MAJORITY ORDERING FOR COLOUR MATHEMATICAL MORPHOLOGY

Alessandro Ledda, and Wilfried Philips

Telin-IPI, Ghent University
St. Pietersnieuwstraat 41, B-9000 Gent, Belgium
phone: + (32) 9 264 89 00, fax: + (32) 9 264 42 95, email: ledda@telin.UGent.be
web: http://telin.ugent.be/∼ledda/

ABSTRACT

Binary and grayscale mathematical morphology have many applications in different domains. On the other hand, colour morphology is not widespread. The reason is the lack of a suitable colour ordering strategy that makes the extension of grayscale morphology to colour images not straightforward. We will introduce a new *majority sorting scheme* (MSS) that can be applied to binary, grayscale and colour images. It is based on the relative importance of each grayscale or colour present in the image, and has the advantage of being independent of the specific grayvalues or colour values. We will compare our ordering technique with a vector based ordering and show the advantages and disadvantages of both methods.

1. INTRODUCTION

1.1 Binary morphology

Mathematical morphology [1] [2] is based on set theory. The shapes of objects in a binary image are represented by object membership sets. Objects are connected areas of pixels with value 1, the background pixels have value 0. Morphological operations can simplify image data, preserving the objects' essential shape characteristics, and can eliminate irrelevant objects.

Binary mathematical morphology is based on two basic operations, defined in terms of a *structuring element*, a small window that scans the image and alters the pixels in function of its window content: a *dilation* of set A with structuring element B ($A \oplus B$) enlarges the objects (more 1-pixels will be present in the image), an *erosion* ($A \ominus B$) shrinks objects (the number of 1-pixels in the image decreases) (see figure 1).

The basic morphological operators on sets A and B are defined as:

$$\begin{array}{rl} \textit{dilation}: & A \oplus B = \bigcup_{b \in B} T_b(A) \\ \textit{erosion}: & A \ominus B = \bigcap_{b \in B} T_{-b}(A) \end{array} \quad (1)$$

with $T_b(A)$ the translation of image A over vector b. This formulation can be rewritten as:

$$\begin{array}{rl} \textit{dilation}: & (A \oplus B)(a) = \max\{A(b) \mid a - b \in B,\ b \in A\} \\ \textit{erosion}: & (A \ominus B)(a) = \min\{A(b) \mid b - a \in B,\ b \in A\} \end{array} \quad (2)$$

The structuring element B can be of any size or shape (square, cross, disk, line, ...). The choice of this element is based on the content of the image and on the purpose of the morphological operation.

Composite operations, like the *opening* (an erosion followed by a dilation, $A \circ B$) and the *closing* (a dilation followed by an erosion, $A \bullet B$), are derived from the basic operators. These operations preserve the overall shape and size of the original objects, but an opening removes isolated pixels and a closing fills small holes in the objects.

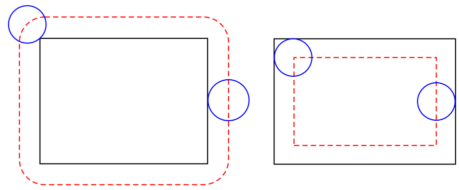


Figure 1: Schematic example of the basic morphological operators. Solid line: original object; Dashed line: result object; Circle: struct. element. Left: *dilation*; Right: *erosion*.

1.2 Grayscale morphology

The morphological theory can be extended to grayscale images with the *threshold* approach or with the *umbra* approach [1]. Here we will use the threshold approach.

For binary images, the union and intersection operation are used for the dilation and erosion, respectively. In the case of grayscale images, the union and intersection of sets are replaced by the maximum and minimum of grayvalues, as in equation 2. A is now a function instead of a set. B is still a binary set, but this restriction is quite acceptable in practice.

1.3 Colour morphology

A colour image is represented in some colour space. A frequently used space is the RGB space, used in computer systems. There are three colour bands (red, green and blue) that together represent the colours of the pixels in the image in the RGB-space. So, the colours in such a colour space can be interpreted as vectors.

There is no absolute unique ordering of the colour vectors, so the max- and min-operation cannot be extended easily to vectors and therefore an extension to colour morphology is not straightforward.

In order to be able to extend the principles of grayscale morphology to colour images, the colours in a colour image have to be ordered in some way. One way of doing this is to transform the vectors into scalars with a *lexicographical ordering scheme* [3], further explained in section 2.1. We present a new technique, the *majority sorting scheme* (section 2.2), that not only can be applied to colour images, but also to binary and grayscale images, or even to multispectral images, in contrast to the technique of [3]. As we will see, this technique does not depend on the colours in the image and is therefore invariant to bijective colour transformations. It is based on the relative importance of colours in the image.

2. METHODOLOGY

2.1 Vector ordering

In [3] the RGB image is transformed into an HSL image. In the HSL space the colour channels are the *luminance* (intensity), the *saturation* (purity), and the *hue* (primary colour).

Specifically in the double-cone HSL representation (figure 2), the luminance L has values from 0 (black) to 1 (white), the hue H lies in the interval $[0°, 360°]$. The saturation S has values between 0 and 1 for $L = \frac{1}{2}$ and the maximum S decreases linearly as L goes to 0 or 1. At $L = 0$ and $L = 1$ the saturation can only be 0.

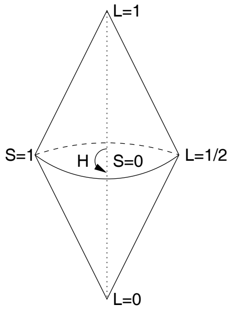


Figure 2: Double-cone HSL representation.

In [3] colour ordering is defined according to a lexicographical ordering rule on the HSL-vector: if a first comparison check is indecisive, then the next comparison is done, etc. Various variants are possible: for instance with the "L-ordering", first the L-values are compared, followed, if necessary, by a comparison of S and H. The results depend on the order chosen. The H-ordering uses a saturation-weighted hue, which means that colours with a high saturation keep their initial hue value and colours with a low saturation value are given a new hue value. This improves the choice of colours by the morphological operations [4].

Hue values are first converted to relative values $H' = |H - H_0|$ for $H \in [0°, 180°]$ and to $H' = |360° - H - H_0|$ for $H \in [180°, 360°]$. The choice of the reference colour H_0 can be arbitrary or calculated in function of the image content (section 2.1.1).

The need for a H_0-value introduces inconsistencies due to the periodic nature of the hue. Values of H' that are almost the same can belong to very different colours. A good choice of H_0 is especially important when using the H-ordering.

2.1.1 Origin of hue

A possible choice for H_0 could be the hue that appears the most in the image. If the background is the most present in the image, then all hue values are referenced to the hue of the background.

Another possibility [4] is to take the hue value of the average chromaticity vector as H_0. To calculate this average, we transform each hue value, represented as a point on a circle with an angle and with radius $r = 1$, in its Cartesian coordinates. Then we average the vectors and transform this average vector back to polar coordinates. The resultant average angle is H_0.

A third alternative is to calculate the average hue value by taking the histogram of the hue in the image and then assign the histogram values to the radius r of the respective hue. Then we also transform to Cartesian coordinates and average the vectors and transform this average vector back to polar coordinates. This approach is less accurate because the histogram consists of a certain number of bins, thus introducing a limited number of possible hue values, while the hue originally could have any value in its domain.

2.2 Majority ordering

In this section we propose a new type of sorting of the colours, the *majority sorting scheme* (MSS). Instead of using the grayvalues or ordered colours, we count the number of pixels present in the image for each colour and order the colours accordingly. We use this ordering for the morphological operations. The original colour values are linked to the MSS-ranking, so the result image won't contain any false colours because we can look them up. See figure 3 for an example.

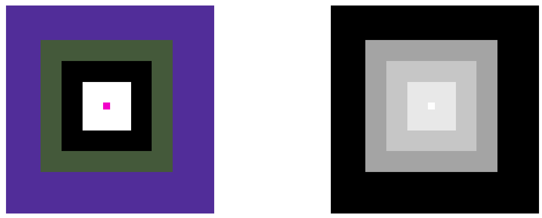


Figure 3: Left: colour image; Right: MSS ordering map.

Morphology in the traditional case is based on the intensity value of the pixels. The background is supposed to be dark and a dilation for example increases the overall intensity of the image.

With the MSS-technique, the background colour is typically the majority colour in the image, but the user can specify another background colour by modifying the MSS-ranking. A dilation decreases the number of pixels with a dominant colour in favour of non-dominant colours.

We still can use the formulation from equation 2 for the morphological operations, but then we first have to invert the image map, since the background has the largest ranking value and in the traditional case the background has the lowest intensity value. The same results can be obtained by using a max-operation for the erosion and a min-operation for the dilation, instead of vice versa. The image A in the equation is now a ranking map and the result is the colour linked to the result of the max/min-operation.

2.2.1 Adjusting the MSS

If different colours have the same number of pixels, then they will be treated equally, which leads to an arbitrary choice of colour if the morphological operation has to choose between several extrema.

In case of a tie, pixels are selected according to their difference with the current pixel. This difference can be a difference in grayvalue or hue, for example. Another possibility is to prefer the pixel closest to the current pixel. A comparison with the background colour is also an option.

For practical reasons, a quantisation of the colours or grayvalues is necessary and can also improve results. If many different colours are present, then many total number of pixels will be low and many colours will be assigned the same ranking, which cancels the effect of the MSS-ordering. Therefore, in most cases a quantisation will have to be done.

In some cases it is an advantage to merge different colour rankings. If the background has two equally prominent colours with almost the same ranking, then one colour would be rated higher than the other. This situation increases the calculation time of the pattern spectrum [5] significantly. If we allow a certain deviation for these ranking values, then, when the rankings with deviation overlap, they can be merged by taking the average of both ranking values. Only small ranking differences will merge the colour rankings (according to an absolute or relative difference tolerance), rankings with big differences will remain different rankings.

2.2.2 Properties of the MSS

This technique has several advantages:

- The MSS-technique can be used for binary, grayscale and colour images; also an extension to multispectral images is possible with the same majority sorting scheme;
- Colour images with only two colours will be treated as if it were binary images;
- The technique doesn't expect a specific colour space;
- We don't have to define a reference colour value H_0 (section 2.1.1);
- The technique is quasi invariant to colour and grayscale transformations (e.g. -compensation); when the transformation is bijective, then there is no difference in the ranking maps;
- The background does not have to be of dark intensity, and the foreground of light intensity.

Some precautions must be taken:

- The background colour must be the colour most present in the image, or must otherwise be chosen by the user;
- Too many colours in the image can make the MSS-technique useless (section 2.2.1);
- If colours have almost the same number of pixels, then the technique can become inaccurate, because maybe one colour is regarded object and another colour noise. The ranking is then not reliable.

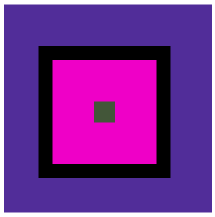
3. EXPERIMENTAL

3.1 Colour morphology

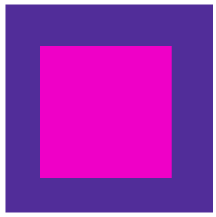
If all objects in the image have the same colour, then the image can be seen as a binary image and the morphological operations will produce the results we expect.

There are some exceptions. For the MSS-ordering, the background colour must be the most dominant colour (this is the equivalent to the darkest value in a grayscale image). With the H-ordering the ranking of the background colour can be higher than that of the object colour. The choice of H_0 is important in this case. In the case of the S- and H-ordering, the comparison of the luminances (see [3]) is referenced to $L = \frac{1}{2}$, which means that there is no difference between colours with L lying the same distance from $L = \frac{1}{2}$.

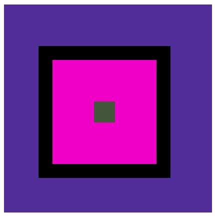
Figure 4 represents a (coloured) object with a blurred edge. A morphological opening should remove the small 3×3-square (regarded as noise) and the border of the big square (blurred edge). The vector ordering technique doesn't remove these artifacts, unless you know the correct H_0-value, which is a potential problem.



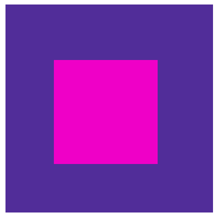
(a) Original image



(b) *MSS*-ordered opening



(c) $L/S/H$-ordered ($H_0 = 0°$)



(d) H-ordered ($H_0 = 105°$)

Figure 4: Opening (with 5×5-square) of figure (a).

Figure 5 shows a colour image (252 colours) subjected to an erosion operation. Our goal here is to extract the yellow letters “IPI”. The MSS-ordering emphasizes the dominant colours (background and yellow), while the results with the L- and S-ordering are disappointing. For the H-ordering with $H_0 = 250°$ (the average hue $+180°$), we also get a nice result.

3.2 Colour reduction

The following is only applied to the MSS. An image with a lot of colours can result into a ranking map that is useless (section 2.2.1). Figure 6 shows the effect of the different number of colours used. An adaptive colour reduction technique has been used [6].

The original images (except the one with only eight colours) are very similar. But the dilated images emphasize the thin lines more when there are less colours in the image. This is useful if we want to extract the roads from the map.

3.3 Colour distinction

When we perform a morphological operation, different pixels in the window of the structuring element can be valid candidates to replace the current pixel. If those pixels have different colours, then a choice has to be made.

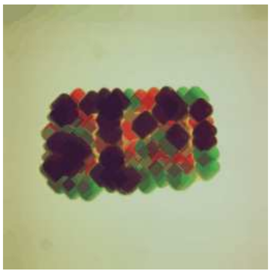
As pointed out in section 2.2.1, we can take the difference of the grayvalue or hue of the pixels with that of the current pixel as an extra criterion to decide which colour will replace the current pixel. Experiments on several images like the ones in figures 5 and 6 show no (significant) visual differences when this extra comparison is used. Also the *Peak Signal-to-Noise Ratio* results in PSNR-values of about 35 or more.

4. CONCLUSION

Morphology on colour images is possible if the colours are ordered in some way. We proposed a *majority sorting scheme* that orders the colours according to the count of pixels of that colour present in the image. This method has some advantages over other schemes. It can be applied to bi-

(a) Example colour image

(b) MSS erosion



(c) L-ordered erosion

(d) S-ordered erosion

(e) H-ordered erosion

Figure 5: Erosion (with cross with diameter 13) of (a).

(a) 244 colours

Dilation of (a)

(b) 101 colours

Dilation of (b)

(c) 32 colours

Dilation of (c)

(d) 8 colours

Dilation of (d)

Figure 6: Dilation (with 5×5-square) of the left figures.

nary, grayscale and multispectral images and is insensitive to colour (bijective) transformations.

The reduction of the colours in the image can improve the morphological operations. Further research is necessary to determine the ideal number of colours.

REFERENCES

[1] R.M. Haralick and L.G. Shapiro, *Computer and Robot Vision*, vol. 1, chapter 5, Addison-Wesley, 1992.

[2] J. Serra, *Image Analysis and Mathematical Morphology*, vol. 1, Academic Press, New York, 1982.

[3] A. Hanbury and J. Serra, “Mathematical Morphology in the HLS Colour Space,” in *BMVC 2001*, Manchester, UK, 2001, pp. 451–460.

[4] A. Hanbury, “Lexicographical order in the HLS Colour Space,” Tech. Rep. N-04/01/MM, Centre de Morphologie Mathématique Ecole des Mines de Paris, 2001.

[5] P. Maragos, “Pattern Spectrum and Multiscale Shape Representation,” *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 11, no. 7, pp. 701–716, 1989.

[6] N. Papamarkos, A.E. Atsalakis, and Ch.P. Strouthopoulos, “Adaptive Color Reduction,” *IEEE Transactions on Systems, Man, and Cybernetics — Part B: Cybernetics*, vol. 32, no. 1, pp. 44–56, 2002.