SINGLE-CARRIER TRANSMISSION WITH ITERATIVE FREQUENCY-DOMAIN DECISION-FEEDBACK EQUALIZATION

Frédérique Sainte-Agathe[(1),(2)] and Hikmet Sari[(1)]

(1) Supélec, Plateau de Moulon, 3 rue Joliot-Curie, F-91192 Gif sur Yvette, France
(2) Thales Land & Joint Systems, 160 boulevard de Valmy, F-92704 Colombes, France
Phone: +33 1 69 85 14 31, Fax: +33 1 69 85 14 69, E-mail: hikmet.sari@supelec.fr

ABSTRACT

This paper addresses single-carrier transmission with frequency-domain equalization (SCT/FDE), which is an alternative to orthogonal frequency-division multiplexing (OFDM). We describe a new iterative frequency-domain decision-feedback equalizer (DFE) with reduced complexity compared to previously published schemes. The simulation results using two channel models indicate that the iterative DFE converges in 3 – 4 iterations and significantly improves performance with respect to linear minimum mean-square error (MMSE) and zero-forcing (ZF) equalizers.

1. INTRODUCTION

Over the past decade, orthogonal frequency-division multiplexing (OFDM) has become very popular in wireless communications. After its adoption in the recently developed wireless local area network (LAN) and broadband wireless access (BWA) standards ([1], [2]), this technique is seen today as a strong candidate for future generations of cellular mobile networks. Although many international standards are purely based on OFDM, the IEEE 802.16 specifications for BWA also include a single-carrier transmission (SCT) mode, which can efficiently compete with OFDM provided it uses frequency-domain equalization (SCT/FDE). This technique was originally proposed in [3] and [4] as an alternative to OFDM, in particular to ease the peak-to-average power ratio (PAPR) and the carrier synchronization problems.

The comparison of OFDM with SCT has been a long standing controversial issue, and virtually any conclusions can be drawn depending on the particular scenarios used in the comparisons. Overall, OFDM requires error-correction coding or some form of precoding to operate on multipath channels, whereas SCT can operate without any coding. Therefore, SCT will work better than OFDM if there is no coding or little coding in the transmitted signal frames, and OFDM may surpass SCT if strong channel coding is used.

The original papers on SCT/FDE considered linear equalizers optimized under the minimum mean-square error (MMSE) or the zero-forcing (ZF) criterion. Obviously, linear equalizers have serious performance limitations on highly distorted channels, and it is highly desirable to use nonlinear equalizers, for instance a decision-feedback equalizer (DFE) instead. Such attempts were made in [5] and [6], but the feedback part of the DFE was kept in the time domain. The number of feedback coefficients in this approach must be kept small, and this limits the performance improvement with respect to linear equalizers. Further work on the subject considered a DFE structure that is fully in the frequency domain and the recently introduced block DFE structure [7] was extended to the frequency domain in [8].

In the present paper, we introduce a new iterative frequency-domain DFE structure and investigate its performance. With respect to the iterative DFE structure described in [7] and [8], the proposed receiver has significantly reduced complexity, because it does not require the computation of the correlation coefficients of the symbol decisions at intermediate iterations.

The paper is organized as follows: In Section 2, we give a brief review of OFDM and SCT/FDE. In Section 3 we describe the new iterative DFE structure. Section 4 reports some computer simulation results to assess the performance of the proposed receiver and compare it to the scheme described in [8]. Finally, we give our conclusions in Section 5.

2. A BRIEF REVIEW OF OFDM AND SCT/FDE

A. OFDM

In an OFDM system with N carriers, the input data stream is partitioned into N-symbol blocks and the resulting blocks are passed to an N-point inverse DFT operator. The inverse DFT output is serially converted and a cyclic prefix is inserted between consecutive blocks before subsequent filtering and modulation operations. At the receiver side, the cyclic prefix is dropped and the resulting signal is passed to an N-point forward DFT operator, which converts the signal back to the frequency domain.

An OFDM system with N carriers thus splits the channel bandwidth into N subchannels and transmits the N symbols of each data block at separate carriers. When N is sufficiently large, each subchannel becomes a flat-fading channel, and channel equalization reduces to a complex multiplication per carrier. The N signal samples at the DFT output during the kth OFDM symbol are:

$$R_n(k) = H_n(k)a_n(k) + w_n(k), n = 1,2,...., N \tag{1}$$

where $H_n(k)$ is the channel transfer function at the nth carrier frequency during the kth OFDM symbol period, and $a_n(k)$ and $w_n(k)$ are the data symbols and the additive noise, respectively. In the sequel, the symbol index k will be dropped for convenience.

From (1), it is clear that OFDM needs only a complex multiplier bank at the DFT output to completely eliminate channel distortion. Denoting by (C_1, C_2, ..., C_N) the set of the multiplier bank coefficients, their values which invert the channel transfer function are given by

$$C_n = \frac{1}{H_n}, n = 1,2,....., N. \tag{2}$$

This is the optimum solution in the absence of additive noise. Since it perfectly equalizes the channel frequency response, it is equivalent to forcing to zero the intersymbol interference (ISI) in the time domain, and therefore it is called ZF equalization.

As is well known, the ZF equalizer suffers from strong noise enhancement on channels with high frequency selectivity. To limit noise enhancement and make a trade off between ISI and noise power, MMSE equalization is usually preferred. Using the MMSE criterion, the optimum coefficients are given by

$$D_n = \frac{H_n^*}{|H_n|^2 + \sigma_w^2 / \sigma_a^2}, n = 1,2,....., N. \tag{3}$$

where the σ_w^2 is the additive noise power and σ_a^2 is the power of the transmitted symbols.

Assuming a ZF equalizer, the threshold detector input at the kth carrier frequency is

$$Y_k = R_k / H_k = a_k + w_k / H_k. \tag{4}$$

The corresponding SNR at the receiver can be written as

$$SNR_k = \sigma_a^2 / (\sigma_w^2 / |H_k|^2) = \sigma_a^2 / \sigma_w^2 |H_k|^2. \tag{5}$$

The bit error rate (BER) being a decreasing function of the SNR, it will be different for symbol streams transmitted at different carrier frequencies. After averaging over the N carriers, the BER of an uncoded OFDM system with QPSK constellation is found to be:

$$BER = \frac{1}{N} \sum_{k=1}^{N} Q(\sqrt{\frac{\sigma_a^2 |H_k|^2}{\sigma_w^2}}). \tag{6}$$

This expression shows that the overall performance of an uncoded OFDM system over a multipath channel is dictated by that of the symbols transmitted at highly attenuated carriers. This is not surprising, because OFDM transmits individual data symbols in a small fraction of the channel bandwidth and as such it destroys the available frequency diversity. The classical approach to overcome this problem is to introduce some form of error correction coding, which restores a large part of the original diversity and makes the system robust to multipath propagation. All of the current industry standards which use OFDM do actually use error correction coding.

Another variant of OFDM that is suitable for multipath channels is the so-called precoded OFDM, where a matrix transformation is made before the inverse DFT at the transmitter [9]. This transformation recovers the loss of frequency diversity that is inherent to the inverse DFT operator and it can also create diversity in the time domain. If we consider a particular type of precoded OFDM, where the original data symbols are spread across the channel bandwidth using Walsh-Hadamard (WH) sequences of length N, we actually get the *point-to-point* version of multi-carrier CDMA (MC-CDMA) with frequency-domain spreading. This operation spreads the symbol energy over the total signal bandwidth resembling single-carrier transmission.

B. SCT/FDE

Single-carrier transmission is the conventional approach to digital communications. With time-domain equalization (TDE), this technique has been used for decades on time-dispersive channels. Despite this, there was a widely shared perception within the digital broadcasting community in the early 1980' that single-carrier transmission would not work for mobile reception and OFDM was viewed as the only realistic possibility for this application.

Then, in [3], [4] and some subsequent papers, H. Sari *et al.* proposed SCT/FDE as an alternative to OFDM and showed that this technique can achieve the performance of OFDM while avoiding its main drawbacks which are its high peak-to-average power ratio (PAPR) and the necessity of local oscillators with significantly reduced phase noise for carrier synchronization. Subsequent work by other authors led to similar conclusions, and SCT was recently adopted in the IEEE 802.16 specifications as one of the modes of operations of broadband wireless access (BWA) systems operating at frequencies between 2 and 11 GHz.

A simple block diagram illustrating OFDM and SCT/FDE is given in Fig. 1. As can be seen from this figure, both systems involve one forward discrete Fourier transform (DFT) and one inverse DFT (IDFT), but unlike OFDM, both operators are at the receiver side in SCT/FDE.

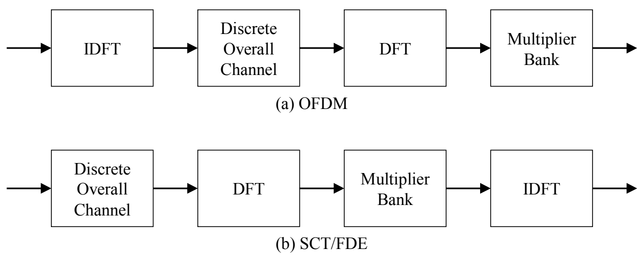

Fig. 1. Transmit and receive block diagrams of OFDM and SCT/FDE.

Furthermore, like OFDM, the SCT/FDE system proposed in [3] and [4] employs a cyclic prefix so as to make the linear convolution of the channel look like the circular convolution performed by the frequency-domain equalizer. Since symbols in SCT/FDE are transmitted over the entire channel bandwidth, this technique can operate without channel coding. What is needed here is to equalize the channel and compensate for ISI.

In order to improve the performance of SCT/FDE, it was proposed in [5] and [6] to use a DFE with time-domain feedback. Although this approach gives some performance enhancement compared to linear equalization, the resulting improvement is limited, because the feedback part has only a small number of coefficients and can only compensate for causal interference. The original papers on the subject restricted their analysis to linear ZF and MMSE equalizers.

More recently, DFE schemes were proposed for SCT/FDE, where both the feedforward and the feedback parts of the equalizer are implemented in the frequency domain [8]. Furthermore, the DFE was made iterative by using the decision block of the previous iteration to compute a new equalizer output. To describe this DFE structure, suppose that (a_1, a_2, ..., a_N) is a symbol block and that (r_1, r_2, ..., r_N) is the corresponding received signal block. The received block is fed to the DFT operator, whose output block is denoted (R_1, R_2, ..., R_N). The equalizer multiplies this signal block with its feedforward coefficients (F_1, F_2, ..., F_N), and the resulting signal block enters an inverse DFT, which yields the output block (y_1, y_2, ..., y_N) on which the threshold detector bases its first decisions for the transmitted signal block.

Once the receiver makes a first set of decisions, the decision block is fed to a feedback filter with coefficients (B_1, B_2, ..., B_N), and an iterative DFE is implemented. At the *k*th iteration, the feedforward and feedback filter block supplies

$$Y_n = \sum_{n=1}^{N} F_n^{(k)} R_n + \sum_{n=1}^{N} B_n^{(k)} A_n^{(k-1)}, n = 1,2,...., N \tag{7}$$

where the $F_n^{(k)}$ and $B_n^{(k)}$ coefficient sets are respectively the feedforward and feedback filter coefficients at the *k*th iteration, and the $A_n^{(k-1)}$ are the frequency-domain decisions at the previous iteration.

The iterative DFE of [8] is optimized under the MMSE criterion. Derivation of the feedforward and feedback filter coefficients requires the computation of the correlation between the transmitted data vector and the decisions from the previous iteration, which is rather involved. In the next section, we derive a simpler frequency-domain iterative DFE, which gives comparable performance results.

3. THE NEW EQUALIZER

The basic idea behind the proposed iterative DFE is to use (after convergence) a matched filter (MF) as the feedforward filter so as to maximize the SNR, and then to restore the ideal channel frequency response by the feedback filter. Obviously, the MF cannot be used as the feedforward filter until a set of decisions is available, because it increases ISI, and compensation of this requires a feedback filter. The second idea used to derive the proposed DFE is to shift smoothly from a feedforward filter optimized under the MMSE (or ZF) criterion to the MF over a few iterations. To describe this equalizer, we assume that the feedforward filter shifts linearly from the MMSE filter at the first iteration to the MF at the last iteration.

At the *k*th iteration, the feedforward and the feedback filter coefficients are respectively given by

$$F_n^{(k)} = \alpha_k \frac{H_n^*}{|H_n|^2 + \sigma_w^2 / \sigma_a^2} + (1 - \alpha_k) H_n^* \tag{8}$$

and

$$B_n^{(k)} = 1 - F_n^{(k)} H_n \tag{9}$$

for $n = 1, 2, ..., N$. The first equalizer decisions are obtained using $\alpha_0 = 1$, i.e., the equalizer is a linear MMSE equalizer. Then, the α parameter decreases linearly as $\alpha_k = 1 - k / K$, where K is the number of iterations. At the last iteration, $\alpha_0 = 0$, and the feedforward filter is a matched filter.

At all iterations, the feedback filter is computed such that the combined channel and equalizer response is ideal (flat frequency response and linear phase) when the decisions from the previous iteration are all correct.

4. PERFORMANCE ANALYSIS

Performance of the proposed frequency-domain iterative DFE was investigated and compared to that of the iterative MMSE DFE of [8] using the quaternary phase-shift keying (QPSK) modulation and two channel models. The first channel is the so-called Proakis-B channel [10], which is characterized by the following discrete impulse response:

$$h(D) = 0.407 D^{-1} + 0.815 + 0.407 D \tag{10}$$

where D denotes unit delay of one symbol period. This channel response corresponds to a very deep fade that is difficult to compensate using a linear equalizer. The second channel is a Rayleigh fading channel with uncorrelated coefficients, which implicitly assumes an infinite interleaver. The equalizer was implemented using 256-point DFTs and the K parameter was 4, indicating that 4 iterations were made after the first set of decisions. In all of the simulations, the channel was assumed perfectly known from the receiver.

The results obtained using the *Proakis-B* channel are depicted in Fig. 2. We can see that the MMSE linear equalizer (MMSE-LE) performance is indeed very limited on this channel, and the BER remains higher than 10^{-2} in the considered range of the transmitted energy per bit to the noise spectral density ratio (E_b/N_0). After the first iteration with our DFE, the BER is reduced to 10^{-3} at the E_b/N_0 value of 16 dB, and after the second iteration, a BER of 10^{-4} is achieved with an E_b/N_0 value of 14.2 dB. The third iteration leads to a gain of 1.2 dB at this BER value, reducing the required E_b/N_0 to 12 dB. Finally, the last iteration leads to a further gain of 1.2 dB at this BER. Performance is better with the more complex MMSE DFE, but the difference between the two structures is very small after the 4th iteration.

Next, the simulation results using the Rayleigh fading channel are reported in Fig. 3. On this channel, the linear MMSE equalizer gives a BER of 10^{-4} at E_b/N_0 = *14.5 dB.* After a first iteration, the proposed DFE gains 3 dB reducing the required E_b/N_0 to *11.5 dB.* After the second iteration, it gains an additional 1.5 dB reducing the required E_b/N_0 to *10 dB.* The third iteration too gains over 1 dB, but with the last iteration, we have diminishing returns. After the last iteration, we only need an E_b/N_0 of 8.5 dB at the BER of 10^{-4}. On this channel too, the MMSE DFE gives better results, but note that the difference between the two schemes is less than half a dB after the third iteration, and it is vanishingly small after the fourth iteration.

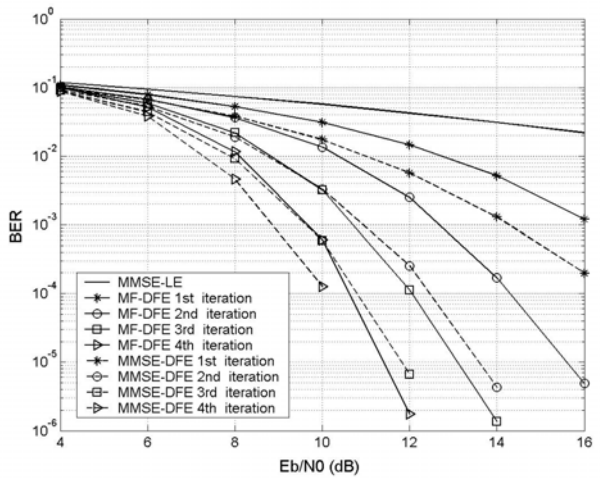

Fig. 2: Performance of the proposed iterative DFE on the Proakis-B channel.

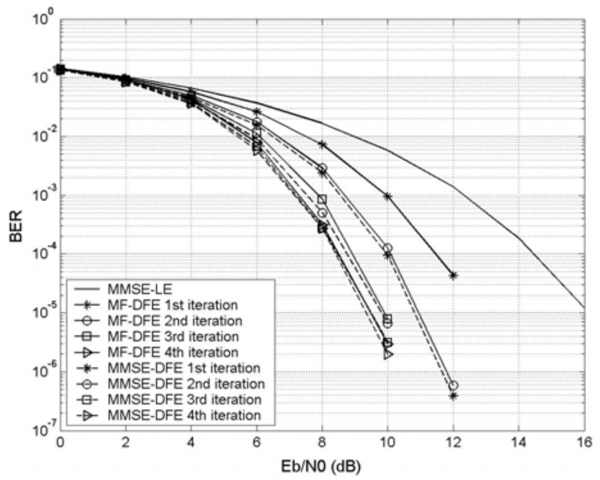

Fig. 3: Performance of the proposed iterative DFE on a Rayleigh fading channel.

5. CONCLUSIONS

We have proposed a new iterative frequency-domain DFE for single-carrier systems. The feedforward filter in this scheme is optimized under the MMSE criterion at the first iteration and shifts linearly from MMSE to reach full matched filtering at the final iteration. The idea is to maximize the SNR by the feedforward filter and to restore by the feedback filter the resulting spectrum. Using two simple channel models, the proposed structure was shown to converge in a few iterations and achieve excellent performance.

Compared to the iterative frequency-domain MMSE DFE, the proposed DFE has a significantly lower complexity. Its performance remains lower, but the difference between the two schemes is negligible after a few iterations.

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