

# GEOMETRICALLY-BASED SPACE-TIME DETERMINISTIC MULTIPATH FADING CHANNEL MODEL WITH APPLICATION TO SPATIAL CORRELATION VERIFICATION OF MULTI-ANTENNA SYSTEMS

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## ABSTRACT

This paper presents a deterministic space-time multipath Rayleigh fading channel model to study space-time correlation of the received signal waveforms to multi-antenna array receiver system. It can be considered as a modification of Lee's model, which is based on vector sum of fading waveforms, incorporating azimuth spread, time delay along the array, scatterers' location geometry and number of scatterers' considerations. Recent field measurements show a Laplacian distribution of power azimuth, suggesting that there are more scatterers or more scatterers' energy closer to the true direction of the mobile transmitter. Therefore number and location of scatterers at the vicinity of the mobile as well as power of contributed signal components should be revisited in spatio-temporally correlated fading models, which is our motivation. Simulation results show fair agreement with recent field measurements in terms of spatial correlation.

## 1. INTRODUCTION

Multiple-antenna in cellular systems have been used in mobile communication systems to provide spatial diversity. In order to evaluate its performance, we need a fading channel simulator to generate the spatiotemporally correlated fading waveforms. This calls for adequate modeling and simulation of the channel. Classically, dense scattering has been viewed as leading to a Rayleigh fading phenomenon for narrowband signals [1]. In [2] a survey of spatial channel models is given where the amplitude and delay of scattered signals were considered as well as the angle of arrival (AOA).

In macrocell environment, the base station antenna is placed at some height and the influence of scatterers close to the base station can be neglected. Because of this some proposed models consider a ring (or disk) of uniformly distributed scatterers around the mobile [3,4], in order to introduce azimuth spread (i.e. AOA). Depending on the spatial distribution of the scatterers, different probability density functions (a  $\cos^n(\varphi)$  function [5], a uniform distribution [6], and a truncated normal distribution [7]) for the azimuth distribution of the incident waves as seen from the base station (BS) have been proposed in the literature. A Gaussian distribution of power azimuth spectrum (PAS) has been consid-

ered in [8], while Kim et al. [9] proposed a spatio-temporal correlated fading model by assuming uniform distribution of power over a fixed span of azimuth, but the high order Doppler filters and the interpolators, which are essential to enhance the temporal correlation, increase computational complexity of the model. Recent field measurements in [10] show a Laplacian distribution of power azimuth suggesting the existence of more scatterers' energy closer to the true direction of the mobile transmitter. In [12], a deterministic model for spatiotemporally correlated fading generation was proposed, and the fading waveforms generated by the vector sum of the complex sinusoids with the Doppler frequencies caused by the scatterers. However, it is difficult to apply the method to the channel with arbitrary spatial correlation. It is not clear what kind of distribution of the scatterers within a cluster area that produce a uniform distribution of arrival angle, which is widely used in the literature.

In this paper we develop a spatio-temporally fading model by incorporating the azimuth of the scatterer signals into Lee's fading model [4]. This produces a set of fading waveforms which is correlated spatially given the desired azimuth spreading. By this method, the power reflected from a particular point will be a function of its distance from the transmitter. Since the circular symmetry of the scatterers about the transmitter is maintained, the appropriate Doppler spectrum and temporal correlation of the waveforms is assured. preparation, submission, and review process.

## 2. CHANNEL MODEL

In a macrocell environment, it is usually assumed that the base station (BS) antennas are set well above the city buildings with no major scatterers nearby and therefore the influence of scatterers close to BS is considered negligible. We assume the mobile station (MS) is frequently immersed in a complex plane scattering environment with no line-of-sight (LOS) between MS and BS, which is typical in an urban environment. The radius of scatterer's location,  $d_0$ , is considerably smaller than the distance between MS and BS,  $d_r$ . The transmitted signal is represented as

$$s_0(t) = a_0 \exp[j(\omega_0 t + \phi_0)], \quad (1)$$

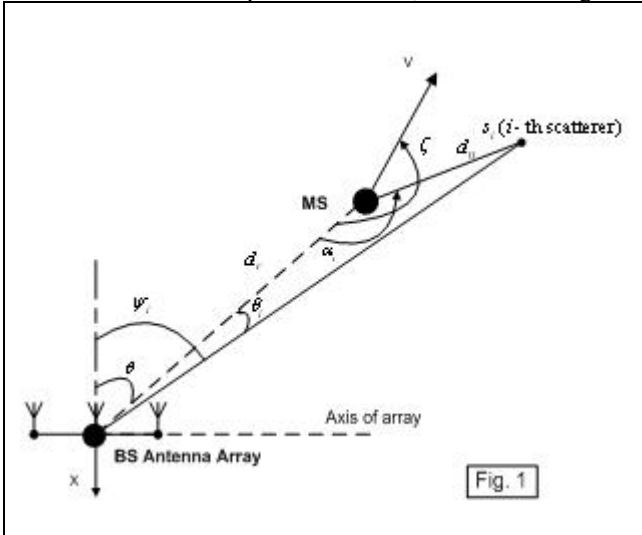
where the angular frequency  $\omega_0$  is equal to  $2\pi f_c$ , and  $f_c$  is the carrier frequency. We restrict ourselves to the case where the scatterers are standing, and the mobile station is moving with a constant velocity  $V$ . The received signal  $s(t)$  at the BS, coming from  $N$  signal paths, can be expressed as

$$s(t) = \sum_{i=1}^N A_i s_0(t - \tau_i), \quad (2)$$

where  $\tau_i$  is delay and complex value  $A_i$  is the  $i$ -th path transmission attenuation factor. We assume  $N$  discrete scatterers are placed uniformly of radial distance  $d_0$  from the mobile transmitter, where the  $i$ -th scatterer is at the angle  $\alpha_i = 2\pi(i-0.5)/N$  with a line-of-sight (LOS) between MS and BS. Then the received signal  $s(t)$  at the receiver can be expressed as

$$s(t) = s_0(t) \left( \sum_{i=1}^N A_i \exp[j2\pi(f_{d_i} t - (f_c + f_{d_i})\tau_i)] \right), \quad (3)$$

where  $f_{d_i} = f_m \cos(\alpha_i - \zeta)$ ,  $f_m (= V/\lambda)$  is the maximum Doppler frequency and  $\zeta$  is the angle of motion of the transmitter with respect to the LOS, as shown in Fig.1.



The angle  $\psi_i$  which is the azimuth of the  $i$ -th scattered signal with respect to broadside of a linear array is equal to  $\theta + \theta_i$ , where  $\theta_i$  is the angle of spreading around some nominal azimuth  $\theta$ . We note that the maximum angle spread ( $\theta_{i_{\max}}$ ) is  $(d_0/d_r)$  and  $\theta_i$  is given by  $\theta_i = \arctan[d_0 \sin(\alpha_i)/(d_r - d_0 \cos(\alpha_i))]$ . To incorporate the azimuth of each scatterer  $\psi_i$ , an array response vector (ARV) is created for each scattered signal. The ARV simply contains the phase shift appropriate for each antenna element due to the azimuth of the signal. For a uniformly spaced linear array (ULA) of  $M$  antenna elements, this is given by

$$\mathbf{V}(\psi_i) = \frac{1}{\sqrt{M}} [1 \exp[-j\delta_0 2\pi \sin \psi_i] \cdots \exp[-j(M-1)\delta_0 2\pi \sin \psi_i]]^T, \quad (4)$$

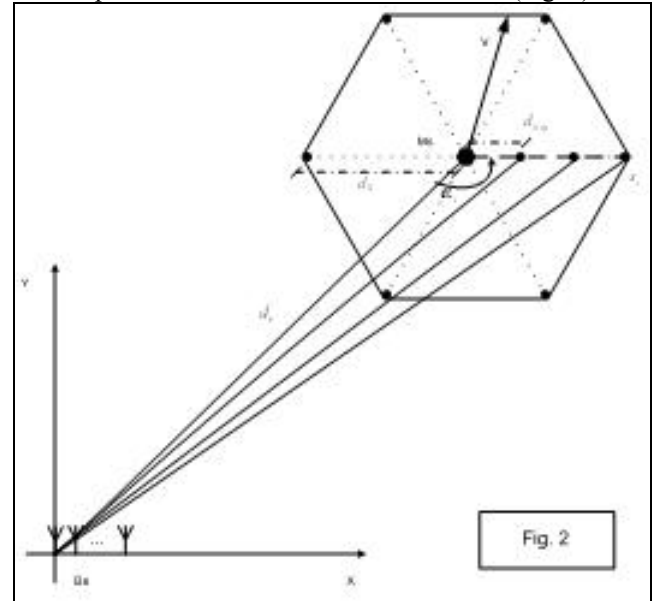
where  $\delta_0$  is the inter-element spacing of the antennas in terms of wavelengths. Each scattered signal is multiplied by its response vector and summed with the other scattered signals to form received signal as

$$\mathbf{s}(t) = \frac{1}{\sqrt{N}} s_0(t) \left( \sum_{i=1}^N A_i \exp[j2\pi(f_{d_i} t - (f_c + f_{d_i})\tau_i)] \mathbf{V}(\psi_i) \right). \quad (5)$$

Now, if we consider  $P$  scatterers on each radial line, then the received signal waveform for NP signal path reflected from NP scatterers is:

$$\mathbf{s}(t) = \frac{1}{\sqrt{N}} \frac{1}{P} s_0(t) \left( \sum_{i=1}^N \sum_{p=1}^P A_{ip} \exp[j2\pi(f_{d_i} t - (f_c + f_{d_i})\tau_{ip})] \mathbf{V}(\psi_{ip}) \right). \quad (6)$$

For all scatterers on radial line  $i$ , ( $i=1, \dots, N$ ), we consider same  $\alpha_i$  (implies the same Doppler shift) and equal angle with respect to the mobile transmitter's motion (Fig. 2).



If we consider scatterers' locations according to  $d_{0_{ip}} = d_0 (p/P)^{q_r}$ , where  $d_{0_{ip}}$  is the distance of the  $p$ -th scatterer in  $i$ -th radial to mobile node (the exponent  $q_r$  reflects the desired distribution of scatterer within the area) and include some scaling factor,  $k_p = (p/P)^{-q_p}$  as a function of distance from the transmitter ( $q_p$  is the degree to which the power reflected from a scatterer tapers off) then the reflected power from a given scatterer is no longer constant but varies as a function of the distance from the transmitter. An 'aggregate' ARV can be used for the  $p$ -th scatterer on  $i$ -th radial line as

$$\mathbf{V}(\psi_{ip}) = \frac{1}{P'} \sum_{p=1}^P \exp[j(\phi_{ip} - \phi_i)] \sqrt{k_p} \mathbf{V}(\psi_{ip}), \quad (7)$$

where  $\phi_{ip} = -2\pi(f_c + f_{d_i})\tau_{ip}$ ,  $\phi_i = \phi_{i1} = -2\pi(f_c + f_{d_i})\tau_i$ .

The factor  $P' = \sum_{p=1}^P \sqrt{k_p}$  assures normalized power in received signal waveform. It is desirable that the phase of each row of  $\mathbf{s}(t)$  be uniformly distributed. This can be accomplished by two conditions such as zero correlation and equal variance between real and imaginary part of each row of

$s(t)$ . By choosing  $A_{ip} = A_i = \exp[j4\pi i / N]$ , the above two conditions are satisfied [11]. By these considerations, the expression for received (Rayleigh fading) signal can be simplified to

$$\mathbf{s}(t) = s_0(t) \frac{1}{\sqrt{N}} \left( \sum_{i=1}^N A_i \exp[j(2\pi f_{d_i} t + \phi_i)] \mathbf{V}(\psi_i) \right). \quad (8)$$

The analytical spatial correlation function corresponding to Laplacian PAS can be derived as  $\rho(\delta) = (1 + (2\pi\delta\sigma_\theta)^2)^{-1}$ , follow in the approach in [12], where  $\sigma_\theta$  and  $\delta$  are azimuth spreading (azimuth deviation) and distance between two receiver sensors (in wavelength) respectively.

### 3. NUMERICAL RESULTS

Simulink was used to simulate the received signal waveforms given by (8) for antenna array  $M$  having 16 elements at wavelength spacing ( $\delta_0$ ) equal to 0.5. The spatial correlation between the waveform for the first antenna and the waveforms generated for each of the other antenna calculated as the time average of the product of the first antenna waveform with the complex conjugate of the waveform for each of the respective antenna. We set the simulation time to 2 second, and then waveforms of 2 second duration were generated for a maximum Doppler shift of 100 Hz. We have used sampling time of  $3.538 \times 10^{-4}$  second. This generates 5653 samples of each fading waveforms within each simulation run. In order to obtain acceptable results, we should use the number of scatterers at the vertices of polygonal cluster ( $N$ ) equal to 64 [11]. The magnitude of the averaged product is plotted versus the separation between the antennas to which the waveforms corresponded. Plotted for comparison is the magnitude of the analytic expression, taken at the appropriate separation ( $\delta$ ), maximum azimuth spread ( $\theta_{i_{\max}}$ ), and nominal azimuth ( $\theta$ ).

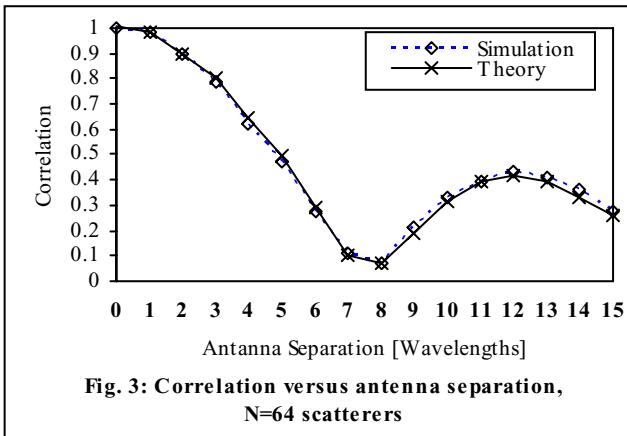


Fig. 3: Correlation versus antenna separation,  $N=64$  scatterers

Correlation as a function of antenna separation is plotted in Fig. 3 for  $N=64$  sources uniformly placed at vertices of a polygonal cluster i.e.  $q_p = 0$ ,  $q_r = 0.5$ . The distance between transmitter and scatterers ( $d_0$ ) as well as the distance between transmitter and receiver ( $d_r$ ) are varied to generate signal with  $5.7^\circ$  maximum angular spread at the receiver. It

can be seen that the spatial correlations are almost the same for different values of  $d_0$  and  $d_r$ . The analytical spatial correlation function is also plotted in Fig. 3. We observe close agreement between the simulated output and the theoretical outputs.

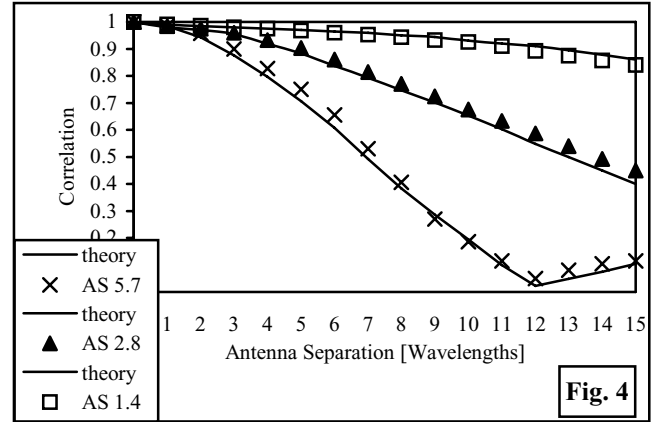


Fig. 4

In Fig. 4, the effect of angle spread on spatial correlation investigated in the case of  $N=64$  sources at vertices and  $P=20$  sources on each radial line between a source and the transmitter node. These sources are uniformly placed in the scattering cluster by considering  $q_r = 0.5$ . We consider all power scattered toward the array receiver to be evenly distributed among all the scatterers in the cluster about the mobile transmitter by considering  $q_p = 0$ . Different values of azimuth spreading considered by increasing the distance between the transmitter and receiver,  $d_r = 2000, 1000, 500$ , with a fixed value  $d_0 = 50$  for different distance between transmitter and scatterer sources ( $\theta_{i_{\max}} = 1.4^\circ, 2.8^\circ, 5.7^\circ$ ). The analytical spatial correlation function is also plotted in Fig. 4. We observe high effect of angular spread on spatial correlation (with smaller angular spread the magnitude of spatial correlation increasing to 1 and with larger angular spread the correlation falling down close to 0). Also good agreement between analytical and simulated outputs is observed.

In Fig. 5, the correlation of generated waveforms by simulation model is compared with measurements reported in [10], taken in cities Stockholm and Aarhus. In those cases, the measured PASs corresponded to parameter values of  $\sigma_\theta = 3.5^\circ, 6.5^\circ$  respectively.

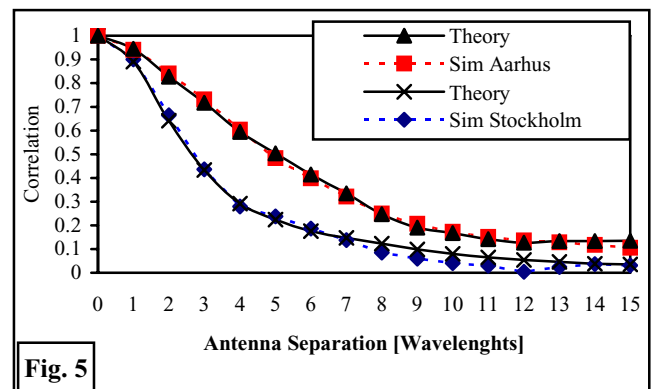


Fig. 5

#### 4. CONCLUSION

This paper presents a spatio-temporally correlated multipath Rayleigh fading model by incorporating azimuth spread in Lee's received signal waveform scattering model. The statistical properties of the model such as spatial correlation of the model-generated signals are in good agreement with theory and measurements. The model can be applied to evaluate the performance of smart antenna systems and beam forming algorithms. By using Walsh-Hadamard code-words wideband channels can be simulated.

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