], and $\xi(n) = [\xi_1^T(n), \cdots, \xi_M^T(n)]^T$ whose covariance matrix is $\sigma_\xi^2 \mathbf{I}_{ML}$.

To get the spatial property of the received signal, the singular value decomposition (SVD) of the covariance matrix, $\mathbf{R}(n) = E[\mathbf{x}(n)\mathbf{x}^H(n)]$ is obtained as

$$\mathbf{R}(n) = [\ \mathbf{U}_s(n) \ \ \mathbf{U}_n(n)\ ] \begin{bmatrix} \mathbf{\Lambda}_s(n) & \mathbf{0} \\ \mathbf{0} & \sigma_\xi^2 \mathbf{I}_{ML-K} \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^H(n) \\ \mathbf{U}_n^H(n) \end{bmatrix}, \tag{3}$$

where $\mathbf{U}_s(n) \in \mathbf{C}^{ML\times K}$ and $\mathbf{U}_n(n) \in \mathbf{C}^{ML\times(ML-K)}$ denote the signal and noise subspaces, respectively, and $\mathbf{\Lambda}_s(n) \in \mathbf{C}^{K\times K}$ contains the K largest eivenvalues of $\mathbf{R}(n)$.

2.2 Blind Linear Multiuser Detectors Using Subspace

A linear multiuser detector for demodulating the kth user's data bit in (2) is obtained as a correlator $\mathbf{h}_k(n) \in \mathbf{C}^{ML\times 1}$ followed by a hard limiter. For instance in BPSK systems, the symbol detection is given as [1]

$$\hat{b}_k(n) = \mathrm{sgn}(\Re\{\mathbf{h}_k^H(n)\mathbf{x}(n)\}), \tag{4}$$

where sgn$[\cdot]$ and $\Re[\cdot]$ denote the sign function and real part, respectively. In this section, we briefly introduce two kinds of blind multiuser detectors using subspace information, which have been reviewed in plenty of works (for instance, in [1]-[4] [6] [7]).

2.2.1 Decorrelating Detector (DD)

The DD is formulated so as to completely eliminate the multiple-access interference (MAI) by the other users [4]. The estimate of the weight vector $\mathbf{w}_k(n) = \mathbf{d}_k(n)$ is given in terms of the signal subspace parameters as

$$\hat{\mathbf{d}}_k(n) = \frac{1}{\tilde{\mathbf{s}}_k^H(n)\hat{\mathbf{U}}_s(n)(\hat{\mathbf{\Lambda}}_s(n)-\sigma_\xi^2\mathbf{I}_K)^{-1}\hat{\mathbf{U}}_s^H(n)\tilde{\mathbf{s}}_k(n)} \times\hat{\mathbf{U}}_s(n)(\hat{\mathbf{\Lambda}}_s(n)-\sigma_\xi^2\mathbf{I}_K)^{-1}\hat{\mathbf{U}}_s^H(n)\tilde{\mathbf{s}}_k(n), \tag{5}$$

where $\tilde{\mathbf{s}}_k(n)$ is known since we assume $\mathbf{a}(\theta_k(n))$ is known. The steering vector $\mathbf{a}(\theta_k(n))$ can be also estimated [7] even if it is unknown.

2.2.2 Minimum Mean-Square Error Detector (MMSED)

In the MMSED, the weight vector is formulated to minimize the mean-square error between the symbol of the user of interest and receiver output [4]. The MMSED in terms of the signal subspace parameters is expressed as

$$\hat{\mathbf{m}}_k(n) = \frac{1}{\tilde{\mathbf{s}}_k^H(n)\hat{\mathbf{U}}_s(n)\hat{\mathbf{\Lambda}}^{-1}(n)\hat{\mathbf{U}}_s^H(n)\tilde{\mathbf{s}}_k(n)} \times\hat{\mathbf{U}}_s(n)\hat{\mathbf{\Lambda}}^{-1}(n)\hat{\mathbf{U}}_s^H(n)\tilde{\mathbf{s}}_k(n). \tag{6}$$

3. ACCELERATED SUBSPACE TRACKER

In order to implement the blind linear multiuser detectors described in the previous section, the estimates of signal subspace parameters, $\hat{\mathbf{U}}_s(n)$ and $\hat{\mathbf{\Lambda}}_s(n)$ in (5) and (6), should be obtained adaptively. We review the PAST and PASTd algorithms [5], and derive an accelerated subspace tracker based on the PASTd algorithm in this section.

3.1 PAST and PASTd Algorithms

In the PAST algorithm [5], the columns of $\mathbf{W}(n)$ which span the signal subspace consist in minimizing the following cost function

$$J(\mathbf{W}(n)) = E\left\{\|\mathbf{x}(n)-\mathbf{W}(n)\mathbf{y}(n)\|^2\right\}, \tag{7}$$

where $\mathbf{x}(n)$ is a noisy data vector described in (2), and $\mathbf{y}(n) = \mathbf{W}^H(n-1)\mathbf{x}(n)$. The solution to minimize (7) is given by $\hat{\mathbf{W}}(n) = \hat{\mathbf{U}}_s(n)\mathbf{Q}$, where $\hat{\mathbf{U}}_s$ is the estimated matrix of signal subspace and $\mathbf{Q}$ is an arbitrary unitary matrix. Then as it can be shown that $\hat{\mathbf{W}}(n)\hat{\mathbf{W}}^H(n) = \hat{\mathbf{U}}_s(n)\hat{\mathbf{U}}_s^H(n)$, the signal subspace in the multiuser detectors of (5) and (6) can be obtained by using $\hat{\mathbf{W}}(n)$ instead of $\hat{\mathbf{U}}_s(n)$. In [5], Yang also proposed the PASTd algorithm which can update each column of $\hat{\mathbf{W}}(n) = [\hat{w}_1(n),\cdots,\hat{w}_K(n)]^T$ via the deflation steps in (12), as an improved PAST algorithm. The update procedure of the ith component of the signal subspace is expressd as follows:

$$y_i(n) = \hat{\mathbf{w}}_i^H(n-1)\mathbf{x}_i(n), \tag{8}$$
$$\mathbf{e}_i(n) = \hat{\mathbf{w}}_i(n-1)y_i(n)-\mathbf{x}_i(n), \tag{9}$$
$$\hat{z}_i(n) = \beta\hat{z}_i(n-1)+|y_i(n)|^2, \tag{10}$$
$$\hat{\mathbf{w}}_i(n) = \hat{\mathbf{w}}_i(n-1)-\mathbf{e}_i(n)y_i^*(n)/\hat{z}_i(n), \tag{11}$$
$$\mathbf{x}_{i+1}(n) = \mathbf{x}_i(n)-y_i(n)\hat{\mathbf{w}}_i(n), \tag{12}$$
$$\text{for } i = 1,\cdots,K$$

where $0<\beta\le 1$ is a forgetting factor and $\mathbf{x}_1(n) \stackrel{\triangle}{=} \mathbf{x}(n)$. $\hat{z}_i(n)$ corresponds to the estimate of the ith largest eigenvalue, thus $\hat{\mathbf{\Lambda}}_s(n) = \mathrm{diag}[\hat{z}_1(n),\cdots,\hat{z}_K(n)]$. The PAST and PASTd algorithms assure global convergence to the signal subspace and low computational complexity.

3.2 Proposed Method by Internal Model Principle

The proposed subspace tracker and multiuser detector is described based on the PASTd algorithm. In the proposed method, (11) and (9) are respectively modified as

$$\hat{\mathbf{w}}_i(n) = -\frac{P(z^{-1})}{(1-z^{-1})^q}\left[\frac{\mathbf{e}_i(n)y_i^*(n)}{\hat{z}_i(n)}\right], \tag{13}$$
$$\mathbf{e}_i(n) = \hat{\mathbf{w}}_i(n)y_i(n)-\mathbf{x}_i(n), \tag{14}$$

where $\mathbf{e}_i(n)$ is a *posteriori* output error. Let the polynomial $P(z^{-1})$ in the numerator be described by

$$P(z^{-1}) = p_0+p_1z^{-1}+\cdots+p_rz^{-r}, \tag{15}$$

and it is introduced to stabilize the system, where $r\le q$, $p_0\ne 0$ and z^{-1} is a time-delay operator. As the unknown time-varying parameter vector $\mathbf{w}_i(n)$ can be regarded as an external disturbance added into the closed-loop system, the effects of the unknown parameters can be cancelled out and the stability can be attained by including the internal model corresponding to the polynomial degree of time function $\mathbf{w}_i(n)$ into the parameter adjusting dynamics.

Since $\mathbf{e}_i(n)$ and $\hat{\mathbf{w}}_i(n)$ appear in the right hand side of (13) and (14) with the same instant, the error cannot be calculated from (14) directly, so $\mathbf{e}_i(n)$ should be rewritten by substituting (13) into (14), as

$$\mathbf{e}_i(n) = \frac{\{1-(1-z^{-1})^q\}[\hat{\mathbf{w}}_i(n)]y_i(n)}{1+p_0y_i^*(n)y_i(n)} - \frac{\{P(z^{-1})-p_0\}[\mathbf{e}_i(n)y_i^*(n)/\hat{z}_i(n)]y_i(n)+\mathbf{x}_i(n)}{1+p_0y_i^*(n)y_i(n)}. \tag{16}$$

Thus, the new subspace tracking algorithm is derived by combinig the PASTd with the proposed adaptive identification of time-varying FIR parameters which is based on the internal model principle. The PASTd algorithm corresponds to the case with $q=1$ in (13). The order q expresses the changing rate of the time-varying parameters. In case of $q=2$, the proposed algorithm can track changing parameters with constant velocity, and in case of $q=3$, it can track changing parameters with constant acceleration. For example, (13) and (15) in the case with $q=2$ in which the parameters change in a linear function of time can be expressed as follows:

$$\hat{\mathbf{w}}_i(n) = 2\hat{\mathbf{w}}_i(n-1)-\hat{\mathbf{w}}_i(n-2)-p_0\mathbf{e}_i(n)y_i^*(n)/\hat{z}_i(n) -p_1\mathbf{e}_i(n-1)y_i^*(n-1)/\hat{z}_i(n-1), \tag{17}$$

$$\mathbf{e}_i(n) = \frac{\{2\hat{\mathbf{w}}_i(n-1)-\hat{\mathbf{w}}_i(n-2)\}y_i(n)}{1+p_0|y_i(n)|^2} - \frac{p_1\mathbf{e}_i(n-1)y_i^*(n)y_i(n-1)/\hat{z}_i(n)+\mathbf{x}_i(n)}{1+p_0|y_i(n)|^2}. \tag{18}$$

In addition, the computational complexity of the proposed subspace tracker at each instant is $(2q+2)MLK+O(K)$, while the PASTd requires $4MLK+O(K)$ [5]. The proposed method for small q can maintain the computational simplicity of the PASTd. One drawback of the PASTd and proposed method is that the orthogonality between the columns of $\hat{\mathbf{W}}(n)$ is not assured. Some solutions have been proposed to combat the problem like in [8], and note that we can apply the orthogonalization scheme to the mentioned subspace trackers in a straightforward manner.

3.3 Convergence Analysis

In this section, we discuss the steady-state convergence of mean and mean-square behaviors to determine the appropriate coefficients $\{p_0,\cdots,p_q\}$ of the polynomial $P(z^{-1})$ in (15), regarding the proposed adaptive algorithm. This analysis is made by the similar approach to the steady-state stability of the recursive least square (RLS) [9]. Thus, it is assumed that $\mathbf{w}_i(n)$ is equivalent to the true impulse response changing in the $(q-1)$th order function of time, and activated by the AWGN $\mathbf{v}(n)$ as

$$\mathbf{w}_i(n) = \frac{1}{(1-z^{-1})^q}\mathbf{v}(n). \tag{19}$$

First we consider the steady-state mean behavior $E[\tilde{\mathbf{w}}_i(n)]$, where $\tilde{\mathbf{w}}_i(n) = \hat{\mathbf{w}}_i(n) - \mathbf{w}_i(n)$. From (8), the output error $\mathbf{e}_i(n)$ is expressed as

$$\begin{aligned} \mathbf{e}_i(n) &= \hat{\mathbf{w}}_i(n)y_i(n) - (\mathbf{w}_i(n)y_i(n) + \eta(n)) \\ &= \tilde{\mathbf{w}}_i(n)y_i(n) - \eta(n), \end{aligned} \tag{20}$$

where $\eta(n)$ is also the AWGN. In addition to these assumptions, the following properties should be exploited

$$E[\hat{z}_i(n)] = E[\sum_{t=1}^{n}\beta^{n-t}\mathbf{x}_i(n)\mathbf{x}_i^H(n)]_i \approx (1-\beta)/\sigma_x^2, \tag{21}$$

$$E[y_i^*(n)y_i(n)] = E[\mathbf{x}_i^H(n)\hat{\mathbf{w}}_i(n)\hat{\mathbf{w}}_i^H(n)\mathbf{x}_i(n)] = \sigma_x^2, \tag{22}$$

where σ_x^2 denotes the variance of $\mathbf{x}_i(n)$. Thus, $E[\tilde{\mathbf{w}}_i(n)]$ can be obtained from (17) in the case of $q = 2$ as

$$\begin{aligned} &\{1 + p_0(1-\beta)\}\cdot E[\tilde{\mathbf{w}}_i(n)] \\ &= \{2 - p_1(1-\beta)\}\cdot E[\tilde{\mathbf{w}}_i(n-1)] - E[\tilde{\mathbf{w}}_i(n-2)], \end{aligned} \tag{23}$$

where $E[\mathbf{v}(n)] = E[\eta(n)] = 0$. Furthermore, (23) can be modified into the following state variable expression as

$$\begin{bmatrix} E[\tilde{\mathbf{w}}_i(n)] \\ E[\tilde{\mathbf{w}}_i(n-1)] \end{bmatrix} = \mathbf{\Phi}\begin{bmatrix} E[\tilde{\mathbf{w}}_i(n-1)] \\ E[\tilde{\mathbf{w}}_i(n-2)] \end{bmatrix}, \tag{24}$$

where $\mathbf{\Phi}$ is a block matrix consisting of the coefficients of (23). The stability condition of the mean behavior is obtained such that the poles of $\mathbf{\Phi}$ are all within a unit circle, by

$$p_0 > 0 \quad , \quad -p_0 < p_1 < p_0 + 4/(1-\beta). \tag{25}$$

Next, a commonly used method for tracking assessment is the mean-square derivation to evaluate the estimation error between the actual weight vector $\mathbf{w}(n)$ of the unknown dynamic system and the estimated weight vector $\hat{\mathbf{h}}(n)$ of the adaptive filter [9]. Let

$$\mathbf{K}_l(n) \triangleq E[\tilde{\mathbf{w}}(n)\tilde{\mathbf{w}}^H(n-l)] \tag{26}$$

denote the covariance matrix of weight error vectors. We discuss the condition to minimize the mean-square error, $\mathrm{tr}\{E\left[\tilde{\mathbf{w}}_i(n)\tilde{\mathbf{w}}_i^H(n)\right]\}$ where $\mathrm{tr}\{\cdot\}$ denotes the trace, under the assumption that the convergence is assured in (25). We assume that the process noise vector $\mathbf{v}(n)$ is independent of both $\mathbf{x}_i(n)$ and $\eta(n)$, and $\mathbf{x}_i(n)$ and $\eta(n)$ are also independent of each other. The following relations of the correlation matrices $\mathbf{K}_l(n)$ are obtained from (17) and (19) to (22), as

$$\begin{aligned} (1+\bar{p}_0)\mathbf{K}_0 &- (2-\bar{p}_1)\mathbf{K}_1 + \mathbf{K}_2 \\ &= \frac{1}{1+\bar{p}_0}\left[(\bar{p}_0^2+\bar{p}_1^2)\frac{\sigma_v^2}{\sigma_u^2} + \sigma_\omega^2\right]\mathbf{I}_M, \end{aligned} \tag{27}$$

$$\begin{aligned} (1+\bar{p}_0)\mathbf{K}_1 &- (2-\bar{p}_1)\mathbf{K}_0 + \mathbf{K}_1 \\ &= \frac{1}{1+\bar{p}_0}\bar{p}_0\bar{p}_1\frac{\sigma_v^2}{\sigma_u^2}\mathbf{I}_M, \end{aligned} \tag{28}$$

$$(1+\bar{p}_0)\mathbf{K}_2 - (2-\bar{p}_1)\mathbf{K}_1 + \mathbf{K}_0 = \mathbf{O} \tag{29}$$

where $\bar{p}_0 \triangleq (1-\beta)p_0$ and $\bar{p}_1 \triangleq (1-\beta)p_1$, and σ_η^2 and σ_v^2 are the variance of $\eta(n)$ and $\mathbf{v}(n)$, respectively. By taking the trace of the both sides in (27) to (29), and eliminating $\mathrm{tr}\{\mathbf{K}_1\}$ and $\mathrm{tr}\{\mathbf{K}_2\}$, we obtain the following cost function $g_{LMS}(\bar{p}_0,\bar{p}_1) = \mathrm{tr}\{\mathbf{K}_0\}$ to be minimized with respect to $\bar{p}_0$ and $\bar{p}_1$ since the mean-square derivation $\mathrm{tr}\{\mathbf{K}_0\}$ should be small for a good tracking performance.

$$\begin{aligned} g_{LMS}(p_0,p_1) &= \frac{M\cdot\sigma_u^2/\sigma_v^2}{\bar{p}_0(\bar{p}_0+\bar{p}_1)(\bar{p}_0-\bar{p}_1+4)} \\ &\times\left[\frac{\bar{p}_0^2}{1+\bar{p}_0}\bar{p}_1(2-\bar{p}_1) + (2+\bar{p}_1)(\bar{p}_0^2+\bar{p}_1^2+\eta)\right], \end{aligned} \tag{30}$$

where we define $\eta \triangleq \sigma_u^2\sigma_\omega^2/\sigma_v^2$. Then taking the derivations, $\partial g/\partial\bar{p}_0 = \partial g/\partial\bar{p}_1 = 0$, we can give the condition on the parameters attaining the minimum mean-square behavior as

$$2\bar{p}_1(2\bar{p}_0+\bar{p}_1) \approx \eta(\bar{p}_0+\bar{p}_1-2). \tag{31}$$

Note that $\bar{p}_0 = (1-\beta)p_0$ and $\bar{p}_1 = (1-\beta)p_1$ are small values since a forgetting factor β is very close to 1. Thus, the p_0 and p_1 to meet the conditioin of both (25) and (31) can attain the convergence in the case ot $q = 2$. The analysis in the cases of $q \geq 3$ can be also derived in a same manner. Note that the number of the estimates does not increase by determining these coefficients in advance.

4. NUMERICAL SIMULATIONS

To describe the effectiveness of the proposed subspace tracker, we conduct some numerical simulations of the blind multiuser detection. In the simulation scenarios, we consider a synchronous DS-CDMA system with four users ($K = 4$) and three-element uniform linear array (ULA, and $M = 3$) with half wavelength interspacing, and binary Gold codes of the processing gain $L = 7$ are employed as spreading code. The source signals are modulated by BPSK through Rayleigh fading channel independent of each user and the total sample number is set at $N = 1022$. The user of interest is assumed $k = 1$ without loss of generality. The SNR of AWGN is set at 20dB to the output of the user 1, and $A_k^2/A_1^2 = 10$, for $k = 2,3,4$. We compare the performance of the MMSED given by (6) using the proposed subspace tracking algorithm and PASTd [5] [6]. The performance measure is the averaged output signal-to-interference-plus-noise ratio (SINR), defined as

$$SINR(n) = \frac{1}{N_0}\sum_{n_0=1}^{N_0}\frac{\|\hat{\mathbf{m}}_1^H(n)\tilde{\mathbf{s}}_1(n)\|^2}{\sum_{k=2}^{4}\|\hat{\mathbf{m}}_1^H(n)\tilde{\mathbf{s}}_k(n)\|^2 + \sigma_\eta^2\|\hat{\mathbf{m}}_1^H(n)\|^2},$$

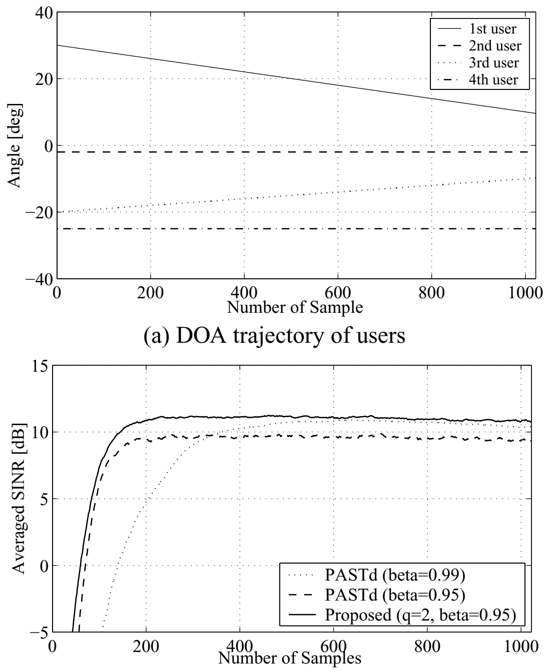
where N_0 denotes the number of trials set at $N_0 = 100$ through the simulations. Assume the orthogonality between the columns of the updated signal subspace is ensured through the orthogonalization method proposed in [8], and we discuss only the tracking ability of the conventional and proposed algorithms through the numerical simulation. The initial values of the parameters are empirically set as follows,

$$\begin{aligned} \hat{\mathbf{W}}(-1) &= \begin{bmatrix} \mathbf{I}_K \\ \mathbf{0}_{(ML-K)\times K} \end{bmatrix}\times 0.3, \\ \hat{\mathbf{W}}(0) &= \begin{bmatrix} \mathbf{I}_K \\ \mathbf{0}_{(ML-K)\times K} \end{bmatrix}\times 0.2, \\ \mathbf{e}(0) &= [1,0,\cdots,0]^T \in \Re^{ML\times 1}, \\ \hat{\mathbf{\Gamma}}(0) &= 100\cdot\mathbf{I}_K,\ \mathbf{y}(0) = [1,\cdots,1]^T \in \Re^{K\times 1}. \end{aligned}$$

We consider the following two simulation scenarios: *slowly* and *rapidly* time-varying cases. Note that we assume the number of users K is known.

4.1 Slowly Time-varying Scenario

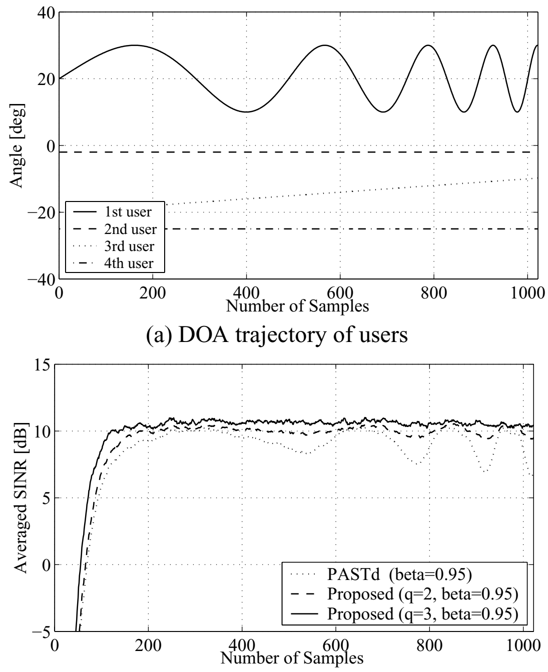
Fig.1 shows the performance comparison of the output SINR in the case that the trajectory of each user's DOA is described as Fig.1(a).



(a) DOA trajectory of users

(b) Performance of SINR

Figure 1: Performance comparison of SINR between the proposed method and PASTd in *slowly* time-varying scenario.



(a) DOA trajectory of users

(b) Performance of SINR

Figure 2: Performance comparison of SINR between the proposed method and PASTd in *rapidly* time-varying scenario.

A forgetting factor is set at $\beta = 0.95$ and 0.99 in the PASTd, and $\beta = 0.95$ in the proposed method with $q = 2$. The coefficients of the polynomial in (13) are chosen as $p_0 = 2.0$ and $p_1 = -1.95$ to meet the convergence conditions described in Section 3.3. From Fig.1(b), the proposed algorithm can detect the user of interest accurately, while the PASTd imposes compromise between high SINR and fast convergence depending on the choice of β.

4.2 Rapidly Time-varying Scenario

Next we evaluate the behavior in a rapidly time-varying case as shown in Fig.2(a). Fig.2(b) illustrates the performance of the PASTd and proposed method with $q = 2$ and $q = 3$ $(p_0 = 1.0, p_1 = 10.0, p_2 = -8.0)$. The proposed method with larger q can detect the user of interest more precisely, even if the users rapidly move, while the PASTd is not able to. On the other hand, the algorithm with $q = 3$ shows a noisy performance compared to $q = 2$, since the forepast estimate errors propagate to the present estimation in (13) and (16). This implies $q = 2$ or 3 is almost sufficient to achieve accurate and stable detection as well as computational efficiency, although the sinusoidal time function theoretically requires an infinite order of the internal model ($q = \infty$).

5. CONCLUSION

A multiuser detection scheme in rapidly time-varying scenarios has been presented by introducing an accelerated subspace tracker based on the internal model principle by approximately expressing the parameter changes in terms of finite order of polynomial time functions. One feature is that the number of estimated parameters does not increase even if a higher-order internal model is adopted to track users. Through some numerical simulations, we can observe that the proposed detector improves the SINR more than up to 3dB, compared to the PASTd algorithm [5].

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