ON THE PERFORMANCE OF STANDARD-INDEPENDENT I/Q IMBALANCE COMPENSATION IN OFDM DIRECT-CONVERSION RECEIVERS

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ABSTRACT

The growing number of different mobile communications standards calls for inexpensive and highly flexible receiver architectures supporting these standards. The direct-conversion receiver is a very attractive candidate for reaching this goal. However, unavoidable imbalances between the I- and the Q-branch of the I/Q demodulator lead to a significant performance degradation at the reception of OFDM signals.

The performance of a novel algorithm for the estimation and compensation of these effects is analyzed in this paper. The novel approach does not depend on any standard-specific signal components, such as pilots or a preamble. Instead, a blind I/Q imbalance parameter estimation is performed during the ordinary receive mode. Therefore, the algorithm is applicable to a wide range of present and future OFDM communications standards.

1. INTRODUCTION

Advanced receiver architectures based on I/Q signal processing are highly attractive because the need for a bulky analog image rejection filter is avoided. However, one of the drawbacks is the so called I/Q imbalance, resulting from imperfect matching of the analog components in the I- and the Q-branch of the receiver [3].

A very promising approach for coping with these analog impairments is to compensate them digitally. The challenge of a digital compensation is an accurate estimation of the parameters of the I/Q imbalance. Several parameter estimation techniques have been proposed, a detailed literature review can be found in [5]. The disadvantage of these approaches is, that the RF part of the receiver has to be feed by some kind of known calibration signal.

These requirements can be dropped by applying a blind I/Q imbalance estimation and compensation scheme, which has been proposed in [5]. Furthermore, this novel approach is suited for multi-standard applications, because no standard-specific structures, such as pilots, are required for the parameter estimation. Instead, only the statistics of the received symbols are evaluated.

In [5] the potential of the novel approach has been demonstrated based on a system level point of view. Considering the IEEE 802.11a WLAN standard, it has been shown that the SER (symbol error rate) can be drastically reduced by using the proposed I/Q imbalance compensation scheme. However, a comprehensive evaluation of the image rejection, which is achievable with and without digital compensation, is still missing. Therefore, this paper aims for a more detailed analysis of the performance of the parameter estimation under different conditions.

The outline as follows: Section 2 introduces a model for the I/Q imbalance, which is used in this paper. The novel approach for the blind estimation and compensation of the I/Q imbalance is described in section 3. Both a theoretical and a simulative performance analysis is presented in section 4, followed by the conclusions in section 5.

2. I/Q IMBALANCE IN OFDM SYSTEMS

The fundamental principle of the so called direct-conversion receiver architecture is to perform the conversion from the radio frequency (RF) down to baseband (BB) using complex (I/Q) signal processing [3]. In two parallel branches, the RF signal is multiplied by two orthogonal phases of a local oscillator (LO) signal. The frequency of the LO f_{LO} is chosen equal to the carrier frequency of the desired RF signal. Ideally, the complex LO signal has the time function $x_{LO}(t) = e^{-j2\pi f_{LO}t}$, which corresponds to the desired down-conversion by f_{LO}.

Unfortunately, a perfect analog I/Q mixing is not achievable in practice. Unavoidable tolerances in the manufacturing process lead to deviations from the desired 90° phase shift and the desired equal gain in the I- and the Q-branch. These imperfections can be modelled by a complex LO signal with the time function $\tilde{x}_{LO}(t) = \cos(2\pi f_{LO}t) - jg\sin(2\pi f_{LO}t + \varphi)$, where g denotes the amplitude imbalance and φ denotes the phase imbalance. Based on g and φ, the complex valued I/Q imbalance parameters

$$K_1 = \frac{1+ge^{-j\varphi}}{2}, \quad K_2 = \frac{1-ge^{+j\varphi}}{2} \tag{1}$$

are defined, in order to rewrite the time function of the complex LO with I/Q imbalance as:

$$\tilde{x}_{LO}(t) = K_1 e^{-j2\pi f_{LO}t} + K_2 e^{+j2\pi f_{LO}t}. \tag{2}$$

Therefore, direct-conversion with I/Q imbalance can be interpreted as a superposition of a desired down-conversion (weighted by K_1) and an undesirable up-conversion (weighted by K_2). The impact of the I/Q imbalance on the transmitted baseband signal depends on the internal structure of the baseband signal. In has been shown in [5], that the receiver I/Q imbalance translates to a mutual interference between symmetric subcarriers in OFDM systems (see Fig. 1). Using matrix notation, this mutual interference can be efficiently modelled by:

$$\begin{bmatrix} Z_m(n) \\ Z^*_{-m}(n) \end{bmatrix} = \mathbf{K} \begin{bmatrix} Y_m(n) \\ Y^*_{-m}(n) \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} K_1 & K_2 \\ K_2{}^* & K_1{}^* \end{bmatrix}. \tag{3}$$

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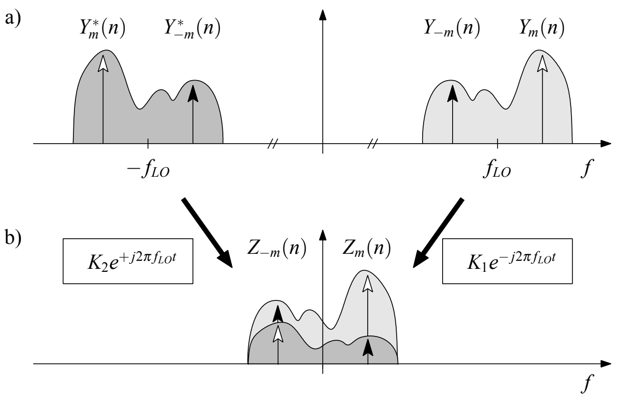

Figure 1: Frequency domain illustration of I/Q imbalance in OFDM direct-conversion receivers: a) Spectrum of the RF signal, b) Spectrum of the base band signal

The asterisk $(\cdot)^*$ denotes complex conjugation. In our notation the subscript m denotes the subcarrier index and the argument n denotes the sample time index of the OFDM symbols. For example, $Z_m(n)$ denotes the demodulated symbol at the m^{th} subcarrier of the n^{th} OFDM symbol. In order to concisely model the effects of the I/Q imbalance effects, the interval of subcarrier indices is set to $m \in [-L_{DFT}/2; L_{DFT}/2-1]$, where L_{DFT} denotes the order of the Discrete Fourier Transform (DFT). The index $m = 0$ corresponds to the DC subcarrier.

The symbols $Y_m(n)$ correspond to the equivalent baseband signal of the received RF signal before down-conversion (see Fig. 1). In the case of an imbalance-free I/Q down-conversion ($K_1 = 1, K_2 = 0$), these symbols will appear at the output of the OFDM demodulator: $Z_m(n) = Y_m(n)$. The key for a digital compensation of the I/Q imbalance lies in the so called mixing matrix $\mathbf{K}$. Because $\mathbf{K}$ is always non-singular for realistic imbalance parameters, the desired OFDM symbols $Y_m(n)$ and $Y_{-m}(n)$ can be perfectly reconstructed out of the interfered symbols $Z_m(n)$ and $Z_{-m}(n)$ by using the inverse $\mathbf{K}^{-1}$.

It should be stressed, that the desired symbols $Y_m(n)$ are not necessarily identical to the transmitted symbols $X_m(n)$. Instead, they might be corrupted by the channel or other RF impairments. The compensation of such distortions is beyond the scope of this paper. We focus on the cancellation of the I/Q imbalance effects, i.e. the goal is to provide OFDM symbols equivalent to those of a perfectly balanced direct-conversion.

3. BLIND I/Q IMBALANCE COMPENSATION

In practice, the challenge of a digital compensation is to gain knowledge about the unknown mixing matrix $\mathbf{K}$. It has been shown in [5], that a completely blind estimation of the I/Q imbalance parameters is possible. The rationale of this novel approach is, that the unknown product $K_1 K_2$ is determined by the statistics of the interfered symbols:

$$K_1 K_2 = \frac{\mathrm{E}\{Z_m(n) Z_{-m}(n)\}}{\mathrm{E}\left\{|Z_m(n) + Z_{-m}^*(n)|^2\right\}}, \tag{4}$$

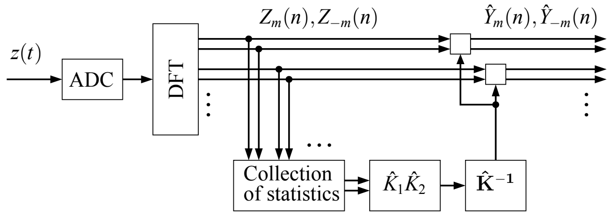

Figure 2: Structure of the proposed I/Q imbalance compensation algorithm

where $\mathrm{E}\{\cdot\}$ denotes expectation. The only assumption that was introduced is, that $\mathrm{E}\{Y_m(n) Y_{-m}(n)\} = 0$ holds at the examined subcarrier index m. In other words, the symbols of at least one pair of symmetric subcarriers Y_m and Y_{-m}^* must be uncorrelated and have zero mean. In practical OFDM systems this assumption is realistic at least for pairs of data-subcarriers, if a proper source and channel coding is applied.

In a practical implementation, the expectation terms of (4) have to be replaced by sample based approximations. This can be done by an averaging operation over multiple pairs of uncorrelated subcarriers. Furthermore, the I/Q imbalance parameters change very slowly with time. Hence, an averaging over time is also reasonable. The estimation can be formally written as:

$$\hat{K}_1 \hat{K}_2 = \frac{\sum_{m \in \mathfrak{M}} \sum_{n \in \mathfrak{N}} Z_m(n) Z_{-m}(n)}{\sum_{m \in \mathfrak{M}} \sum_{n \in \mathfrak{N}} |Z_m(n) + Z_{-m}^*(n)|^2}. \tag{5}$$

$\mathfrak{M}$ denotes the chosen subset of M (positive) subcarrier indices, $\mathfrak{N}$ denotes the chosen subset of N sample time indices. Obviously, the accuracy of the estimation will be affected by the number of incorporated sample pairs MN. An increased subcarrier block size M raises the computational effort at each time instant n, whereas an increased temporal block size N raises the duration of the parameter estimation. Hence, the proposed parameter estimation allows for a flexible tradeoff between accuracy, computational effort and measurement time.

In order to determine an estimate of the inverse $\hat{\mathbf{K}}^{-1}$, the estimated product $\hat{K}_1 \hat{K}_2$ has to be split into its composing factors. Originally, it has been suggested to perform the splitting via the estimated parameters $\hat{g}$ and $\hat{\varphi}$ [5]. However, this approach requires the calculation of trigonometric functions, which raises the computational effort in a practical implementation. The need for trigonometric functions can be avoided, if the splitting procedure is done via the more abstract parameters $\hat{\alpha} = \hat{g}\cos(\hat{\varphi})$ and $\hat{\beta} = \hat{g}\sin(\hat{\varphi})$ instead. By adapting the definition of the actual I/Q imbalance parameters (1) to the corresponding estimates

$$\begin{aligned} \hat{K}_1 &= \frac{1 + \hat{g}e^{-j\hat{\varphi}}}{2} = \frac{1 + \hat{\alpha} - j\hat{\beta}}{2}, \\ \hat{K}_2 &= \frac{1 - \hat{g}e^{+j\hat{\varphi}}}{2} = \frac{1 - \hat{\alpha} - j\hat{\beta}}{2}, \end{aligned} \tag{6}$$

it can be easily shown, that

$$\hat{K}_1 \hat{K}_2 = \tfrac{1}{4}(1 - \hat{\alpha}^2 - \hat{\beta}^2 - j2\hat{\beta}) \tag{7}$$

holds.

Hence, given the estimated complex-valued product $\hat{K}_1\hat{K}_2$, it can be split into the real-valued parameters

$$\begin{aligned}\hat{\beta} &= -2\,\mathfrak{Im}\left\{\hat{K}_1\hat{K}_2\right\}\\ \hat{\alpha} &= \sqrt{1-\hat{\beta}^2-4\,\mathfrak{Re}\left\{\hat{K}_1\hat{K}_2\right\}},\end{aligned} \tag{8}$$

where $\mathfrak{Re}\{\cdot\}$ and $\mathfrak{Im}\{\cdot\}$ denotes the real and the imaginary part, respectively. By using (6), the estimated I/Q imbalance compensation matrix can be determined as follows:

$$\hat{\mathbf{K}}^{-1} = \frac{1}{|\hat{K}_1|^2 - |\hat{K}_2|^2}\begin{bmatrix} +\hat{K}_1^* & -\hat{K}_2 \\ -\hat{K}_2^* & +\hat{K}_1 \end{bmatrix}. \tag{9}$$

Based on this blindly gained compensation matrix, a reconstruction of the desired symbols is possible:

$$\begin{bmatrix} \hat{Y}_m(n) \\ \hat{Y}_{-m}^*(n) \end{bmatrix} = \hat{\mathbf{K}}^{-1}\begin{bmatrix} Z_m(n) \\ Z_{-m}^*(n) \end{bmatrix} = \hat{\mathbf{K}}^{-1}\mathbf{K}\begin{bmatrix} Y_m(n) \\ Y_{-m}^*(n) \end{bmatrix}. \tag{10}$$

Note, that the estimation of the compensation matrix $\hat{\mathbf{K}}^{-1}$ is restricted to uncorrelated pairs of symmetric subcarriers. In contrast, the subsequent compensation (10) can be applied to all subcarrier indices m. The overall structure of the I/Q imbalance estimation and compensation scheme is summarized in Figure 2.

4. PERFORMANCE ANALYSIS

4.1 Definitions

The goal of this paper is to analyze the quality of the blind parameter estimation under different conditions. It is reasonable to define such a quality measure based on the elements of the effective mixing matrix

$$\mathbf{K_{eff}} = \hat{\mathbf{K}}^{-1}\mathbf{K} = \frac{1}{|\hat{K}_1|^2 - |\hat{K}_2|^2}\begin{bmatrix} a_{11} & a_{12} \\ a_{12}^* & a_{11}^* \end{bmatrix}, \tag{11}$$

where

$$a_{11} = K_1\hat{K}_1^* - K_2^*\hat{K}_2, \tag{12}$$

$$a_{12} = K_2\hat{K}_1^* - K_1^*\hat{K}_2. \tag{13}$$

In the case of a perfect estimation, $\mathbf{K_{eff}}$ will be the identity matrix. A non-perfect estimation leads to non-zero non-diagonal elements, i.e an undesirable mutual interference between the symmetric subcarriers persists, as one can see from (10). The reconstructed symbol $\hat{Y}_m(n)$ is a linear combination of desired symbol $Y_m(n)$ and the interfering image symbol $Y_{-m}(n)$. The power ratio of the desired and the undesirable signal component is determined by the ratio a_{11}/a_{12}. Following the analysis framework presented in [4], we define the normalized image power gain with compensation:

$$G_C = \left|\frac{a_{12}}{a_{11}}\right|^2 = \left|\frac{K_2\hat{K}_1^* - K_1{}^*\hat{K}_2}{K_1\hat{K}_1^* - K_2{}^*\hat{K}_2}\right|^2. \tag{14}$$

G_C is zero in the case of a perfect estimation and non-zero for a non-perfect estimation. For reference, we also define the image power gain of the analog part only (no digital compensation)

$$G_A = \left|\frac{K_2}{K_1}\right|^2, \tag{15}$$

which is calculated based on (3). G_A is the inverse of what is generally referred to as image rejection ratio (IRR). For example, an IRR of 30 dB corresponds to an image power gain of -30 dB.

Because G_A depends on the I/Q imbalance parameters only, it is deterministic. In contrast, G_C (with compensation) depends on the quasi-random realization of the samples incorporated for the parameter estimation. Therefore, instead of a single realization G_C, it is more reasonable to consider its expectation $\mathrm{E}\{G_C\}$.

4.2 Theoretical considerations

A comprehensive analysis of the properties of $\mathrm{E}\{G_C\}$ has been presented in [4] for Low-IF receivers. With respect to impairments due to I/Q imbalance, the Low-IF receiver is very closely related to the multi-carrier direct-conversion receiver [5]. By adapting the results of [4] to the notations used in this paper, an approximation for the parameter estimation based on a single pair of symmetric subcarriers can be derived:

$$\mathrm{E}\{G_C\} \approx \frac{1}{N}\frac{P_m P_{-m}}{(P_m + P_{-m})^2}. \tag{16}$$

$P_m = \mathrm{E}\{Y_m(n)Y_m^*(n)\}$ denotes the power of subcarrier m. Most practical OFDM systems are designed such that symmetrical subcarriers are transmitted with the same power, i.e. $P_m = P_{-m}$ is a realistic assumption. Furthermore, in the case of P_m being constant for all $m \in \mathfrak{M}$, (16) can be generalized to a parameter estimation based on multiple pairs of subcarriers:

$$\mathrm{E}\{G_C\} \approx \frac{1}{4}\frac{1}{MN}. \tag{17}$$

Note, that the performance after the digital compensation is independent of the analog I/Q imbalance parameters. Therefore, the demands to image rejection capabilities of the analog part of the receiver can be reduced without any loss of performance.

4.3 Simulation results

In this subsection, the theoretical results are validated using computer simulations. We considered the IEEE 802.11a WLAN standard [2], which is a widely used OFDM-based wireless communications standard. The highest modulation order (64-QAM), which is also most sensitive to I/Q imbalances, is used in the simulations.

4.3.1 Single pair of subcarriers

We start our investigations with the case $M = 1$, i.e. the parameter estimation is done based on a single pair of subcarriers with indices m and $-m$. The temporal block size was set to $N = 1000$. An I/Q imbalance of $g = 1.05$, $\varphi = 5°$ was assumed.

Figure 3 shows the performance of the parameter estimation as a function of the subcarrier index m, both for an exemplary single realization and for the average of 1000 independent realizations. A perfect match with the performance predicted by (17) can be ascertained. Interestingly, the parameter estimation conforms also for the zero-subcarriers ($m = 27\ldots31$). Zero-subcarriers are unused for data transmission and carry channel noise only. This fact stresses the property of the blind parameter estimation of being independent from any special signal form, as long as the assumption of uncorrelated symmetric subcarriers holds.

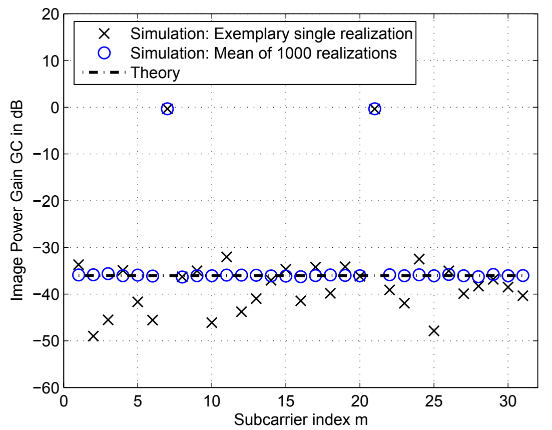

Figure 3: I/Q imbalance parameter estimation based on a single pair of subcarriers (AWGN channel, SNR=30dB, N=1000, g=1.05, φ=5°)

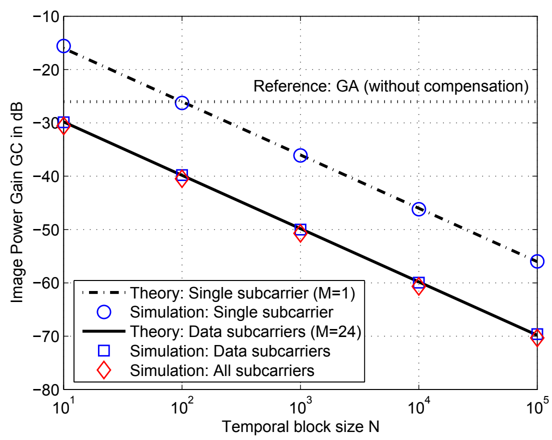

Figure 4: I/Q imbalance parameter estimation based on multiple pairs of subcarriers (AWGN channel, SNR=30dB, g=1.05, φ=5°)

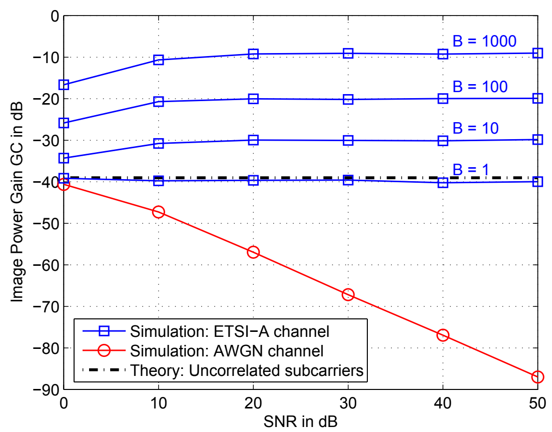

Figure 5: I/Q imbalance parameter estimation based on two pairs of pilot subcarriers under different channel conditions (N=1000, g=1.05, φ=5°)

In contrast, the estimation fails for the subcarrier positions $m = 7$ and $m = 21$. In the IEEE 802.11a standard, these are subcarriers carrying pilot symbols. By definition [2], the transmitted pilot symbols are related by

$$+X_{-21}(n) = +X_{-7}(n) = +X_7(n) = -X_{21}(n). \qquad (18)$$

Depending on the transmission channel, a correlation between the received symbols may persist, violating the fundamental assumption of the parameter estimation scheme. Hence a parameter estimation based on a single pair of subcarriers requires a careful choice of the subcarrier index m.

4.3.2 Multiple pairs of subcarriers

Next we move on on to $M > 1$, i.e. a parameter estimation using multiple pairs of subcarriers. We compare the reference case of a single data-subcarrier ($M = 1$) to two practically reasonable choices: All data-subcarriers ($M = 24$), and all available subcarriers ($M = 31$). Figure 4 shows the quality of the parameter estimation as a function of the temporal block size N. The simulation results are the mean of 1000 independent realizations.

For $M = 24$ a perfect match with the theory can be ascertained. Hence, the number of samples in time N and the number of incorporated subcarriers M are exchangeable. A reduced length of the measurement time can always be compensated by an increased number of incorporated subcarriers and vice versa. By using all data-subcarriers, 100 OFDM symbols (equivalent to 0.4 milliseconds measurement time) are sufficient in order to reach a mean image power gain of less than -40 dB.

For $M = 31$ the theoretical analysis (17) is not applicable anymore. Because both data- and zero-subcarriers are evaluated, P_m is not constant for all $m \in \mathfrak{M}$. A comparison of the simulation results to the case $M = 24$ yields only a small gain of 0.7 dB. Interestingly, the parameter estimation does not fail, even though the set of evaluated subcarriers included the correlated pilots. This phenomenon is worth a more detailed analysis.

4.3.3 Pilot subcarriers only

Therefore, we finally analyze the special case of a parameter estimation based on the 4 pilot subcarriers only, i.e. $\mathfrak{M}$=[7,21]. Figure 5 shows, that the performance of the estimation strongly depends on the channel conditions.

First, we consider an AWGN channel. Here the results are orders of magnitudes better than in the reference case of an estimation based on 2 uncorrelated pairs of subcarriers. The high accuracy is a consequence of the special structure of the pilots in an IEEE 802.11a system. In general, a parameter estimation based on a single pair of correlated subcarriers introduces additive error terms in both the numerator and the denominator of (5). Consequently, the estimation fails for a *single* pair of pilots. However, because of the opposite sign in definition (18), the additive error terms mutually eliminate each other if the sum over $m = 7$ and $m = 21$ is taken. This phenomenon is discussed in more detail in the appendix of this paper.

Hence, the estimation generates excellent results, if *both* pairs of pilots are used, especially for a high SNR. For example, at an SNR of more than 40 dB one single OFDM symbol ($N = 1$) is sufficient in order to reach a mean image power gain of less that -45 dB.

In contrast, in the case of a frequency selective channel, a perfect cancellation of the error terms does not hold anymore. In our simulations we used the ETSI channel A model [1], which is most frequently used for the analysis of IEEE 802.11a systems. The time-variant character of the channel is approximated by block fading. For the exemplary estimator block size of $N = 1000$, we considered 4 different settings of the length B of the block fading. In the case of a time-invariant randomly generated channel ($B = N$), the parameter estimation performs worst. For the asymptotic case of a channel, which changes with every OFDM symbol ($B = 1$), the performance is equivalent to a parameter estimation based on uncorrelated subcarriers.

It should be mentioned in this context, that the performance of a parameter estimation based on data subcarriers or zero subcarriers is not affected by the channel conditions. Symmetric pairs of subcarriers, which are uncorrelated at the transmitter side, remain uncorrelated at the receiver side, even in the case of a frequency-selective fading channel.

The results of the pilot subcarrier analysis can be generalized as follows: Any correlation between the subcarriers, which is introduced by the communications standard at the transmitter side, can be partly or fully removed at the receiver side due to the individual fading processes in each of the subcarriers. Hence, even pairs of pilot subcarriers can be treated as uncorrelated, if the coherence time of the fading channel is small compared to the estimation time.

5. CONCLUSION

The performance of a novel algorithm for the blind estimation and compensation of I/Q imbalance in OFDM direct conversion receivers has been analyzed in this paper. We derived a formula for an analytic evaluation of the I/Q imbalance compensation using the proposed parameter estimation scheme.

The validity of this formula has been verified exemplary for the IEEE 802.11a WLAN standard. It has been shown, that the parameter estimation does not require any standard-specific components, such as pilots. However, available pilots can significantly enhance the performance of the parameter estimation under certain channel conditions.

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A. APPENDIX

Let $\mathscr{N}_{\mathfrak{M},\mathfrak{N}}$ denote the numerator and $\mathscr{D}_{\mathfrak{M},\mathfrak{N}}$ denote the denominator of (5). Definition (3) yields:

$$\mathscr{N}_{\mathfrak{M},\mathfrak{N}} = \sum_{m\in\mathfrak{M}}\sum_{n\in\mathfrak{N}} Z_m(n)Z_{-m}(n) \tag{19}$$

$$= \underbrace{\sum_{m\in\mathfrak{M}}\sum_{n\in\mathfrak{N}} K_1K_2\left[|Y_m(n)|^2+|Y_{-m}(n)|^2\right]}_{\text{desired term}} \tag{20}$$

$$+ \underbrace{\sum_{m\in\mathfrak{M}}\sum_{n\in\mathfrak{N}} {K_1}^2 Y_m(n)Y_{-m}(n) + {K_2}^2 Y_m^*(n)Y_{-m}^*(n)}_{\text{undesirable error term}}.$$

Similarly, an analysis of the denominator yields:

$$\mathscr{D}_{\mathfrak{M},\mathfrak{N}} = \sum_{m\in\mathfrak{M}}\sum_{n\in\mathfrak{N}} |Z_m(n)+Z_{-m}^*(n)|^2 \tag{21}$$

$$= \underbrace{\sum_{m\in\mathfrak{M}}\sum_{n\in\mathfrak{N}} |Y_m(n)|^2+|Y_{-m}(n)|^2}_{\text{desired term}} \tag{22}$$

$$+ \underbrace{\sum_{m\in\mathfrak{M}}\sum_{n\in\mathfrak{N}} Y_m(n)Y_{-m}(n) + Y_m^*(n)Y_{-m}^*(n)}_{\text{undesirable error term}}.$$

If the error terms in both the numerator and the denominator are zero, (5) results in a perfect estimation, i.e. $\hat{K}_1\hat{K}_2 = \mathscr{N}_{\mathfrak{M},\mathfrak{N}}/\mathscr{D}_{\mathfrak{M},\mathfrak{N}} = K_1K_2$. Otherwise, non-zero error terms result in an erroneous parameter estimation. Clearly, the undesirable error terms vanish under the condition:

$$\sum_{m\in\mathfrak{M}}\sum_{n\in\mathfrak{N}} Y_m(n)Y_{-m}(n) \equiv 0. \tag{23}$$

The contrary behavior of a parameter estimation based on one versus two pairs of pilot subcarriers can be easily understood by considering the simple case of an ideal channel, i.e. $Y_m(n) = X_m(n)$. By using property (18), a parameter estimation based on a single pair of pilot symbols results in

$$\sum_{m\in\{7\}}\sum_{n\in\mathfrak{N}} Y_m(n)Y_{-m}(n) = \sum_{n\in\mathfrak{N}} {X_7}^2(n) \neq 0, \tag{24}$$

$$\sum_{m\in\{21\}}\sum_{n\in\mathfrak{N}} Y_m(n)Y_{-m}(n) = \sum_{n\in\mathfrak{N}} -{X_{21}}^2(n) \neq 0, \tag{25}$$

respectively. Consequently, the undesirable error terms in (20) and (22) will persist, resulting in an erroneous parameter estimation. In contrast, using both pairs of pilot subcarriers yields:

$$\sum_{m\in\{7,21\}}\sum_{n\in\mathfrak{N}} Y_m(n)Y_{-m}(n) = \sum_{n\in\mathfrak{N}} \underbrace{{X_7}^2(n) - {X_{21}}^2(n)}_{0} = 0. \tag{26}$$

Again, this property is a consequence of the specific structure of the pilots in an IEEE 802.11a symbol, as described by (18). Hence, the undesirable error terms in (20) and (22) will vanish, resulting in a perfect parameter estimation.