A GENERALIZED DUAL MODE BLIND EQUALIZATION SCHEME WITH CARRIER RECOVERY

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ABSTRACT

A generalized blind equalization scheme, insensitive to phase shifts introduced by the channel and small carrier phase offsets, is derived. Multiple constraints optimization techniques have been used in the development of the proposed algorithm. This scheme is an improved version of a dual mode modified constant modulus algorithm (MCMA). It uses the principle of minimal disturbance to induce robustness and stability by avoiding the gradient noise amplification problem. Better performance is obtained when compared to Lin's algorithm and that of the MCMA with decision directed mode (MCMA-DD).

1. INTRODUCTION

Blind equalization is a process of determining and equalizing the channel response without the aid of a training sequence. The constant modulus algorithm (CMA) happens to be an effective member of the Godard [1] class of blind equalization algorithms. However, the main drawbacks of CMA are the presence of local minima and a slow rate of convergence [2, 3]

The method of constrained optimization, to obtain a normalized version of CMA which provided a better convergence rate when compared to the CMA, was introduced in [4]. Tanrikulu $et.al.$ [5] used the concept of multiple constraints by utilizing the past inputs to the equalizer to further enhance the convergence rate. However, CMA and its normalized version lacked the capability to recover from the arbitrary phase shift due to the channel and to handle carrier phase offsets; apart from giving higher error floors.

Oh and Chin [6] modified the CMA (MCMA) cost function by separating it into the real and the imaginary parts and obtained a new cost function that could recover from the phase shift due to the channel and also could handle small carrier phase offsets. In another paper [7], the same authors again modified the previously introduced MCMA by infusing the decision directed (DD) mode (MCMA-DD) to achieve better convergence rate and reduced error floors.

Lin [8] proposed a new scheme which utilizes the principle of minimal disturbance to introduce a normalized version of the MCMA-DD that resulted in better performance than MCMA-DD.

This work generalizes that of Lin's [8] to achieve better convergence rate and better error floor at a slightly increased complexity using the principle of multiple constraints [5].

2. THE NEW GENERALIZED SCHEME

Consider the baseband representation for digital data transmission in Figure 1. The received signal can be modeled as [6]:

$$x(n) = \sum_{i=0}^{L-1} c(i)a(n-i)e^{j\phi(n)} + v(n), \qquad (1)$$

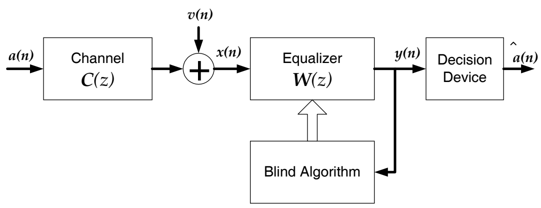

Fig. 1. Blind equalization in the baseband.

where $c(i)$ $\{0 \leq i \leq L-1\}$ are the complex channel tap weights, L is the length of the channel, $\{a(n)\}$ are the complex data symbols, and $e^{j\phi(n)}$ is caused by a carrier phase error given by $\phi(n) = 2\pi\Delta f/R$ where $\Delta(f)$ is the frequency shift and R is the dispersion constant. In order to remove the channel distortion an equalizer with tap weight vector $\mathbf{w}(n) = [w_0(n), w_1(n), \ldots, w_{N-1}(n)]^T$ is chosen.

The cost function as given by Lin [8] can be modified by utilizing the behavior of the past inputs and adding multiple constraints [5]. Thus the new constrained problem can be formulated as:

$$J = \min_{\mathbf{w}(n+1)} \Big\{ \|\mathbf{w}(n+1) - \mathbf{w}(n)\|_2^2 + \sum_{k=1}^{m} [\lambda_{1k} s_R(n-k+1)(s_R^2(n-k+1) - R_R^2)] + \sum_{k=1}^{m} [\lambda_{2k} s_I(n-k+1)(s_I^2(n-k+1) - R_I^2)] \Big\}, \quad (2)$$

where

$$R_R = \frac{E[|a_R(n)|^2]}{E[|a_R(n)|]}, \qquad R_I = \frac{E[|a_I(n)|^2]}{E[|a_I(n)|]}, \quad (3)$$

λ_{1k}, λ_{2k} $\{1 \leq k \leq m\}$ are the Lagrange multipliers, and the a posteriori output $s(n)$ is defined as:

$$s(n) = s_R(n) + j s_I(n) = \mathbf{w}^H(n+1)\mathbf{x}(n). \quad (4)$$

If *hard constraints* are enforced, the above equation becomes:

$$s(n) = R_R \text{sgn}[y_R(n)] + j R_I \text{sgn}[y_I(n)]. \quad (5)$$

Note that for $m = 1$, Equation (2) reduces to that of Lin's [8].

By using the method of Lagrange multipliers, Equation (2) can be solved and the following tap update equation is obtained:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mathbf{X}\Omega^{-1}[\mathbf{s} - \mathbf{y}]^H, \quad (6)$$

where $\mathbf{s}$, $\mathbf{y}$, $\mathbf{X}$, Ω, and $y(n)$ are defined, respectively, as:

$$\begin{aligned} \mathbf{s} &= \begin{bmatrix} s(n) & s(n-1) & \dots & s(n-k+1) \end{bmatrix}, \\ \mathbf{y} &= \begin{bmatrix} y(n) & y(n-1) & \dots & y(n-k+1) \end{bmatrix}, \\ \mathbf{X} &= \begin{bmatrix} \mathbf{x}(n) & \mathbf{x}(n-1) & \dots & \mathbf{x}(n-k+1) \end{bmatrix}, \\ \Omega &= \mathbf{X}^H \mathbf{X}, \text{ and} \\ y(n) &= \mathbf{w}^H(n)\mathbf{x}(n). \end{aligned}$$

Equation (6) can be modified by replacing the difference of the vectors $\mathbf{s}$ and $\mathbf{y}$ by a single vector $\mathbf{E}$, and using a learning parameter μ to become:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \mathbf{X}\Omega^{-1}\mathbf{E}^H. \quad (7)$$

For the case of $m = k$ constraints, $\mathbf{E}$ can be written as

$$\mathbf{E} = \begin{bmatrix} E_1 & E_2 & \dots & E_k \end{bmatrix}, \quad (8)$$

where each E_i $(1 \leq i \leq k)$ can be written as:

$$E_i = R_R \text{sgn}[y_R(n-i+1)] + j R_I \text{sgn}[y_I(n-i+1)] - y(n-i+1). \quad (9)$$

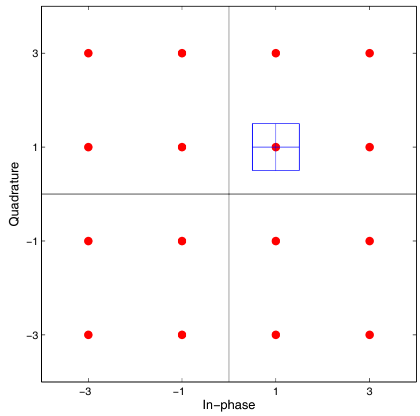

Fig. 2. 16-QAM constellation decision boundaries representing $\hat{A}$.

2.1. Decision Directed Mode

Lin [8] incorporated the decision directed mode in his algorithm in order to obtain a higher convergence rate and low steady state error. The above derived algorithm can also be switched into the decision directed mode once the error drops below a particular threshold value. The threshold value can be chosen as shown in Figure 2.

Once the equalizer output, $y(n)$, reaches within the boundary levels as specified in Figure 2, the algorithm can be switched to the decision directed mode. Thus the values of E_i's $(1 \leq i \leq k)$ in Equation (9) are switched to the decision directed mode accordingly:

$$E_i = \begin{cases} R_R \text{sgn}[y_R(n-i+1)] + j R_I \text{sgn}[y_I(n-i+1)] - y(n-i+1), & y(n-i+1) \not\subseteq C \\ R_R \text{sgn}[y_R(n-i+1)] + j R_I \text{sgn}[y_I(n-i+1)] - \hat{A}, & y(n-i+1) \subseteq C, \end{cases} \quad (10)$$

where $\hat{A}$ is the desired output as shown in Figure 2. One good aspect of this algorithm is the smooth shift from the blind mode to the decision directed mode. This is in accordance to the magnitude of the equalizer output error without any specific detection mechanism, because the algorithm can cluster the output signals at the right positions. Hence, the equalization, both in the blind mode and the decision directed mode, operates with the same stability as the normalized least mean square algorithm [8]. The main difference between the new scheme and the one proposed by Lin [8] with respect to the decision directed mode is that, in the new scheme unless and until both the real and the imaginary parts of the output signal are inside the bounded box, simultaneously, as shown in Figure 2, the algorithm doesn't shift to the decision directed mode, whereas in case of the

Lin's algorithm the decision directed mode is applied to the real and the imaginary parts independently. Applying the decision directed mode in a way as used in the newly proposed scheme, gives more scope for the algorithm in the blind mode and it performs in a better fashion when dealing with the signals infected by phase offset. Simulation results carried out for this purpose, but unfortunately due to space limitations not reported here, show the superiority of our algorithm over Lin's one.

3. SIMULATION RESULTS

In this section, we illustrate the performance of the proposed blind equalization algorithm with $m = 1, m = 2$, and $m = 3$ corresponding, respectively, to single, double, and triple constraints with that of MCMA-DD algorithm. Three different scenarios on two different channels are considered for this purpose. The signal-to-noise-ratio was taken as 30 dB. The residual inter symbol interference (ISI) as well the mean-square error (MSE) are used as performance indices. The residual ISI is defined as [8]:

$$ISI = \frac{\sum_k |c(k) * w^*(k)|^2 - |c(k) * w^*(k)|^2_{max}}{|c(k) * w^*(k)|^2_{max}}.$$

The ensemble average residual ISI's and MSE are obtained from 100 Monte Carlo runs. The *a priori* inputs to the equalizer are chosen in such a way that the auto-covariance matrix Ω is non singular.

In the first part of the simulations, a complex 9-tap transversal equalizer is used and it is initialized so that the center tap is set to 1 and the other taps are set to zero. The channel (channel I) used is taken from [9]. The value of $\phi(n)$ here is assumed to be zero. As it is clearly evident from Fig. 3 and Fig. 4 that as the number of constraints increases the convergence rate is enhanced. The MCMA-DD algorithm converges slower than even the algorithm with single constraint.

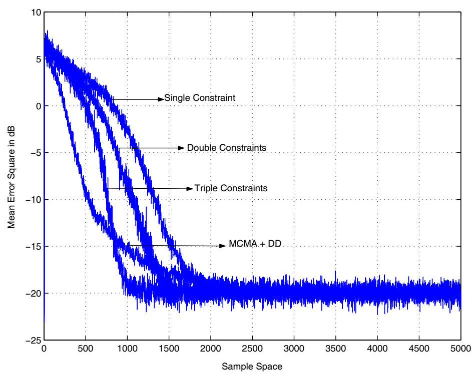

Fig. 3. MSE comparison for different constraints for complex channel I.

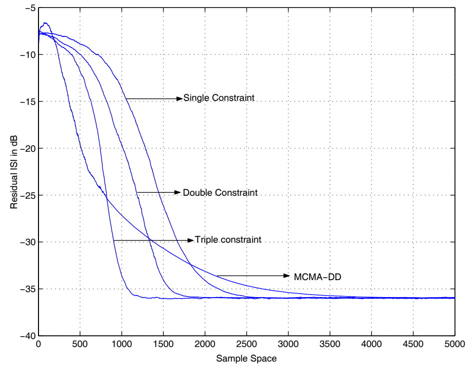

Fig. 4. Residual ISI comparison for different constraints for complex channel I.

In the second part of the simulations, a complex 25-tap transversal equalizer is used and it is initialized so that the center tap is set to one and other taps are set to zero. The channel (channel II) used is taken from [10]. Even though this channel is very severe and introduces a large amount of ISI, the proposed algorithm is able to withstand the severity of the channel and gives better performance than that of MCMA-DD as depicted in Fig. 5 and Fig. 6; also as can be noticed these figures the performance of MCMA-DD degrades when a severe channel (channel II) is used.

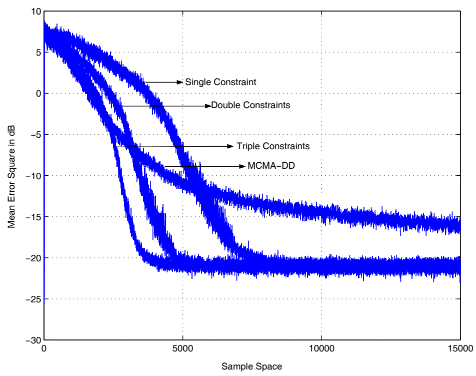

Fig. 5. MSE comparison for different constraints for complex channel II.

Finally, in this part of the simulations a phase error is deliberately introduced into channel I by setting the value of $\phi(n) = 10^{-5}$. The other conditions are set the same as assumed in the in the first part of the simulations. As can be seen from Fig. 7 and Fig. 8, as the number of constraints increases the convergence rate of the proposed algorithm is improved whereas the performance of the MCMA-DD degrades.

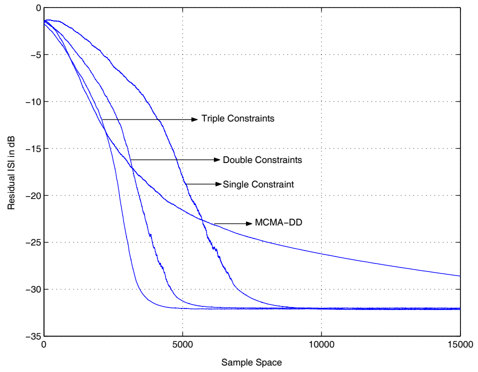

Fig. 6. Residual ISI comparison for different constraints for complex channel II.

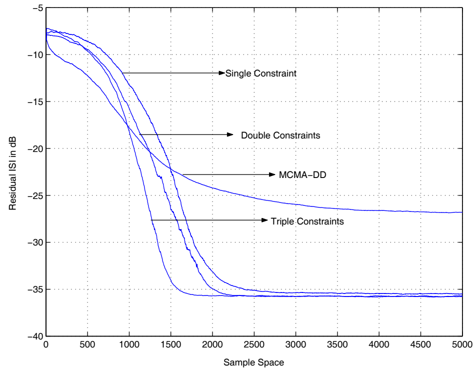

Fig. 8. Residual ISI comparison for different constraints for complex channel I affected by phase offset.

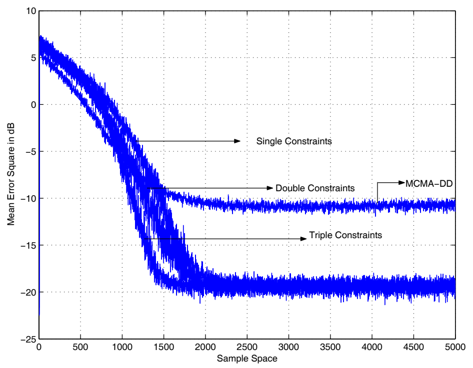

Fig. 7. MSE comparison for different constraints for complex channel I affected by phase offset.

4. CONCLUSION

An improved and generalized version of a dual mode modified constant modulus algorithm (MCMA-DD) using the idea of multiple constraints optimization techniques has been successfully derived which is insensitive to small carrier phase offsets. The principle of minimal disturbance has been used in the derivations. Better performance is obtained when compared to Lin's algorithm and that of the MCMA-DD when increasing the number of constraints at the expense of extra computational load.

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5. REFERENCES

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