LOW-COMPLEXITY METHOD FOR PAPR REDUCTION IN OFDM BASED ON FRAME EXPANSION PARAMETER SELECTION

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ABSTRACT

This paper proposes a low-complexity scheme for PAPR reduction in OFDM based on the Erasure Pattern Selection (EPS) method. EPS has been recently proposed [10] for joint BER and PAPR reduction by using frame expansion in combination with erasures in the OFDM framework. In this paper we discuss the selection of parameters in the EPS method that makes the erasure patterns tight subframes. Based on this selection we develop a low-complexity implementation of the reconstruction algorithm. We compare both complexity and PAPR performance of the proposed scheme with other probabilistic schemes. A key result presented in this paper is that the low-complexity EPS scheme can be effectively combined with existing probabilistic methods to provide improved performance. The combinations have the same complexity of the existing probabilistic methods, but simulation results show a significant improvement in PAPR reduction.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation method in which multiple symbols are transmitted in parallel using different subcarriers. Important advantages of OFDM are bandwidth efficiency, lower intersymbol interference and easier equalization [1]. However, OFDM modulation also exhibits drawbacks, the major one being a high peak-to-average power ratio (PAPR) that necessitates the usage of a highly linear amplifier [2].

The solutions proposed to alleviate this problem can be classified into three categories [2]: *block coding techniques* [3, 4], *clip effect transformations* [5], and *probabilistic solutions* [6, 7].

Probabilistic methods are most commonly employed in practice for PAPR reduction. The principle underlying these methods is to reduce the probability of high PAPR by examining alternate signal representations for blocks of transmitted information and selecting one with the lowest PAPR. Partial Transmit Sequence (PTS) and Selected Mapping (SLM) are the most well-known probabilistic methods. PTS partitions the OFDM symbol vector into V non-overlapping subvectors, each of which is multiplied by a rotation factor. The rotation factor that produces the lowest PAPR is selected for transmission [6]. SLM, like PTS, also examines different representations of the information and transmits one that has the lowest PAPR. The original OFDM signal is multiplied carrierwise with U distinct vectors composed of rotation factors to obtain U different representations [7].

In OFDM, channel coding is generally used to introduce redundancy in the data in order to combat impairments during transmission and reduce the bit-error-rate (BER). Also, in signal design for PAPR reduction, some redundancy in representation is required in order to allow the selection of one among several alternate representations so that the PAPR is low. In this paper we consider use of low-complexity methods of inserting redundancy in OFDM using frames in order to accomplish both tasks of PAPR reduction and error protection. We focus on the properties of the DFT frame in order to reduce complexity of the PAPR reduction scheme and make it suitable for real-time applications.

2. FRAME EXPANSION THEORY AND ERASURES

We briefly describe frame expansion here, restricting attention to frames in $\mathbf{R}^K$ instead of general Hilbert spaces, where $\mathbf{R}$ denotes the set of real numbers. The set of vectors is called a frame if there exist $0 < A \le B < \infty$ such that

$$A\|x\|^2 \le \sum_{k=1}^{N} |\langle x, \phi_k \rangle|^2 \le B\|x\|^2, \text{ for all } x \in \mathbf{R}^K, \quad (1)$$

where $\|x\|$ is the norm of vector x and $\langle x, y \rangle$ is the inner product between vectors x and y. A *frame* is therefore a collection of N K-dimensional vectors that satisfy equation 1. A and B are called the frame bounds, and if they coincide the frame is called *tight*. A frame introduces redundancy in the data equal to $r = N/K$. Each frame is associated with a frame matrix F whose rows are the frame vectors ϕ_k. If the frame is tight, then $F^t F = A I_K$, where I_K is the identity matrix of order K.

Frames have desirable properties for reconstruction when erasures occur [8, 9]. Since redundancy is introduced into the data, the erased samples can be recovered from the received samples. Here we assume that the location of the erased samples is known. In this case matrix F_R is formed by selecting the rows of F corresponding to the received samples. Then the original transmitted signal x can be recovered as [9]:

$$x = (F_R^h F_R)^{-1} F_R^h y_R, \quad (2)$$

where y_R is the vector composed of the R received samples.

When erasures occur in a noisy environment, the average reconstruction error depends not only on the number of erased (and received) samples, but also on their location. In [9] it was shown that if the sets of erased and received samples form tight subframes, then the mean-squared-error (MSE) is minimized for that number of received samples. Multiple subframes produce this minimum MSE, and they are called *equivalent* [9].

These properties of frame expansion will now be applied in the OFDM framework, in order to design a transmission system which is robust to the errors introduced by the channel and at the same time has lower value of PAPR of the transmitted signals.

3. FRAME EXPANSION IN OFDM FRAMEWORK

An approach to invoke frame expansion theory in OFDM has been proposed in a novel probabilistic method called Erasure Pattern Selection (EPS). Redundancy is introduced in EPS for providing both error correction and PAPR reduction. The approach consists of expanding data with a frame, followed by partially erasing the redundancy with different

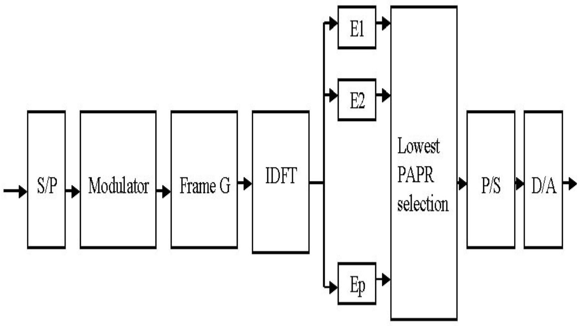

Figure 1: Transmitter for EPS scheme

erasure patterns. Multiple erasure patterns produce different signal representations, and the one with lowest PAPR is selected for transmission. The number and location of the erasures is chosen according to the properties of frame theory in order to minimize the reconstruction error. Since part of the redundancy is left in the data, it can be exploited for error correction. Therefore EPS does not need an extra channel coder. The block diagram of the EPS scheme transmitter is shown in Figure 1.

We start by converting serial data into parallel form, with K output groups of $log_2(M)$ bits. Here M is the size of the constellation used and in our simulations we use $M = 4$. Then, the K groups of bits are modulated into K symbols. The K symbols are expanded through a frame G which generates N outputs. The N-sample vector is transformed using an IDFT block. Finally E samples are erased from the OFDM symbol with an algorithm that will be explained later, and the parallel data are transformed into serial form for transmission. Side information is sent along with the data, in order to identify the erasure pattern that was selected at the transmitter. The erasure pattern selection can be viewed as a time-varying puncturing scheme.

The receiver performs the inverse of operations carried out at the transmitter. The DFT and the IDFT blocks can be represented by matrices W and its conjugate transpose W^h. The cascade of G and W^h, the matrix F, is a frame too. Since tight frames have many useful properties, it is desired that frame the F is tight. A key task in the EPS scheme is the selection of the erasure pattern. The novelty of this method is that the number and position of the erased samples are selected at the transmitter in order to minimize the mean squared reconstruction error (MSE).

4. LOW COMPLEXITY IMPLEMENTATION

4.1 DFT frames

An (N, K) DFT code is a code with generator matrix composed of any K columns of the IDFT matrix W^h. Any DFT code is a tight frame [9]. In our simulation we focus on DFT codes because by exploiting the properties of the Fourier transform the complexity of the EPS scheme can be significantly reduced. If frame F is a DFT code, then matrix G is simply an $N \times K$ matrix formed by the K columns of the identity matrix I_N with indices equal to the indices of the K selected columns of W^h that form F. Multiplying a K-element vector with this matrix G is equivalent to inserting $N - K$ zeros in the vector.

4.2 Issues in the selection of parameters

Once data are expanded with the DFT frame, multiple erasure patterns are applied and the one that produces the lowest PAPR is selected for transmission. The number and the location of the erasures are chosen according to frame theory. In [9] it is shown that in order to minimize the reconstruction error the erased and received samples need to form tight subframes, according to which $N \geq 2K$ and $K \leq E \leq N - K$.

We wish to compare the performance of EPS to other probabilistic schemes, like PTS and SLM, when they are combined with a convolutional coder. The redundancy of the coder and of the frame has to be the same. Since R samples are transmitted in EPS, with $R = N - E$, then $R/K = r$, where r is the redundancy of the convolutional coder, chosen equal to 2 in our analysis. Since $R = 2K$ and $R = N - E$, we therefore require $2K = N - E$. The minimum number of erasures is K, therefore $2K \leq N - K$, which implies $N \geq 3K$. In our simulation we choose $N = 3K$, $E = K$ and $R = 2K$.

Once the number of erasures per pattern has been selected, we need to establish the location of these erasures in each pattern and the total number of erasure patterns to be investigated. Clearly the larger the number of erasure patterns investigated, the lower the resulting PAPR, since the lowest value is selected among a larger set. However we need to consider that larger numbers of patterns require larger side information to be sent to the receiver and that not all erasure patterns produce the same BER. In our simulations $N/E = 3$, therefore there exist 3 *equivalent* erasure patterns. If we consider any number of patterns larger than 3, the extra patterns will not be equivalent to these 3 and on average they produce a larger BER. In the results section we will show simulations both for $P = 3$ and $P > 3$ erasure patterns.

In [10] we have investigated the application of frame expansion in OFDM with a low number of erasures in each pattern. Good PAPR reduction is achieved by including a projection-onto-convex set (POCS) algorithm in that scheme. Here we focus on erasure patterns that are also tight subframes and we propose a low-complexity scheme for PAPR reduction.

4.3 Low-complexity reconstruction algorithm

After the R samples are received, the receiver recovers the original K-element vector as in equation 2. If a direct implementation of equation 2 is used then the reconstruction process could be extremely time consuming, due to the matrix inversion required. Here we focus on the task of reducing complexity of the EPS scheme by exploiting the properties of frame theory and of the Fourier transform.

First, let us rewrite matrix F_R as $F_R = M_1 W^h M_2$, where M_1 is the $R \times N$ matrix obtained from the identity matrix I_N by removing the E rows corresponding to the selected erasure pattern, and M_2 is the $N \times K$ matrix obtained from the identity matrix I_N by removing the $N - K$ columns with indices equal to the unselected columns of W^h in forming F. With no loss of generality, we can assume that F is composed of the first consecutive K columns of W^h. Let us now focus on the matrix to be inverted in equation 2,

$$F_R^h F_R = M_2^T W M_1^T M_1 W^h M_2. \quad (3)$$

Matrix $M_1^T M_1$ is equal to a $N \times N$ diagonal matrix with entries along the diagonal equal to 1 in the locations corresponding to the received samples and equal to 0 in the locations corresponding to the erased samples. This matrix can also be written as $M_1^T M_1 = I_N - Q$, where matrix Q is a $N \times N$ diagonal matrix with E entries along the diagonal equal to 1 in the locations of the erased samples, and the other entries equal to 0. The locations of the non-zero entries depends on the selected erasure pattern, but in any case they are equally spaced with spacing equal to N/E. Equation 3 therefore becomes

$$F_R^h F_R = M_2^T W(I_N - Q)W^h M_2 =$$
$$= M_2^T W W^h M_2 - M_2^T W Q W^h M_2 = I_K - M_2^T W Q W^h M_2. \quad (4)$$

The first term in 4 follows from

$$M_2^T W W^h M_2 = \begin{bmatrix} I_K & O_{K-N} \end{bmatrix} \begin{bmatrix} I_N \end{bmatrix} \begin{bmatrix} I_K \\ O_{N-K} \end{bmatrix} = I_K. \quad (5)$$

Let us now focus on the last term in equation 4, starting with matrix multiplication WQ. The $ih-th$ element of the resulting matrix is

$$(WQ)_{ih} = \begin{cases} \frac{1}{\sqrt{N}} e^{-j2\pi ih/N}, & h \in S_E \\ 0, & h \in S_R. \end{cases} \quad (6)$$

Here S_E is the set of indices corresponding to the erased samples and S_R is the set of indices corresponding to the received samples. Therefore matrix WQ has the same columns of W for column indices equal to the indices of the erased samples, with the remaining columns equal to 0.

Next consider the matrix multiplication $(WQ)W^h$:

$$(WQW^h)_{ik} = \sum_{h \in S_E} \frac{e^{-j2\pi ih/N}}{\sqrt{N}} \frac{e^{j2\pi hk/N}}{\sqrt{N}} = \frac{1}{N} \sum_{h \in S_E} e^{-j2\pi h(i-k)/N} \quad (7)$$

We mentioned earlier that the indices in S_E are equally spaced with spacing equal to N/E. Therefore index h can be written as $h = \frac{N}{E}m + h_0$, with m going from 0 to $E-1$ and h_0 is the offset due to the index of the selected pattern, with $1 \leq h_0 \leq N/E$. With this change of variable, equation 7 becomes

$$(WQW^h)_{ik} = \frac{1}{N} e^{-j\frac{2\pi}{N}h_0(k-i)} \sum_{m=0}^{E-1} e^{-j\frac{2\pi}{E}m(k-i)} =$$
$$= \begin{cases} \frac{E}{N} e^{-j\frac{2\pi}{N}h_0 nE}, & k-i = nE \\ 0, & k-i \neq nE, \end{cases} \quad (8)$$

with $n = 0, 1, ..., N/E - 1$. Matrix WQW^h becomes equal to

$$WQW^h = \begin{bmatrix} c_1 I_K & c_2 I_K & \dots & c_{N/E} I_K \\ c_2 I_K & c_1 I_K & \dots & c_{N/E-1} I_K \\ \dots & \dots & \dots & \dots \\ c_{N/E} I_K & c_{N/E-1} I_K & \dots & c_1 I_K \end{bmatrix}, \quad (9)$$

with $c_i = \frac{E}{N} e^{-j\frac{2\pi}{N}Eh_0(i-1)}$. Let us now post-multiply matrix WQW^h with matrix M_2.

$$WQW^h M_2 = (WQW^h) \begin{bmatrix} I_K \\ O_{N-K} \end{bmatrix} = \begin{bmatrix} c_1 I_K \\ c_2 I_K \\ \dots \\ c_{N/E} I_K \end{bmatrix}. \quad (10)$$

Finally we pre-multiply this resulting matrix with M_2^T:

$$M_2^T WQW^h M_2 = \begin{bmatrix} I_K & O_{K-N} \end{bmatrix} \begin{bmatrix} c_1 I_K \\ c_2 I_K \\ \dots \\ c_{N/E} I_K \end{bmatrix} = c_1 I_K = \frac{E}{N} I_K. \quad (11)$$

Therefore from equation 4 the matrix to be inverted is $F_R^h F_R = I_K - \frac{E}{N} I_K = \frac{R}{N} I_K$, and its inverse is equal to $\frac{N}{R} I_K$.

So far we have shown that it is not necessary to compute the inverse matrix in the reconstruction formula. Rewriting equation 2 with the results in equation 11 we have

$$x = \frac{N}{R} F_R^h y_R = \frac{N}{R} M_2^T W M_1^T y_R. \quad (12)$$

Now, $M_1^T y_R$ is an N-element vector with entries equal to 0 for all indices included in set S_E and equal to the received samples for all indices included in set S_R. Next the DFT is computed, and finally the resulting vector is multiplied by M_2^T. This is equivalent to selecting the K "used" samples from the resulting vector. Therefore the complexity of the receiver is simply the computation of the DFT.

In comparing complexity and performance of EPS with existing schemes, we note that the IDFT and DFT in EPS are performed over N samples, whereas in other schemes they are performed over $R < N$ samples. However complexity can be reduced by noting that $N-K$ samples out of N are equal to 0, and by exploiting the properties of the FFT algorithm. Therefore the complexity of the two different DFTs are comparable.

5. RESULTS, COMPARISON AND COMBINATION WITH EXISTING SCHEMES

Before comparing the performance of the proposed implementation of EPS with the performance of existing probabilistic schemes like PTS and SLM, let us compare the complexity of the transmitter and of the receiver for the different schemes. In PTS (or SLM) V (or U) IDFTs are required at the transmitter and one DFT is required at the receiver. In EPS one IDFT is required at the transmitter and one at the receiver. Therefore EPS has lower complexity than PTS and SLM even with $V = U = 2$.

In order to compare the performance of PTS and SLM with EPS, we use a combination of PTS or SLM and a convolutional coder with redundancy equal to 2. For our simulations we chose a convolutional coder with encoder memory equal to 3. The coder does not affect the PAPR reduction, but provides error correction. On the other hand, EPS provides both error correction and PAPR reduction.

Figure 2 shows (a) the BER and (b) the complementary cumulative distribution function (CCDF) of PAPR for PTS and SLM combined with the convolutional coder and EPS with $P = 3, 8$ erasure patterns investigated. In these simulations we consider the case $N = 256$, $K = N/2$ and $R = 2N/3$. Results similar to those shown in this section are obtained for any number of subcarriers. In the computation of the PAPR oversampling is required in order to reliably estimate the peak value. In our simulations an oversampling factor of 4 is considered.

As expected PTS and SLM perform identically in BER, because the BER reduction is only due to the convolutional coder. EPS slightly outperforms the other schemes in BER reduction. Figure 2(b) shows the CCDF of PAPR for PTS and SLM with $V = U = 2$ and of EPS with $P = 3, 8$. Note that the complexity of the EPS scheme is lower than the one of the other schemes. Comparing EPS with SLM and PTS we see that EPS performs comparably to SLM but it is outperformed by PTS, since PTS computes and compares a larger number of PAPRs. Given the lower complexity of EPS, we can conclude that EPS performs better than SLM, but worse than PTS. At CCDF of 10^-3 the PAPR improvement for the EPS scheme is 1.1 dB, whereas it is equal respectively to 1.2 dB and 1.8 dB for the more complex SLM and PTS schemes.

Figure 2 also shows simulation results for EPS in the case of $P = 3$ and $P = 8$ erasure patterns investigated. In the case with $P = 8$, the erasure patterns do not have equally spaced erasure locations, therefore they do not minimize the

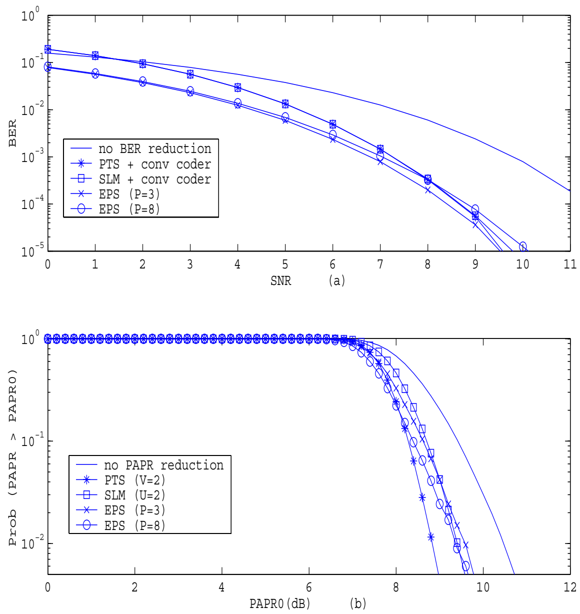

Figure 2: BER and CCDF of PAPR for EPS, PTS and SLM

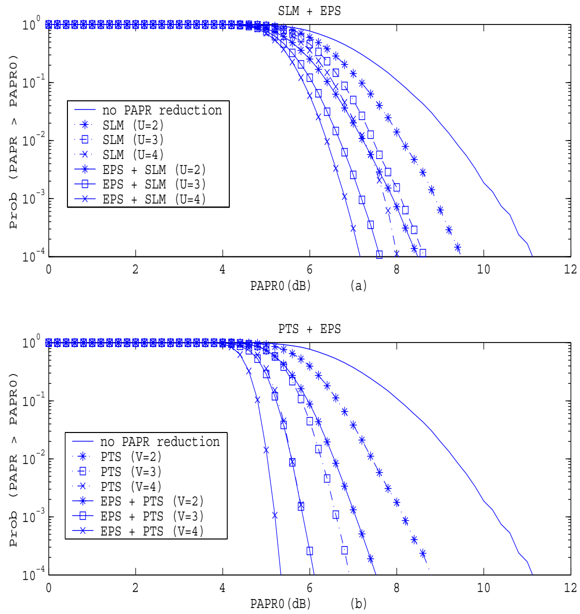

Figure 3: CCDF of PAPR for combination schemes

mean squared reconstruction error. From Figure 2 we see that the case with $P = 3$ performs better than the case with $P = 8$ in BER, and its PAPR performance is only slightly worse. Furthermore, a larger number of erasure patterns investigated would require larger side information. Therefore the case $P = 3$ is preferable.

The main result presented in this paper is the improvement in PAPR reduction achieved when the low-complexity EPS scheme is combined with existing probabilistic schemes, like PTS and SLM. First, we examine how EPS is combined with PTS. The OFDM vector is divided into V subvectors, and the IDFT of each subvector is computed. All combinations of rotation factors as well as all erasure patterns are investigated and the rotation factor-erasure pattern pair that produces the lowest PAPR is selected for transmission. The complexity of the transmitter is mainly given by the V IDFTs, while the receiver only requires one DFT. Therefore the combination EPS-PTS has the same complexity of PTS only. Similarly, when EPS is combined with SLM, the vector of rotation factors-erasure pattern pair that produces the lowest PAPR is selected for transmission, with an overall complexity comparable to conventional SLM. Figure 3 shows the CCDF of PAPR for (a) the EPS + PTS scheme and (b) the EPS + SLM scheme for V,U = 2,3 and 4 and $N = 32$. In both cases we see that the combination schemes significantly outperform conventional PTS or SLM, even though they have the same complexity.

6. CONCLUSION

In this paper we focus on reducing the computation complexity of the Erasure Pattern Selection (EPS) scheme. EPS is a probabilistic method that combines BER and PAPR reduction by exploiting the properties of frame expansion with erasures. When comparing the low-complexity EPS scheme with existing probabilistic methods like PTS and SLM we notice that EPS performs comparably to SLM, but worse than PTS. However, its lower complexity makes it more suitable for real time applications. A key result presented in this paper is that EPS can be combined with existing methods. The combination schemes have the same complexity of PTS and SLM only and simulation results show that the PAPR is significantly reduced in both cases.

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