TIME-FREQUENCY ESTIMATION IN THE COSPAS/SARSAT SYSTEM USING ANTENNA ARRAYS: VARIANCE BOUNDS AND ALGORITHMS

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ABSTRACT

This paper deals with the signal processing techniques to be applied in the reception of the Search And Rescue (SAR) system COSPAS/SARSAT distress beacons. The receiver unit has to estimate time delays and Doppler shifts of a set of satellite-relayed replicas of the original beacon in order to compute the position where the device has been activated. The Cramér-Rao Bound (CRB), which determines the minimum variance achievable for any unbiased estimator, is computed for the problem at hand considering a receiver provided with an antenna array. Finally, we propose a new sort of beamforming which exploits temporal and spatial references achieving a performance close to the CRB with a moderate implementation effort.

1. INTRODUCTION

Cospas-Sarsat is a satellite system designed to provide distress alert and location data to assist search and rescue (SAR) operations, using spacecraft and ground facilities to detect and locate the signals of distress beacons operating on 406 MHz or 121.5 MHz. This work deals with the design of ground receivers, called Local User Terminals (LUTs), considering the existing digital 406 MHz beacon and a recently proposed new signal structure. The novelty relies in the use of an antenna array in the LUT, instead of the more common (and more expensive to maintain) mechanically steerable dishes.

Emergency locator beacons are activated in emergency situations emitting a signal to the satellite constellation of the system, then this signal is retransmitted to the LUT, a ground station that is in charge of processing the signal received and determining the beacon's position using signal processing techniques, mainly Time and Frequency of Arrival (TOA and FOA) techniques. When the position is already estimated, it is passed to the Mission Control Center (MCC) which automatically sends the information to the nearest Rescue Coordination Center (RCC), that will coordinate the SAR forces to find out what had happened and rescue possible survivors.

The extension of the Cospas-Sarsat Search and Rescue system to medium earth orbit satellites, as a payload in the forthcoming Galileo satellites, and the tight power budget restrictions foreseen in this system, point to the need of antenna arrays in order to achieve the required accuracy in position estimation [1]. This paper investigates the theoretical bounds of variance of time and frequency estimators and proposes a kind of beamforming which exploits time and spatial diversity and is well–suited to the problem at hand because of system's particularities.

2. SIGNAL MODEL

This section presents a brief analysis in terms of signal structure, both for the existing beacon used in the Cospas–Sarsat system and for a recently proposed beacon.

2.1 Existing SARSAT beacon

Complete specification can be found in [2]. The period between transmissions shall be 50 s ± 5 percent. The repetition period shall not be stable to avoid burst collision between two activated beacons.

There are two types of messages that a distress beacon can send: the short and the long one. Both, which modulate a 406 MHz carrier at 400 bps $\pm$ 1 percent (in a BPSK modulation), have the same format for the message with only different lengths. The Cospas–Sarsat message is characterized for being extremely brief, the short message duration is 440 ms $\pm$ 1 percent and 520 ms $\pm$ 1 percent for the long message.

Although the emitted 406 MHz distress beacon is relayed by the satellites to the Local User Terminals at a frequency of 1544 MHz, the analysis presented hereafter is performed in baseband without loss of generality. The whole baseband signal can be expressed as the sum of three factors: the pure tone $s_{\mathrm{T}_1}(t)$, being $t_1 = 160$ ms:

$$s_{\mathrm{T}_1}(t) = A \quad \left(\frac{t-\frac{t_1}{2}}{t_1}\right) \quad \text{where} \quad \left(\frac{t}{T}\right) = \begin{cases} 1 & -\frac{T}{2} \leq t \leq \frac{T}{2}, \\ 0 & \text{otherwise.} \end{cases}$$

the preamble $s_{\mathrm{P}_1}(t)$, where $\{a_i\} = \pm 1$ is the sequence 000101111, $t_2 = 60$ ms, $\quad = \cos(1.1)$ and $\quad = sin(1.1)$:

$$s_{\mathrm{P}_1}(t) = A\left(\quad + j \quad \sum_{k=1}^{24} a_k p_{man}(t - t_1 - kT_b)\right) \quad \left(\frac{t-t_1-\frac{t_2}{2}}{t_2}\right),$$

where $p_{man} = \quad \left(\frac{t-\frac{T_b}{4}}{\frac{T_b}{2}}\right) - \quad \left(\frac{t-\frac{3T_b}{4}}{\frac{T_b}{2}}\right)$, $T_b = 2.5$ ms; and the data $s_{\mathrm{D}_1}(t)$, where $\{d_k\} = \pm 1$ is the user data:

$$s_{\mathrm{D}_1}(t) = A\left(\quad + j \quad \sum_{k=1}^{L} d_k p_{man}(t - t_1 - t_2 - kT_b)\right) \quad \left(\frac{t-t_1-t_2-\frac{t_3}{2}}{t_3}\right)$$

being $L = 88/120$, $t_3 = 220/300$ ms (short / long message), resulting on the following model:

$$s_{\mathrm{SARSAT}}(t) = s_{\mathrm{T}_1}(t) + s_{\mathrm{P}_1}(t) + s_{\mathrm{D}_1}(t)$$

The message is filtered with a signal mask in order to avoid out–of–band spurious emissions or, at least, reduce them to acceptable levels. The bandwidth of the filter mask is set to 3 kHz. This filter has been implemented following specifications in [2] with a linear–phase FIR design using least–squares error minimization criteria with 256 coefficients.

2.2 Proposed SARSAT beacon

General specification of this new beacon can be found in [3]. It also follows the structure of tone–preamble–data similar to the existing beacon but with the main difference of being QPSK modulated.

The pure tone $s_{\mathrm{T}_2}(t)$

$$s_{\mathrm{T}_2}(t) = A \quad \left(\frac{t-\frac{t_1}{2}}{t_1}\right) \qquad t_1 = 82 \text{ ms}$$

The preamble $s_{\mathrm{P}_2}(t)$, where $\{a_k\} = \pm 1$ and $\{b_k\} = \pm 1$ are the sequences of In–phase and Quadrature known preamble bits, and the pulses are defined as $p(t) = \quad \left(\frac{t-\frac{T_s}{2}}{T_s}\right)$, the Manchester pulse as $p_{man} = \quad \left(\frac{t-\frac{T_s}{4}}{\frac{T_s}{2}}\right) - \quad \left(\frac{t-\frac{3T_s}{4}}{\frac{T_s}{2}}\right)$ and $T_s = 2.5$ ms:

$$\begin{aligned} s_{\mathrm{P}_2}(t) &= A \sum_{k=1}^{15} (a_k p(t-t_1-kT_s) + jb_k p_{man}(t-t_1-kT_s)) \cdot \\ &\cdot \quad \left(\frac{t-t_1-\frac{t_2}{2}}{t_2}\right) \qquad t_2 = 37.5 \text{ ms} \end{aligned}$$

The data $s_{\mathrm{D}_2}(t)$, where $\{c_k\} = \pm 1$ and $\{d_k\} = \pm 1$ are the convolutionally encoded and scrambled Inphase and Quadrature user data:

$$\begin{aligned} s_{\mathrm{D}_2}(t) &= A \sum_{k=1}^{L} (c_k p(t-t_1-t_2-kT_s) + \\ &+ jd_k p_{man}(t-t_1-t_2-kT_s)) \quad \left(\frac{t-t_1-t_2-\frac{t_3}{2}}{t_3}\right) \end{aligned}$$

$L = 96/129$, $t_3 = 240/322.5$ ms (short / long message. In case of long message, $d_{129} = 0$)

Resulting on the following emitted signal:

$$s_{\mathrm{SARSAT2}}(t) = s_{\mathrm{T}_2}(t) + s_{\mathrm{P}_2}(t) + s_{\mathrm{D}_2}(t)$$

A 10-kHz bandwidth filter mask, wider than the existing SARSAT filter, is suggested in [3] and has been also implemented in simulations presented in section 5. As will be shown, this filter determines the accuracy achievable in synchronization, mainly in time delay estimation.

2.3 Array signal model

The problem under study concerns the extraction of information from measurements using an array of antennas. The measurements are considered to be a superposition of plane waves corrupted by noise and, eventually, interferences and multipath. Given the measurements, the objective is to estimate a set of parameters associated with the wavefronts. An N element antenna array receives M scaled, time–delayed and Doppler–shifted signals with known structure. The receiving complex baseband signal can be modeled as

$$x(t) = \sum_{i=1}^{M} a_i s(t - \ _i) \exp\{j2 \ \ f_i t\} + n(t)$$

where a_i is the complex amplitude of each signal, $s(t)$ stands for $s_{SARSAT}(t)$ or $s_{SARSAT2}(t)$, $\ _i$ is the delay, f_i is the Doppler shift, and $n(t)$ is additive white Gaussian noise. Each antenna receives a different replica of this set of signals, with a different phase depending on the array geometry and the Directions Of Arrival (DOA). This can be expressed by a vector signal model, where each row corresponds to one antenna:

$$\mathbf{x} = \mathbf{GAd} \ \ + \mathbf{n} \tag{1}$$

where

- $\mathbf{x} \in \mathbb{C}^{N \times 1}$ is the observed signal vector,
- $\mathbf{G}(\ , \ ) \in \mathbb{C}^{N \times M}$ contains in its columns the spatial signatures of the received replicas, and depends on the azimuth $\ \in \mathbb{R}^{M \times 1}$ and elevation $\ \in \mathbb{R}^{M \times 1}$ angles of the incoming signals,
- $\mathbf{A} \in \mathbb{C}^{M \times M}$ is a diagonal matrix with the elements of the amplitude vector $\mathbf{a} = [\ a_1 \ \ \dots \ \ a_M\ ]^T$ along its diagonal,
- $\mathbf{d} = [\ s(t- \ _1)e^{j2 \ f_1 t} \ \ \dots \ \ s(t- \ _M)e^{j2 \ f_M t}\ ]^T$ the delayed Doppler shifted narrowband signals envelopes, thus $\mathbf{d} \in \mathbb{C}^{M \times 1}$, and
- $\mathbf{n}(t) \in \mathbb{C}^{N \times 1}$ represents additive noise and all other disturbing terms, like multipath of each signal or interferences. This gathering, although its simplicity, captures the statistical behavior of multipath (as shown in [4]) when an arbitrary spatial covariance matrix $\mathbf{Q} = E\{\mathbf{n} \ \ ^H\}$ is considered.

This model is built upon the narrowband array assumption, consisting of taking the time required for the signal to propagate along the array as much smaller than its inverse bandwidth. Thus, a phase shift can be used to describe the propagation from one antenna to another. In the same way, we have assumed that the Doppler effect can be modeled by a frequency shift, which is commonly referred as the narrowband signal assumption.

Suppose that K snapshots of the impinging signal are taken at a suitable sampling interval T_s. Then the sampled data can be expressed as

$$\mathbf{X} = \mathbf{GAD} \ \ + \mathbf{N} \tag{2}$$

using the following definitions:

- $\mathbf{X} = [\ \mathbf{x}(t_0) \ \ \dots \ \ \mathbf{x}(t_{K-1})\ ] \in \mathbb{C}^{N \times K}$, referred as the spatiotemporal data matrix,
- $\mathbf{D} = [\ \mathbf{d}(t_0) \ \ \dots \ \ \mathbf{d}(t_{K-1})\ ] \in \mathbb{C}^{M \times K}$, known as the basis–function matrix, and
- $\mathbf{N} = [\ \mathbf{n}(t_0) \ \ \dots \ \ \mathbf{n}(t_{K-1})\ ] \in \mathbb{C}^{N \times K}$, a matrix containing all the undesired inputs.

3. CRAMÉR-RAO BOUND FOR SEVERAL SIGNALS

According to signal model (1) and taking into account that K *independent* snapshots have been recorded, the probability density function is

$$p_{\mathbf{X}}(\mathbf{X}) = \prod_{k=0}^{K-1} \frac{\exp\{-(\mathbf{x}(t_k) - \mathbf{GAd} \ \ (t_k, \ \ ))^H \mathbf{Q}^{-1} (\mathbf{x}(t_k) - \mathbf{GAd} \ \ (t_k, \ \ ))\}}{\det[\ \ ^N \mathbf{Q}]}$$

where the term $\mathbf{GAd}$ depends on the vector parameter $\ = [\ \ \{\mathbf{a}\}^T \ \ \{\mathbf{a}\}^T \ \ ^T \ \ ^T \ \ ^T \ \ \mathbf{f}^T\ ]^T$. This PDF leads to the log-likelihood function defined as

$$\begin{aligned} \mathbf{x}(\ ) &= \ln p_{\mathbf{X}}(\mathbf{X})) = -K \ln\det\left[\ \ ^N \mathbf{Q}\right] + \\ &- \sum_{k=0}^{K-1} (\mathbf{x}(t_k) - \mathbf{GAd} \ \ (t_k, \ \ ))^H \mathbf{Q}^{-1} (\mathbf{x}(t_k) - \mathbf{GAd} \ \ (t_k, \ \ )) \end{aligned}$$

Which leads to the following Fisher Information Matrix (FIM):

$$\begin{aligned} J_{ij} &= -E\left[\frac{\ ^2 \ \mathbf{x}(\mathbf{X})}{\ _i \ \ _j}\right] = \\ &= K \operatorname{tr}\left\{\mathbf{Q}^{-1} \frac{\ \mathbf{Q}}{\ _j} \mathbf{Q}^{-1} \frac{\ \mathbf{Q}}{\ _i}\right\} + \\ &+ 2 \ \ \left\{\sum_{k=0}^{K-1} \frac{\ (\mathbf{GAd} \ \ (t_k, \ \ ))^H}{\ _i} \mathbf{Q}^{-1} \frac{\ \mathbf{GAd} \ \ (t_k, \ \ )}{\ _j}\right\} \end{aligned} \tag{3}$$

In the proposed signal model the covariance matrix $\mathbf{Q}$ is arbitrary and thus independent of the parameters of interest contained in $\ $, nulling the trace term in equation 3. The generic vector parameter can be splitted in amplitudes, Directions of Arrival and synchronization parameters: $\ = [\ \ Re\{\mathbf{a}\}^T \ \ \{\mathbf{a}\}^T\ ]^T$, $\ = [\ \ ^T \ \ ^T\ ]^T$ and $\ = [\ \ ^T \ \ \mathbf{f}^T\ ]^T$. Therefore, the FIM can be expressed with submatrices, each one of $2M \times 2M$:

$$\mathbf{J} = \begin{pmatrix} \mathbf{J} & \mathbf{J}^T & \mathbf{J}^T \\ \mathbf{J} & \mathbf{J} & \mathbf{J}^T \\ \mathbf{J} & \mathbf{J} & \mathbf{J} \end{pmatrix}$$

The elements of such submatrices can be computed using the definition obtained in equation (3):

$$\mathbf{J}_{\ i\ j} = 2 \left\{ \sum_{k=0}^{K-1} \mathbf{d}^H \frac{\mathbf{A}^H}{i} \mathbf{G}^H \mathbf{Q}^{-1} \mathbf{G} \frac{\mathbf{A}}{j} \mathbf{d} \right\}$$

$$\mathbf{J}_{\ i\ j} = 2 \left\{ \sum_{k=0}^{K-1} \mathbf{d}^H \mathbf{A}^H \frac{\mathbf{G}^H}{j} \mathbf{Q}^{-1} \mathbf{G} \frac{\mathbf{A}}{j} \mathbf{d} \right\}$$

$$\mathbf{J}_{\ i\ j} = 2 \left\{ \sum_{k=0}^{K-1} \frac{\mathbf{d}^H}{i} \mathbf{A}^H \mathbf{G}^H \mathbf{Q}^{-1} \mathbf{G} \frac{\mathbf{A}}{j} \mathbf{d} \right\}$$

$$\mathbf{J}_{\ i\ j} = 2 \left\{ \sum_{k=0}^{K-1} \mathbf{d}^H \mathbf{A}^H \frac{\mathbf{G}^H}{i} \mathbf{Q}^{-1} \frac{\mathbf{G}}{j} \mathbf{A}\mathbf{d} \right\}$$

$$\mathbf{J}_{\ i\ j} = 2 \left\{ \sum_{k=0}^{K-1} \frac{\mathbf{d}^H}{i} \mathbf{A}^H \mathbf{G}^H \mathbf{Q}^{-1} \frac{\mathbf{G}}{j} \mathbf{A}\mathbf{d} \right\}$$

$$\mathbf{J}_{\ i\ j} = 2 \left\{ \sum_{k=0}^{K-1} \frac{\mathbf{d}^H}{i} \mathbf{A}^H \mathbf{G}^H \mathbf{Q}^{-1} \mathbf{G}\mathbf{A} \frac{\mathbf{d}}{j} \right\}$$

Since we are only interested in the estimation of the synchronization parameters (in fact, DOA parameters can be computed a priori since the LUT and the satellites are well positioned), the Cramér-Rao matrix for synchronization can be obtained applying the inverse partitioning matrix:

$$CRB \quad = \left[\mathbf{J} \quad - \mathbf{J} \quad \mathbf{J}^{-1} \mathbf{J}^T \quad \right]^{-1} \tag{4}$$

The minimum achievable variance obtained by any unbiased estimator of time delays and Doppler shifts are found in the main diagonal of CRB .

4. HYBRID SPACE–TIME REFERENCE BEAMFORMING

Space reference can be combined with time reference in order to obtain an improved performance. This section describes a type of beamforming which exploits both diversities with a modular and parallelized structure. Given that the computation of the weighting vectors takes now into account temporal information and, as will be shown immediately, it needs the incoming sampled signal stored in matrix $\mathbf{X}$, a totally digital weighting architecture such as Figure 1 seems a suitable structure for the implementation.

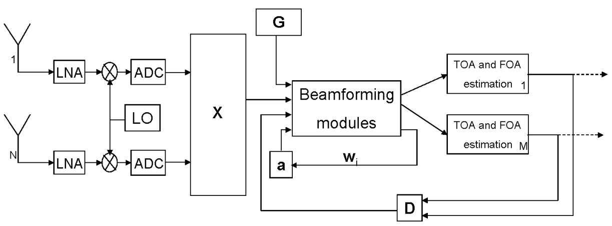

Figure 1: Proposed block diagram of the hybrid beamforming

The derivation of the beamforming is as follows. Firstly, we define the following notation based on the signal model (2):

$$\hat{\mathbf{R}}_{XX} = \frac{1}{K}\mathbf{X}\mathbf{X}^H \qquad \hat{\mathbf{R}}_{XD} = \frac{1}{K}\mathbf{X}\mathbf{D}^H$$
$$\hat{\mathbf{R}}_{DX} = \hat{\mathbf{R}}_{XD}^H \qquad \hat{\mathbf{R}}_{DD} = \frac{1}{K}\mathbf{D}\mathbf{D}^H$$

and

$$\hat{\mathbf{W}} = \hat{\mathbf{R}}_{XX} - \hat{\mathbf{R}}_{XD}\hat{\mathbf{R}}_{DD}^{-1}\hat{\mathbf{R}}_{XD}^H$$

The mean square error (MSE) between the output of a beamformer with weights $\mathbf{w}$ and a temporal reference signal $\mathbf{a}^T\mathbf{D}$ is

$$J_1(\mathbf{w}) = \frac{1}{K}\left\| \mathbf{w}^H\mathbf{X} - \mathbf{a}^T\mathbf{D} \right\|^2 \tag{5}$$

In this case, the temporal reference is not completely known but parameterized by the amplitudes $\mathbf{a}$, the Doppler shifts $\mathbf{f}$ and the time delays . In order to take advantage of the *a priori* knowledge of the steering matrix, as justified at the end of previous section, a spatial constraint is imposed to force the beamformers to always point the desired signal while nulling other directions of arrival. For each beamforming module $\mathbf{w}_i$, with $i = 1, ..., M$ and M being the number of tracked satellites or other interferers, the criterion combining temporal and spatial information could be stated as follows:

$$\min_{\mathbf{w}_i} J_1(\mathbf{w}_i) \tag{6}$$

$$\text{subject to} \qquad \mathbf{w}_i^H\mathbf{G} = \mathbf{e}_i \tag{7}$$

where $\mathbf{e}_i$ is a $1 \times M$ vector with all zeros except a one in the i^{th} component. The amplitudes-vector components that minimize J_1 for fixed $\mathbf{w}_i$, $\mathbf{f}$ and can be computed as

$$\hat{a}_i = \left(\mathbf{D}^*\mathbf{D}^T\right)^{-1}\mathbf{D}^*\mathbf{X}^T\mathbf{w}_i^*$$

where matrix $\mathbf{D}$ is computed from previous estimations of $\mathbf{f}$ and or proper initializations. If there are more tracking channels than tracked satellites, or an unknown interference signal is wanted to be rejected, the corresponding row in $\mathbf{D}$ can be filled with zeros. Constructing $\hat{\mathbf{a}} = [\ \hat{a}_1 \quad \cdots \quad \hat{a}_M\ ]^T$ and replacing it in (5) we obtain a new cost function that has to be minimized for every beamforming module:

$$J_2(\mathbf{w}_i) = \mathbf{w}_i^H\hat{\mathbf{W}}\mathbf{w}_i \tag{8}$$

This is a well-known M linear–constrained (7) quadratic–form (8) optimization problem. Applying Lagrange's multipliers technique, a very interesting expression for the weight vectors is obtained:

$$\begin{aligned} \hat{\mathbf{w}}_{hybrid,i} &= \hat{\mathbf{R}}_{\mathbf{XX}}^{-1}\hat{\mathbf{R}}_{\mathbf{XD}}\hat{\mathbf{a}}^* + \\ &+ \quad \hat{\mathbf{R}}_{\mathbf{XX}}^{-1}\mathbf{G}\left(\mathbf{G}^H\hat{\mathbf{R}}_{\mathbf{XX}}^{-1}\mathbf{G}\right)^{-1}\left(\mathbf{e}_i - \mathbf{G}^H\hat{\mathbf{R}}_{\mathbf{XX}}^{-1}\hat{\mathbf{R}}_{\mathbf{XD}}\hat{\mathbf{a}}^*\right) \end{aligned} \tag{9}$$

This result is a multiple beamforming which is a linear combination of two previously known results. On one hand,

$$\hat{\mathbf{w}}_{TE} = \hat{\mathbf{R}}_{\mathbf{XX}}^{-1}\hat{\mathbf{R}}_{\mathbf{XD}}\hat{\mathbf{a}}^*$$

is the multiple beamforming under the MSE criterion taking into account only the temporal reference. On the other hand,

$$\hat{\mathbf{w}}_{MVB} = \hat{\mathbf{R}}_{\mathbf{XX}}^{-1}\mathbf{G}\left(\mathbf{G}^H\hat{\mathbf{R}}_{\mathbf{XX}}^{-1}\mathbf{G}\right)^{-1}\mathbf{e}_i$$

is the minimum variance beamforming considering only the spatial information. These solutions have a different behavior against multipath and interferences: while $\hat{\mathbf{w}}_{TE}$ tries to combine constructively the desired signal with the other replicas in order to increase the SNIR, $\hat{\mathbf{w}}_{MVB}$ combines destructively such signals to minimize the output signal power [4]. The presented hybrid beamforming combines these two behaviors to mitigate multipath and interferences. In addition, each beamforming module takes advantage of the estimations performed in other channels since, although the output of the ith beamforming module is used to compute the corresponding $\hat{f}_i$, $\hat{}_i$ and $\hat{a}_i$ with single antenna techniques applied to a *cleaned* signal, the computation of the weighting vector $\mathbf{w}_i$ makes use of the

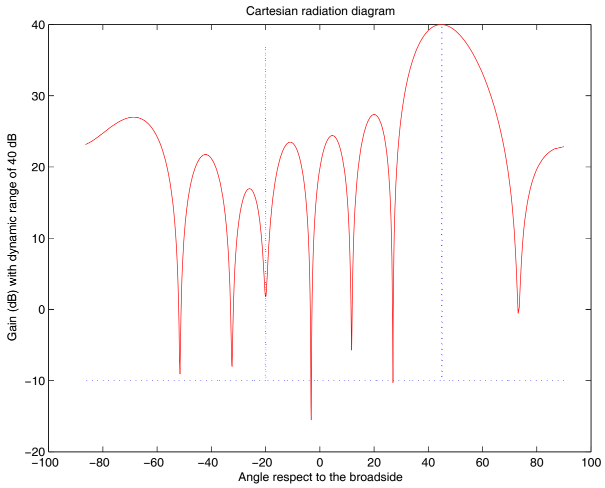

Figure 2: Beam pattern of the Hybrid Space-Time Reference Beamforming in the presence of a desired signal and an interference due to a secondary path

whole estimation vectors $\hat{\mathbf{f}}$, ˆ and $\hat{\mathbf{a}}$, exploiting all the information available to model the scenario.

The behavior of a single beamforming module is shown in Figure [2], where the radiation pattern points to the signal impinging at 45^o respect to the array broadside while nulling another incoming signal at -20^o. This strategy helps to overcome the masking effect and the mutual interference between relayed replicas of the same distress beacon, two major issues in single-antenna approaches.

5. SIMULATIONS

The proposed FOA estimation method is the FFT plus a Blackman–Harris data windowing to avoid the effect of having a finite set of samples recorded. On the other hand, the proposed TOA estimation method is performed as a 2-stage algorithm, setting a matched filter for the Tone and the Preamble, which will yield to a rough time–delay estimation, performing a decision about the received user data and repeating the process with an extended matched filter taking into account the Data.

In order to prove the robustness of the estimation methods proposed and the antenna array techniques capabilities to reject multipath, a scenario with a single desired signal impinging an 8–element uniform linear array with an additional interference due to the multipath effect. The interferer is assumed to have a Carrier to Noise density ratio (usually referred to as CN0) 3 dB lower than the direct signal. Other parameters are the differential time-delay which is 10 samples, at a sampling frequency of $f_s = 25.6$ kHz, the Doppler shift is -1 kHz for the direct signal and 1 kHz for the multipath and, finally, impinging angles of $_s = 45^o$ and $_m = -20^o$ respect the broadside.

Figures 3 and 4 show the Mean Squared Error obtained with the FOA and TOA estimation methods for the existing and the proposed beacon structure, with the minimum bound given by the CRB derived in the section 3. From the plots it arises that the proposed beacon achieves slightly better TOA estimation performance at high CN0 because of its wider filter mask, which allows faster bit transitions, and the bit codification used, which is designed to have more transitions than the existing one does. The results in FOA estimation are worse in the proposed beacon due to its shorter tone. These facts are also reflected in the CRB. A total of 2000 realizations have been simulated and averaged.

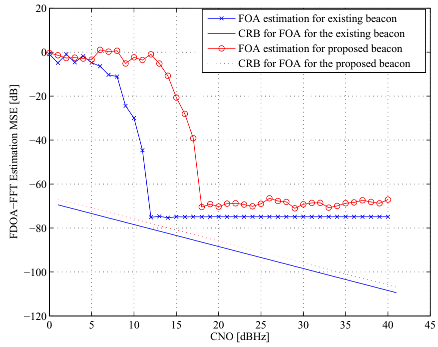

Figure 3: FOA estimation error and CRB when the scenario is composed of a line-of-sight signal and a secondary path.

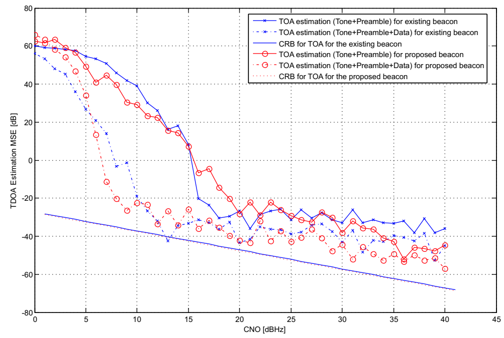

Figure 4: TOA estimation error and CRB when the scenario is composed of a line-of-sight signal and a secondary path.

6. CONCLUSIONS

This paper proposes the use of antenna arrays in the Local User Terminals of the COSPAS/SARSAT Search and Rescue system. The main contributions are the theoretical variance bounds for the estimation of time delays and Doppler shifts, both for the existing beacon structure and for a recently proposed new structure, and a new type of beamforming which combines spatial and temporal references with an intrinsic behavior against multipath and interferences, while overcoming problems such as the self–interference of the system. Simulation results show the performance of such beamforming, close to the Cramér-Rao bound with a reasonable computation effort.

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