IMPROVED BEARING ESTIMATION IN OCEAN BY NONLINEAR WAVELET DENOISING UNDER NON-GAUSSIAN NOISE CONDITIONS

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ABSTRACT

Bearing estimation of underwater acoustic sources is an important aspect of passive localization in the ocean. The performance of all bearing estimation techniques degrades under conditions of low signal-to-noise ratio (SNR) at the sensor array. The degradation may be arrested by denoising the array data before performing the task of bearing estimation. In the last few years, there has been considerable progress in the use of the wavelet transform for denoising signals. It is known that the traditional wavelet transform, which is a linear transformation, can be used for denoising signals in Gaussian noise; but this method is not suitable if the noise is strongly non-Gaussian. Statistical measurements of ocean acoustic ambient noise data indicate that the noise may have a significantly non-Gaussian heavy-tailed distribution in some environments. In this work, we have explored the possibility of employing nonlinear wavelet denoising [1, 2], a robust technique based on median interpolation, to improve the performance of bearing estimation techniques in ocean in a strongly non-Gaussian noise environment. We propose the application of nonlinear wavelet denoising to the noisy signal at each sensor in the array to boost the SNR before performing bearing estimation by known techniques such as MUSIC and Subspace Intersection Method [3]. Simulation results are presented to show that denoising leads to a significant reduction in the mean square errors (MSE) of the estimators, and enhancement of resolution of closely spaced sources.

1. INTRODUCTION

Bearing estimation of underwater acoustic sources is a problem of great interest in the area of ocean acoustics. Popular direction-of-arrival (DOA) estimation techniques such as MUSIC, ESIPIT and min-norm algorithms, developed for plane wave DOA estimation, yield biased bearing estimates in the ocean due to the multimode nature of acoustic propagation in ocean. Unbiased bearing estimate can be obtained using matched field processing techniques [4], but these techniques involve a computationally expensive search in a 3-dimensional space. Recently Lakshmipathi and Anand [3] have developed a subspace intersection method(SIM) which requires only a one-dimensional search. But all these methods provide reliable bearing estimates only if the SNR is sufficiently high and the data model is accurate. There is a significant degradation in the performance if the SNR is low. Therefore, SNR enhancement becomes a necessary part of array precessing at low SNR.

In this paper, the problem of improving the performance of the bearing estimator in shallow ocean in strongly non-Gaussian noise, with heavy tailed distribution, is considered. The motivation for this noise model is provided by the fact that ocean acoustic noise is known to have a strongly non-Gaussian heavy-tailed distribution under many conditions of practical interest [5, 6]. Conventional wavelet denoising techniques [7] based on a linear wavelet transform do not work well when the noise is strongly non-Gaussian. Hence we propose the use of nonlinear wavelet transform based on median interpolation[1, 2] for denoising the array data before performing the bearing estimation. Simulation results are presented to show that even under very low SNR conditions the bearing estimation performance of MUSIC and SIM can be improved significantly by using the proposed method.

This work was supported by the Defense Research and Development Organisation, Ministry of Defence, Govt of India, under the DRDO-IISc Joint Programme on Advanced Research in Mathematical Engineering.

2. ACOUSTIC PROPAGATION MODEL FOR SHALLOW OCEAN

Modeling of acoustic propagation in a shallow ocean has to account for the effects of the ocean boundaries and medium inhomogeneity on the propagation of acoustic waves. A simple and widely used acoustic propagation model for shallow ocean is the Pekeris model: a homogenous water layer of depth Δ, density ρ and sound speed c is assumed to be resting over a homogenous fluid half-space of density ρ_b and sound speed $c_b > c$. For a point source of frequency $\frac{\omega}{2\pi}$, in the water at a depth $z_s < \Delta$, the far-field approximation to the complex envelope of the acoustic pressure at a range r and depth $z < \Delta$, assuming normal mode propagation [8] , is given by

$$p(r,z) = \frac{2\sqrt{2\pi}\, e^{j\frac{\pi}{4}}}{\Delta} \sum_{m=1}^{K} \alpha_m^2 \, sin(\gamma_m z_s)\, sin(\gamma_m z) \frac{e^{jk_m r}}{\sqrt{k_m r}}\ , \quad (1)$$

where K is the number of propagating modes and k_m is the wavenumber of the m^{th} normal mode. The quantity α_m is defined as

$$\alpha_m^2 = \left[1 + \frac{\rho(k^2 - k_b^2) sin^2(\gamma_m \Delta)}{\rho_b \gamma_m^2 \beta_m \Delta}\right]^{-1}, \quad (2)$$

with $k = \omega/c$, $k_b = \omega/c_b$, $\beta_m = \sqrt{k_m^2 - k_b^2}$, and $k_m = \sqrt{k^2 - \gamma_m^2}$. Let J narrowband sources of center frequency $\frac{w}{2\pi}$ be located at depths z_i and ranges r_i, $i = 1, 2, \ldots, J$, with respect to the first sensor of a horizontal uniform linear array(ULA). The ULA consists of M narrowband sensors located at depth z_h and having intersensor spacing $d = \frac{\pi}{k}$. Let the bearing angle of the i^{th} source with respect to the endfire direction of the array be denoted by θ_i. The noisy data vector at the array at time instant t can be written as

$$\mathbf{y}(t) = \mathbf{P}\mathbf{s}(t) + \mathbf{n}(t) \; , \tag{3}$$

where $\mathbf{s}(t)$ is the $J \times 1$ source signal amplitude vector, $\mathbf{n}(t)$ is the array noise vector that is spatially *white*, and $\mathbf{P}$ is the $M \times J$ *steering vector* matrix given by

$$\mathbf{P} = [\; \mathbf{p}(r_1, \theta_1, z_1) \; \mathbf{p}(r_2, \theta_2, z_2) \; \ldots \; \mathbf{p}(r_J, \theta_J, z_J)] \, . \tag{4}$$

The i^{th} column of $\mathbf{P}$ can be written as

$$\mathbf{p}(r_i, \theta_i, z_i) = \mathbf{A}(\theta_i)\mathbf{x}_i(r_i, z_i) \; , \tag{5}$$

where $\mathbf{A}(\theta_i)$ is an $M \times K$ matrix consisting of the steering vectors for each mode as given below

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{jk_1 dcos(\theta_i)} & e^{jk_2 dcos(\theta_i)} & \cdots & e^{jk_K dcos(\theta_i)} \\ e^{j2k_1 dcos(\theta_i)} & e^{j2k_2 dcos(\theta_i)} & \cdots & e^{j2k_K dcos(\theta_i)} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(M-1)k_1 dcos(\theta_i)} & e^{j(M-1)k_2 dcos(\theta_i)} & \cdots & e^{j(M-1)k_K dcos(\theta_i)} \end{bmatrix} , \tag{6}$$

and $\mathbf{x}_i(r_i, z_i) = [x_{i1} \;\; x_{i2} \;\; \ldots \;\; x_{iK}]^T, i = 1, 2, \ldots, J$. For the Pekeris channel, x_{im} is given by [see Eq. (1)]

$$x_{im} = \frac{2\sqrt{2\pi}\, e^{j\frac{\pi}{4}}}{\Delta} \, \alpha_m^2 \, sin(\gamma_m z_i) \, sin(\gamma_m z_h) \frac{e^{jk_m r_i}}{\sqrt{k_m r_i}} \, . \tag{7}$$

The sources are assumed to be mutually uncorrelated with mean zero, and with identical exponentially decaying autocorrelation functions

$$E[s_i(t) s_l^*(u)] = \sigma_s^2 \; \delta_{i,l} \; e^{-\alpha|t-u|} \, . \tag{8}$$

It is also assumed that the signal and noise are uncorrelated. The covariance matrix of the array data vector is

$$\mathbf{C} \;\; = \;\; E[\mathbf{y}(t)\mathbf{y}^H(t)] \;\; = \;\; \mathbf{P}\mathbf{\Sigma}\mathbf{P}^H + \sigma^2 \mathbf{I} \, , \tag{9}$$

where $\mathbf{\Sigma} = diag[\sigma_1^2, \; \sigma_2^2, \; \ldots \; \sigma_J^2]$ is the source correlation matrix.

3. BEARING ESTIMATION

3.1 MUSIC

The MUSIC algorithm for 3-dimensional source localization is a generalisation of the MUSIC algorithm for plane wave DOA estimation. If the columns $\mathbf{p}(r_1, \theta_1, z_1), \ldots \mathbf{p}(r_J, \theta_J, z_J)$ of the matrix $\mathbf{P}$ in Eq. 4 are linearly independent, they span the signal subspace S defined as

$$S \;\; = \;\; span\{\mathbf{u}_1, \; \mathbf{u}_2, \; \ldots \mathbf{u}_J\}, \tag{10}$$

where $\mathbf{u}_1, \mathbf{u}_2 \ldots \mathbf{u}_J$ are the signal eigenvectors (corresponding to the J largest eigenvalues) of the data eigenvector matrix C. A sufficient condition for linear independence of the vectors $\mathbf{p}(r_i, \theta_i, z_i), i = 1, 2, \ldots J$ is that [3] the array length M is greater than or equal to K(J+1). Let $\mathbf{V} \; = \; [\mathbf{u}_{J+1} \ldots \mathbf{u}_M \,]$ be the matrix whose columns are the noise eigenvectors $\mathbf{u}_{J+1}, \ldots \mathbf{u}_M$. The steering vector $\mathbf{p}(r, \theta, z)$ is orthogonal to the columns of V if and only if (r, θ, z) is an element of the set $\{(r_1, \theta_1, z_1), \ldots (r_J, \theta_J, z_J)\}$, or equivalently,

$$\mathbf{p}^H(r, \theta, z)\mathbf{V}\mathbf{V}^H\mathbf{p}(r, \theta, z) = 0 \, , \tag{11}$$

if and only if $(r, \theta, z) \in \{(r_1, \theta_1, z_1), \ldots (r_J, \theta_J, z_J)\}$. Hence the estimates of the source coordinates (r_i, θ_i, z_i), $i = 1, \ldots, J$ are provided by the J largest peaks of the MUSIC spectrum defined as

$$B_{MUSIC}(r, \theta, z) \;\; = \;\; \left[\frac{\sum_{m=J+1}^{M} |\mathbf{p}^H(r, \theta, z) \; \mathbf{u}_m|^2}{||\mathbf{p}(r, \theta, z) \,||^2} \right]^{-1} \tag{12}$$

MUSIC provides high resolution estimates of all the source coordinates. However, it involves a computationally expensive 3-dimensional search which is redundant if only the bearing estimates are required.

3.2 Subspace Intersection Method

The subspace intersection method (SIM) is an elegant and simple bearing estimation technique that requires only a one-dimensional search [3]. The steering vector defined in Eq (5) is a linear combination of columns of the matrix $\mathbf{A}(\theta_i)$. If we define the modal subspace $\mathscr{M}(\theta_i)$ as the span of the columns of $\mathbf{A}(\theta_i)$, then $\mathbf{p}(r_i, \theta_i, z_i) \in \mathscr{M}(\theta_i)$. If $M \geq K(J+1)$, if the bearing angles $\theta_1, \theta_2, \ldots, \theta_J$ are distinct, it can be shown [3] that the modal subspace $\mathscr{M}(\theta)$ and the signal subspace S intersect if and only if $\theta \in \{\theta_1, \theta_2, \ldots, \; \theta_J\}$.

To obtain a bearing estimator that uses the above property, define the $M \times L$ matrix $\mathbf{D}(\theta)$ as

$$\mathbf{D}(\theta) = [\mathbf{d}_1(\theta) \; \mathbf{d}_2(\theta) \; \ldots \; \mathbf{d}_L(\theta)] \, , \tag{13}$$

where $L = K + J$,

$$\mathbf{d}_i(\theta) \;\; = \;\; \begin{cases} \frac{\mathbf{a}(\theta, k_i)}{\sqrt{M}} & \text{if } 1 \leq i \leq K \\ \mathbf{u}_{i-K} & \text{if } K+1 \leq i \leq K+J \end{cases} ,$$
$$\mathbf{a}(\theta, k_i) \;\; = \;\; [1 \; e^{jk_i dcos(\theta)} \; \ldots \; e^{j(M-1)k_i dcos(\theta)}]^T \, . \tag{14}$$

Hence, $\theta \in \{\theta_1, \theta_2, \ldots, \theta_J\}$ if and only if

$$\mathbf{d}_i(\theta) \in span\{\mathbf{d}_1(\theta), \; \mathbf{d}_2(\theta), \ldots, \; \mathbf{d}_{i-1}(\theta)\} \, , \tag{15}$$

for some $i \in \{K+1, \; K+2, \ldots, \; K+J\}$. Let the matrix $\mathbf{D}(\theta)$ be Q-R decomposed as

$$\mathbf{D}(\theta) = \mathbf{Q}(\theta)\bar{\mathbf{R}}(\theta) \, , \tag{16}$$

where $\mathbf{Q}(\theta) = [\mathbf{q}_1(\theta) \; \mathbf{q}_2(\theta) \; \ldots \; \mathbf{q}_L(\theta)]$ is an $M \times L$ matrix whose columns $\mathbf{q}_i(\theta)$ are orthonormal vectors and the matrix $\bar{\mathbf{R}}(\theta)$ is an $L \times L$ upper triangular matrix with elements $\bar{r}_{mi}(\theta)$. The columns of $\mathbf{D}(\theta)$ are related to the columns of $\mathbf{Q}(\theta)$ through the equation

$$\mathbf{d}_i(\theta) = \sum_{m=1}^{i} \bar{r}_{mi}(\theta)\mathbf{q}_m(\theta) \, , \; i = 1, 2, \ldots, L \, . \tag{17}$$

From Eqs. (14), (15) and (17), it follows that

$$\min_{K+1 \leq i \leq L} |\bar{r}_{ii}(\theta)| = 0 \text{ if and only if } \theta \in \{\theta_1, \theta_2, \ldots, \theta_J\} \, . \tag{18}$$

This condition can be utilized to obtain an estimator which constructs $\mathbf{D}(\theta)$ and calculates $\bar{r}_{ii}(\theta)$ for each i. At the true bearing angles, Eq (18) holds. The SIM spectrum is defined as

$$B_{SI}(\theta) = \left[\min_{K+1 \leq i \leq L} |\bar{r}_{ii}(\theta)| \right]^{-1} . \tag{19}$$

4. NONLINEAR WAVELET TRANSFORMS BASED ON MEDIAN INTERPOLATION

Nonlinear wavelet transform is also called Median Interpolating Pyramid Transform (MIPT). Central to this approach is the notion of the Median Interpolating (MI) refinement scheme [2].

4.1 Median-Interpolating Refinement

Given a function f on an interval I,let med($f|I$) denote a median of f for the interval I, defined by

$$med(f|I) = inf\{\mu : m(t \in I : f(t) \geq \mu) \geq m(t \in I : f(t) \leq \mu)\}, \quad (20)$$

where $m()$ denotes the Lebesgue measure on $\Re$. Now suppose we are given a triadic array $\{m_{j,k}\}_{k=0}^{3^j-1}$ of numbers representing the medians of f on the triadic intervals $I_{j,k} = [k3^{-j},(k+1)3^{-j})$:

$$m_{j,k} = med(f|I_{j,k}) \quad 0 \leq k < 3^j, j \geq 0. \quad (21)$$

The goal of median-interpolating refinement is to use the data at scale j to infer behavior at the finer scale $j+1$, obtaining imputed medians of f on the intervals $I_{j+1,k}$[2].

1.**Interpolation** : For each interval $I_{j+1,k}$, find a quadratic polynomial $\pi_{j,k}$ satisfying the condition:

$$med(\pi_{j,k}|I_{j,k+l}) = m_{j,k+l} \; for \; -1 \leq l \leq 1. \quad (22)$$

2.**Imputation** : Obtain approximate medians at the finer scale by setting

$$\tilde{m}_{j+1,3k+l} = med(\pi_{j,k}|I_{j+1,3k+l}) \; for \; l = 0,1,2. \quad (23)$$

4.2 Pyramid Algorithm

Given a discrete data set $y_i, i = 0,\ldots,n-1$ where $n = 3^J$ is a triadic number, we use the nonlinear refinement scheme to decompose and reconstruct such sequences.The algorithms for computing the forward MIPT and inverse MIPT are given below.

ForwardMIPT: Pyramid Decomposition
1. Initialization: Set j=J and $j_0 \geq 0$.
2. Calculate block medians $m_{j,k} = med(y_i : i/n \; \varepsilon \; I_{j,k})$
3. Calculate $\tilde{m}_{j,k} = Q(m_{j-1,k})$, where Q, called the refinement operator, is a map for predicting the medians at finer scale from the medians at coarser scale as described in the previous section.
4. Calculate *detail corrections* $\alpha_{j,k} = m_{j,k} - \tilde{m}_{j,k}$
5. If $j = j_0+1$, set $m_{j_0,k} = med(y_i : i/n \; \varepsilon \; I_{j_0,k})$ and terminate the algorithm.Else set $j = j-1$ and go to 2.

InverseMIPT: Pyramid Reconstruction
1. Initialization: Set $j = j_0+1$.
2. Reconstruction by refinement: $m_{j,k} = Q(m_{j-1,k}) + \alpha_{j,k}$
3. Iteration: If $j = J$ go to 4,else set $j = j+1$ and go to 2.
4.Termination: Set $y_i = m_{J,i}$, i = 0,1,....n-1.

4.3 Denoising by Thresholding

Denoising of a signal is done in 3 steps:
1. Pyramid Decomposition(FMIPT) of the signal.
2. Thresholding coefficients: Coefficient amplitudes smaller than t_j at scale j are judged negligible,as noise rather than signal. It has been proved [2] that, for any noise distribution, maximum amplitude of coefficients at scale j have a very high probability of being below

$$t_j = \sqrt{3^{J-j}} F^{-1}\left(\frac{1}{2} + \frac{1}{2}\sqrt{1 - \left(\frac{1}{2J3^J}\right)^{\frac{2}{3^{J-j}}}}\right), \quad (24)$$

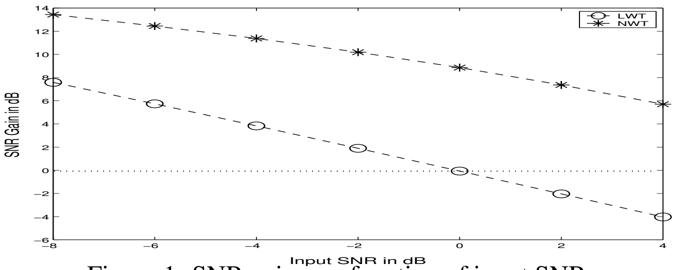

Figure 1: SNR gain as a function of input SNR.

where F is CDF of the noise distribution.
3. Pyramid Reconstruction (IMIPT) of the signal.

5. SIMULATION RESULTS

Simulation results on the performance of the bearing estimation techniques using data preprocessed by Nonlinear Wavelet Denoising (NWD) is presented in this section. The ocean is modeled as a Pekeris channel with : $\rho = 1000 \; kg/m^3$, $\rho_b = 1500 \; kg/m^3$, $c = 1500 \; m/s$, $c_b = 1700 \; m/s$, $\Delta = 100 \; m$. A Horizontal ULA of M sensors with $\lambda/2$ spacing is assumed to be present at depth $z_h = 50 \; m$. We consider 729 snapshots(a triadic number is considered for carrying out the MIPT) of the signal at each sensor. All sources are assumed to have a narrow bandwidth with $\alpha = \frac{1}{200}$and a common centre frequency $\frac{\omega}{2\pi} = 100 \; Hz$. At this frequency 6 normal modes propagate in the medium. The sources are assumed to be mutually uncorrelated. It is also assumed that the signal and noise are uncorrelated. We consider iid noise with Generalized Gaussian (GG) pdf

$$f(x) = a \; exp(-b|x|^p), \; 0 < p \leq 2, \quad (25)$$

where $a = \frac{p\Gamma^{1/2}(3/p)}{2\Gamma^{3/2}(1/p)}$, $b = \left[\frac{\Gamma(3/p)}{\Gamma(1/p)}\right]^{p/2}$.

Consider two sources at $(r_1,\theta_1,z_1) = (4000\,m, 50^\circ, 20\,m)$ and $(r_2,\theta_2,z_2) = (5000\,m, 60^\circ, 30\,m)$. Fig. 1 shows plots of SNR gain Vs. input SNR for both linear wavelet denoising (LWD) and (NWD). The noise is assumed to have generalised Gaussian pdf with $p = 0.5$. The results are calculated by averaging over 100 Monte-Carlo simulations. It is observed that NWD outperforms LWD. SNR gain increases as input SNR is reduced.

Fig. 2 illustrates the improvement in the performance of the bearing estimator MUSIC after denoising. The figure shows plots of MUSIC spectrum obtained using a 15 element array. The source locations were $(r_1,\theta_1,z_1) = (4000 \; m, 70^\circ, 20 \; m)$ and $(r_2,\theta_2,z_2) = (4000 \; m, 74^\circ, 20 \; m)$. Noise has generalised gaussian distribution with $p = 0.5$ and SNR_{in} = -5 dB. It is observed that the NWD preprocessed data resolves the sources while the undenoised data does not.

Similarly Fig. 3 shows the plots of SIM spectrum with a 60 element array, with and without denoising.Two sources are assumed to be at bearing angles $\theta_1 = 40^\circ$ and $\theta_2 = 45^\circ$, other parameters are the same as fig. 2. Once again, an improved performance due to NWD can be seen.

Tables 1 lists the root mean square errors ($RMSE$) of bearing estimates calculated over 100 Monte-Carlo simulations for SNR=-5 dB with two different GG noise distributions with p=0.5 and p=1. Three sources with source locations $(r_1,\theta_1,z_1) = (4000 \; m, 50^\circ, 20 \; m)$,$(r_2,\theta_2,z_2) = (5000 \; m, 60^\circ, 30 \; m)$

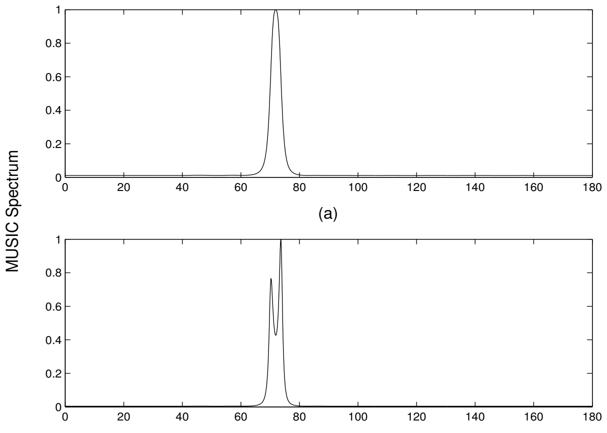

Figure 2: Spectral MUSIC for generalised Gaussian noise (p=0.5) using (a)undenoised signal,(b)nonlinear wavelet denoised signal. 2 sources are present at 70^0 and 74^0

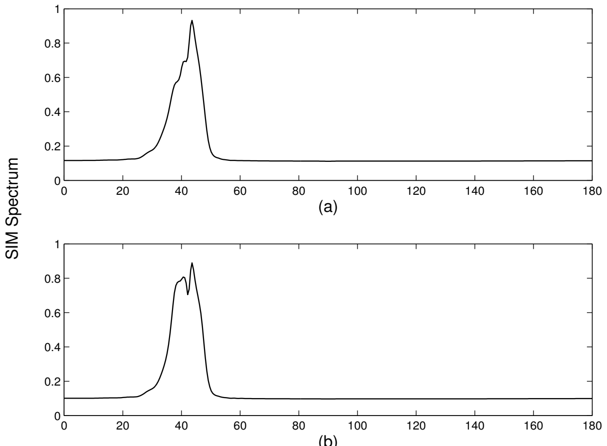

Figure 3: SIM for generalised Gaussian noise (p=0.5) using (a)undenoised signal,(b)nonlinear wavelet denoised signal. 2 sources are present at 40^0 and 45^0

Table 1: RMSE of DOA Estimates

MUSIC		$\theta_1 = 50^0$	$\theta_2 = 60^0$	$\theta_3 = 70^0$
GGN(p=0.5)	Undenoised	0.974	4.52	0.153
	Denoised	0.481	2.809	0.098
GGN(p=1)	Undenoised	1.254	4.409	0.115
	Denoised	1.040	3.313	0.115
SIM		$\theta_1 = 50^0$	$\theta_2 = 60^0$	$\theta_3 = 70^0$
GGN(p=0.5)	Undenoised	2.043	2.431	1.456
	Denoised	0.827	1.862	1.320
GGN(p=1)	Undenoised	2.492	2.990	1.451
	Denoised	1.851	2.881	1.155

and $(r_3, \theta_3, z_3) = (6000\ m, 70°, 40\ m)$were considered. Computational results clearly show that denoising leads to a reduction in the *RMSE* of the bearing estimates.

Figures 4 and 5 show the plots of RMSE of bearing estimate (of the middle source at 60°) versus input SNR in dB. These figures clearly show that nonlinear wavelet denoising improves the performance of the bearing estimators by decreasing the mean square errors of the bearing estimates even at input SNR as low as -10 dB.

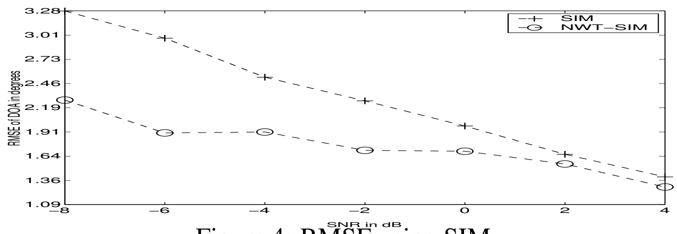

Figure 4: RMSE using SIM

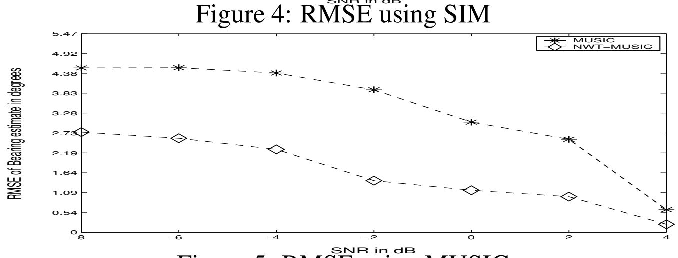

Figure 5: RMSE using MUSIC

6. CONCLUSIONS

We have shown,in this paper, that the application of nonlinear wavelet transform for denoising the signal received at the sensor array in ocean in a strongly non-Gaussian noise environment leads to an enhancement in the performance of the bearing estimators. The above denoising technique can enhance the noisy signal even at very low SNRs. We have shown through simulations that there is a SNR gain of about 15 dB at an input SNR of about -10 dB.This helps us to enhance the capability of the bearing estimators to resolve two closely spaced sources in ocean and also reduces RMSE of the bearing angle estimates of the sources. This technique can be readily extended to obtain improved range-depth estimation of acoustic sources in the ocean.

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