MSE OPTIMUM ZONE AND MULTI-ZONE FILTERING

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ABSTRACT

We present a new approach for optimum filtering which we call as the zone filtering. In zone filtering, one is only interested on the optimal filtering of a part of the finite length signal. Therefore error is only computed for that part of the signal. When the MSE optimum filtering formulation is done, the result turns out to be the Wiener filtering with covariance method. We show that as long as the length of the FIR zone Wiener filter is the same as the zone length and the optimum delay is introduced, MSE performance is slightly better than that of the IIR Wiener filter. Therefore with zone filtering, one obtains a performance, which can be achieved by an IIR filter, by using an FIR filter. Proposed approach is suitable for parallel implementations and is computationally more efficient than a FIR or Block Wiener filter with comparable performance.

1. INTRODUCTION

In general, there are three main classes of Wiener filters, namely, FIR, IIR and Block Wiener filters [1]. It is known that the MSE performance of the FIR Wiener filter is inferior to that of IIR and Block Wiener filters [2]. IIR and Block Wiener filters have similar MSE performances and they are noncausal filters requiring a large latency for the implementation . Therefore we have causal and stable FIR filters on one hand, and better performance noncausal IIR and Block Wiener filters on the other hand. Latency is an important factor in certain applications such as equalization when forward error correction is applied. Therefore it is important to find a middle way between two extremes where we have the same MSE performance as the IIR or Block Wiener filters while the latency is kept as minimum.

An obvious solution to this problem is to divide the input signal into smaller blocks and apply the noncausal IIR or Block Wiener filter to these smaller blocks. It turns out that the MSE performance of these filters for such a case is higher than the case where the full length of the input signal is considered.

In this paper, we propose zone filtering approach for the solution of the problem. In zone filtering, we consider the optimal filtering of only a segment of the finite length signal. For multi-zone approach, more than one segment is considered for optimal filtering. More explicitly, we desire to have the MSE optimum filtering of a part of the finite length signal and we do not care about the rest of the signal. Therefore error is computed for only the part of the signal that we consider as the zone. Note that the above idea can be extended to the optimal filtering of the whole finite length signal by dividing the signal into nonoverlapping zones of interest and applying the overlap-and-save method. When the outputs of each zone filter are combined, the resulting signal has the same MSE as that of the one when an IIR or Block Wiener filter is used for the whole finite length signal. Previously, a multistage Wiener filter approach is proposed in order to deal with a reduced rank problem [5] for a better computational complexity and performance. Zone filtering also has similar advantages by employing a more simple approach to the problem.

In addition to latency, the proposed filter has another advantage over the other filter types based on system delay. The delay is an important parameter in FIR type optimum filtering and its effect on MSE is shown in [4]. On the other hand delay has no effect for the IIR and Block Wiener filters. This is due to the fact that delay term either can not be included into the formulation as in the Block Wiener or it does not make a difference as in the case of IIR Wiener filter. Unlike the classical FIR Wiener filter, in zone filtering we can select the best delay for each zone part independently. For the equal length FIR Wiener and zone filter, although the number of possible delay is the same, zone filter has a more degree of freedom due to independent delay selection for each zone part. Therefore smaller MSE can be obtained in zone filtering.

When the zone filtering approach is formulated, we obtain an MSE optimum FIR Wiener filter designed by using the covariance method. At this point, the comparison between the covariance and autocorrelation methods of finding the correlation function naturally arises. These two methods have been discussed in detail in the literature before [3]. Nevertheless, we will summarize some important properties of the covariance method in order to clarify the proposed zone filtering approach.

In this paper, we show that we obtain a slightly better MSE performance of an IIR Wiener filter, by employing an FIR filter. When the zone filtering approach is applied into the nonoverlapping zones with independent delay selection, where the FIR filter length is the same as the zone length, we have the MSE optimum filtering of the whole finite length signal. In this case, we have P filters of length N/P for an N-point signal and the MSE performance is slightly better than that of a length N FIR Wiener filter. Obviously, it may seem that there is no gain for the zone approach for this application. However, the design of a length N FIR Wiener filter requires a matrix inverse with a complexity of about $O(N^2)$ operations. Zone filtering requires only $PO((N/P)^2)$ operations for this case. In other words, zone filtering is computationally more efficient. Furthermore, latency is N/P samples only and zone filtering approach is suitable for parallel implementations when all the signal samples are acquired. Zone filtering approach can be applied to a variety of applications just like the classical Wiener filtering including noise filtering and deconvolution. As a result, zone filtering opens up a new perspective in optimum filtering.

2. ZONE FILTERING RATIONALE

In this part, we will present the basis of zone filtering by using a theorem. The idea is based on the covariance method and its application in smoothing problem. In smoothing problem, we have past, present and future samples of a signal, and we try to estimate the signal from its noisy observation. Therefore, smoothing can be seen as a noncausal operation. However note that smoothing is certainly a practical operation since usually a finite length signal is given and we are required to remove the noise from the signal. In the following part, we will first deal with noiseless signal and prove that the covariance method can perfectly estimate the signal model.

Lemma 1: Let $x(n)$ be an $N+1$ point finite-length signal. $x(n)$ can be modeled as an AR process which has an order of at most N.

Proof: According to Paley-Wiener condition and innovations representation, any process can be modeled as the output of a minimum-phase filter when the input is a white noise. This minimum-phase filter can be constrained to be an allpole filter with an order of at

most N and a set of linear equations can be found for the N coefficients of the allpole filter and they can be solved perfectly as in the case of Pade approximation.
Theorem 1: Assume that $x(n)$ is a $2N+1$ point finite-length signal which is due to an AR(N) process. Suppose that we are trying to find the AR coefficients of this process given the signal samples. If the error is defined as,

$$e(n) = x(n) + \sum_{k=1}^{N} a_k x(n-k) \qquad N \leq n \leq 2N \tag{1}$$

then a_k model coefficients can be found perfectly (with zero error) when the covariance method is used.
Proof: We assume that $x(n)$ is an AR(N) process. An AR(N) process satisfies the following equation by definition.

$$\mathbf{X}\mathbf{a} = \mathbf{0} \tag{2}$$

$$\begin{bmatrix} x(N) & x(N-1) & \dots & x(0) \\ x(N+1) & x(N) & \ddots & x(1) \\ & \vdots & & \\ x(2N) & x(2N-1) & \dots & x(N) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

If we multiply the above equation from right by $\mathbf{X}^H$, we have

$$\mathbf{X}^H\mathbf{X}\mathbf{a} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} S \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{3}$$

which is the same as the least-squares Yule-Walker equations in [2]. In the above equation, $S = 0$ is the sum of the squared errors and the estimate for the prediction error variance is given as,

$$\sigma_\varepsilon^2 = \frac{S}{N+1} \tag{4}$$

Therefore we perfectly find the a_k coefficients and $\mathbf{R} = \mathbf{X}^H\mathbf{X}$ is the correlation matrix computed by using the covariance method.

It is known that linear prediction and AR modeling have similar set of equations and the resulting coefficients are complex conjugates of each other [2]. For real case, those two problems are equivalent to each other. Similarly forward and backward prediction coefficients are same when the process is real. Therefore it turns out that for real random processes, forward, backward prediction and AR modeling are all same problems with the same solutions [2]. Furthermore, it can be shown that noise removal problem by using an FIR Wiener filter can be converted to a recursive filtering problem which involves linear prediction [2] as long as the signal is generated by an AR process. In Theorem 1, we have shown that covariance method solves the AR modeling or prediction problem in MSE optimum manner. In this respect, covariance method is expected to solve the noise removal problem with the same performance. In zone filtering, a finite-length signal is divided into certain zones and best delay optimum filtering is performed for the specified zones. When the formulation of this approach is done, we end up having the covariance method for the computation of signal correlations.

3. ZONE FILTERING IN NOISE REMOVAL

In this part, we will present the derivation of the zone filtering for the noise filtering problem. Note that a similar approach can be followed for the other problem settings as well. Let us assume that a signal, $s(n)$ is corrupted by noise $v(n)$ and the observed signal is $x(n)$ is given as,

$$\mathbf{x} = \mathbf{s} + \mathbf{v} \tag{5}$$

We would like to design an FIR filter $\mathbf{h}$ such that the selected zones in the finite length signal $\mathbf{x}$ are filtered from noise in MSE optimum manner. Note that if we specify more than one zone for noise removal, the performance may be worse than the case of single zone as long as the filter length is constant and multi-zone length is larger than the single zone. The zone filter output can be expressed in matrix-vector notation as a convolution,

$$\mathbf{y} = \mathbf{S}\mathbf{h} + \mathbf{V}\mathbf{h} \tag{6}$$

where $\mathbf{S}$ and $\mathbf{V}$ are the appropriate Toeplitz matrices. Since the filter output is longer than the desired signal, the signal estimate can be obtained by using an appropriate windowing matrix $\mathbf{C}$ at the output, which also introduces a delay in the system,

$$\hat{\mathbf{s}} = \mathbf{C}\mathbf{y} = \mathbf{C}\mathbf{S}\mathbf{h} + \mathbf{C}\mathbf{V}\mathbf{h} \tag{7}$$

The error is defined as,

$$\mathbf{e} = \mathbf{s} - \hat{\mathbf{s}} = \mathbf{s} - \mathbf{C}\mathbf{y} \tag{8}$$

Since in zone filtering, we only consider the error within a zone and do not care about the rest, we use a weighting matrix, $\mathbf{W}$ in order to select only the portion of the error corresponding to the desired zone, i.e.,

$$\hat{\mathbf{e}} = \mathbf{W}\mathbf{e} \tag{9}$$

Note that $\mathbf{W}$ is a diagonal matrix where nonzero diagonal elements dictate the zone region. Then the MSE for the weighted error can be expressed as,

$$MSE = \frac{1}{M}E\{\hat{\mathbf{e}}^H\hat{\mathbf{e}}\} = \frac{1}{M}E\{\mathbf{e}^H\mathbf{T}\mathbf{e}\} \tag{10}$$

$$= \frac{1}{M}E\{(\mathbf{s} - \mathbf{C}\mathbf{y})^H\mathbf{T}(\mathbf{s} - \mathbf{C}\mathbf{y})\} \tag{11}$$

where $\mathbf{T} = \mathbf{W}^H\mathbf{W}$. If we assume uncorrelated noise and discard the $1/M$ factor since it does not contribute to the result, we can write the MSE as,

$$MSE = \mathbf{R}_s^w + \mathbf{h}^H\mathbf{R}_s^w\mathbf{h} + \mathbf{h}^H\mathbf{R}_v^w\mathbf{h} - \mathbf{h}^H\mathbf{r_s}^w - (\mathbf{r_s}^w)^H\mathbf{h} \tag{12}$$

If we take the derivative of the above expression we have,

$$\frac{dMSE}{d\mathbf{h}^H} = \mathbf{R}_s^w\mathbf{h} + \mathbf{R}_s^w\mathbf{h} - \mathbf{r_s}^w = 0 \tag{13}$$

Then the FIR zone filter is given as,

$$\mathbf{h} = (\mathbf{R}_s^w + \mathbf{R}_v^w)^{-1}\mathbf{r_s}^w \tag{14}$$

It turns out that the correlation matrices in the above equation are equivalent to the correlation matrices found by covariance method. There are certain properties of the covariance method which results a performance for the above FIR filter equivalent to that of the IIR Wiener filter. Covariance method does not have the windowing effect as the autocorrelation method. It yields an unbiased estimate and the expected value of the correlation matrix is Toeplitz which makes the above formulation well suited to the classical Wiener formulation. Also it is known that covariance estimate is related to the maximum likelihood estimate with some mild assumptions [2].

4. CLASSICAL WIENER FILTERS FOR NOISE REMOVAL

It is possible to design Block or IIR Wiener filters in order to remove the noise from the signal. In this case, all the signal samples are used and processed by the filter as opposed to the zone filtering. Since the design of these filters are well known, we will only present their formula for comparison. If we consider the Block Wiener filter $\mathbf{G}$, it is given as,

$$\mathbf{G} = (\mathbf{R}_s + \mathbf{R}_v)^{-1}\mathbf{r}_s \tag{15}$$

The noncausal IIR Wiener filter is implemented in Fourier domain and it is given as,

$$H^{IIR}(e^{jw}) = \frac{S_s(e^{jw})}{S_s(e^{jw}) + S_v(e^{jw})} \tag{16}$$

Note that in the above formulations $\mathbf{R}_s$ and $S_s(e^{jw})$ as well as the corresponding terms for noise are found by using the autocorrelation method in general which has the windowing effect. As the signal length goes to infinity, this windowing effect may be negligible. However for finite length signals, there is a significant amount of error due to windowing. Obviously, we can use a similar strategy as in the case of zone filtering and divide the signal into zones and use covariance method estimates for the computation of IIR Wiener filter for the specified zones. Unfortunately, the result is not as good as the case where the whole signal samples are used for the design of the IIR filter. It turns out that IIR and Block Wiener filters do the best MSE filtering on each sample already and it is not possible to do a better job by using only a portion of the signal. Therefore zone filtering approach does not work for the IIR and Block Wiener filtering.

5. PERFORMANCE EVALUATIONS

In this part, we will present our results for the proposed zone filtering approach. Figure 1 shows an example of the zone filtering approach where the error square is plotted. The zone region is indicated by a box and the error is significantly lower than the region outside the zone. Therefore we can see that zone filtering sacrifices the error outside the zone region in favor of the specified zone.

In the following part, the experiments are done for different signal to noise ratios (SNR) and at each SNR, 100 trials are done with different signal and noise sequences. Input signal is selected as an AR process and the noise is white Gaussian uncorrelated with the desired signal. The length of the signal is selected as 128. In order to compare the zone filtering approach with the IIR Wiener filter, we divided the input signal into nonoverlapping zones and designed the appropriate zone filter for each part. The optimum delay constraint in the filter design is considered independently for each part. The length of the zone filter is selected as the length of the zone. Then the output signals are combined as in the case of overlap and save method. Therefore the MSE for both IIR Wiener filter and the zone filter are compared for the full length signal. We have also implemented the zone filter approach using the autocorrelation method for the computation of signal correlations. In addition, we have implemented the FIR Wiener filter with a length same as the input signal. Figure 2 shows the result for such an experiment. In this case, zone length is 8, and there are 16 nonoverlapping zones. The input signal is an AR(8) process. As it seen from this figure, zone filter performes better than FIR Wiener especialy at lower SNR and both filters have better MSE performance than that of IIR Wiener filter. Note that zone filter requires the matrix inverse of a 8×8 matrix whereas the full length FIR needs 128×128 matrix inverse. The latency of the zone filter is much less than the IIR Wiener filter. Zone filter requires 8 sample latency whereas IIR Wiener filter needs 128 samples for the implementation. As expected, autocorrelation method has slightly worse performance than the covariance method.

In the second experiment, we had a similar case as in the previous experiment. But, we increased the order of the AR process to 32. Now we are looking at the signals with nonoverlapping zone of length 8. The MSE performances of the all three filters are given in Figure 3. Again zone filter performance is better than the FIR and IIR Wiener filter. In this case, the difference between the autocorrelation and covariance approaches is more obvious. Figure 4 shows a similar experiment where the idea of using IIR filtering together with zone approach is employed for noise removal. In this case, we see that IIR zone filtering does not result a better performance than the standard IIR Wiener filtering and IIR zone filtering with covariance method has some stability problems.

As a third experiment we have shown the importance of delay selection in MSE performance. We had the same case as in experiment two and the MSE performances of zone and FIR Wiener filters are compared for optimum and constant delay selections. In constant delay case, the delay is selected as 4 for each zone filter and 64 for FIR Wiener filter. As it is seen, optimum delay selection improves the MSE performances of both filters. But it is important to state that the performance improvemet is much more for zone filter than that of FIR Wiener filter especially at lower SNR. This is due to the fact that, in zone filtering the optimum delay can be selected for each zone part independently as opposed to FIR Wiener filter, which selects only one delay. Therefore zone filter has more degree of freedom in delay selection and it results a lower MSE.

Zone filtering idea can be applied for nonstationary signals as well. In fact, this case is better suited for zone filtering than other types of filters. For this case, we have generated a 128 point chirp signal, whose frequency is changing from DC to 30 Hz in one second and the zone length is chosen as 16. The MSE performance of the three filters is given in Figure 6. As it is seen from this figure, zone filtering can perform approximately 5-6 dB better than the FIR and IIR Wiener filtering when the signal of interest is nonstationary.

6. CONCLUSION

We presented a novel approach for Wiener filtering of finite length signals which we call as the zone filtering. The idea is based on weighting the error vector such that only a part of the signal is filtered in MSE sense and the rest is disregarded. Therefore this is a kind of trade off where a part of the signal error is traded with the rest. When the desired FIR filter for the selected zone is designed, it turns out that the formulation is equivalent to that of the Wiener filtering where covariance method is used for the correlation functions. Since covariance method is unbiased and has no windowing effects, we get a MSE performance which is equivalent to that of the IIR Wiener filter. When the zone filter length is at most the length of the zone, we have some gains on the computational efficiency in comparison to that of a full length FIR or Block Wiener filter. Furthermore latency is less than the IIR Wiener or Block Wiener filters. Another advantage of zone filtering is having a more degree of freedom in selecting the best delay, which results a better MSE performance. Zone filtering approach can be applied to multi-zones arbitrarily selected within the finite length signal. Furthermore zone filter results better MSE performance than the other Wiener filters when the input signal is nonstationary.

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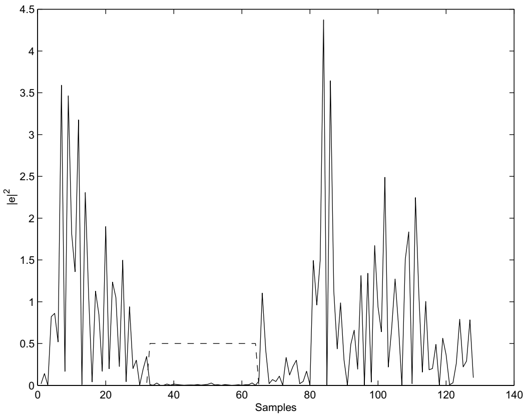

Figure 1: Error square for zone filtering (pointed by a box).

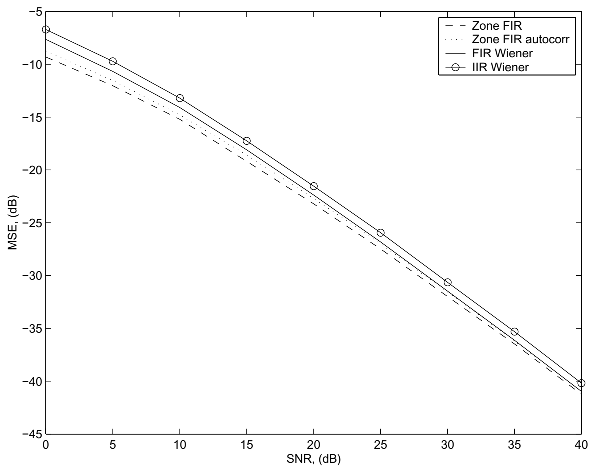

Figure 2: MSE performance of zone filtering, FIR and IIR Wiener filters for noise removal problem for AR(8) process.

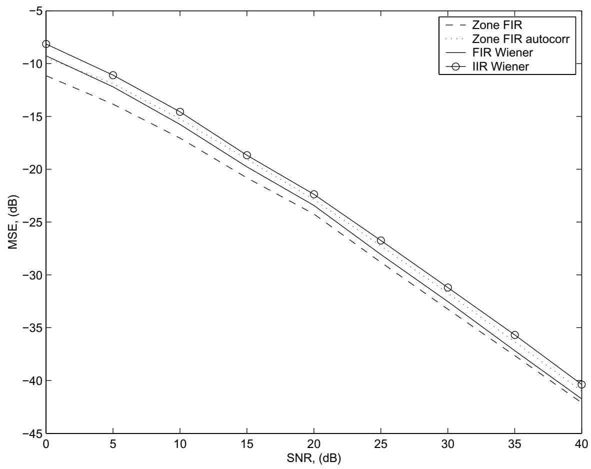

Figure 3: MSE performance of zone filtering, FIR and IIR Wiener filters for noise removal problem for AR(32) process.

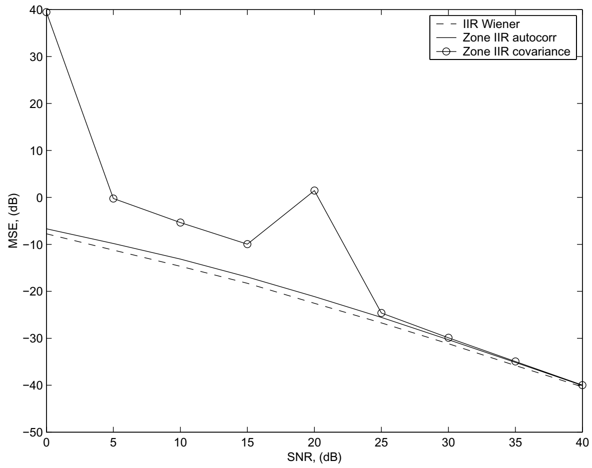

Figure 4: MSE performance of IIR Wiener filter and zone IIR with autocorrelation and covariance methods.

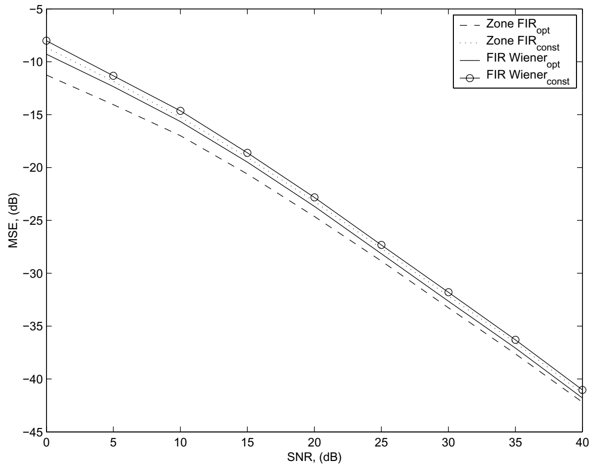

Figure 5: MSE performance of zone and FIR Wiener filters with optimum delay and constant half length delay constraints for noise removal problem for AR(32) process.

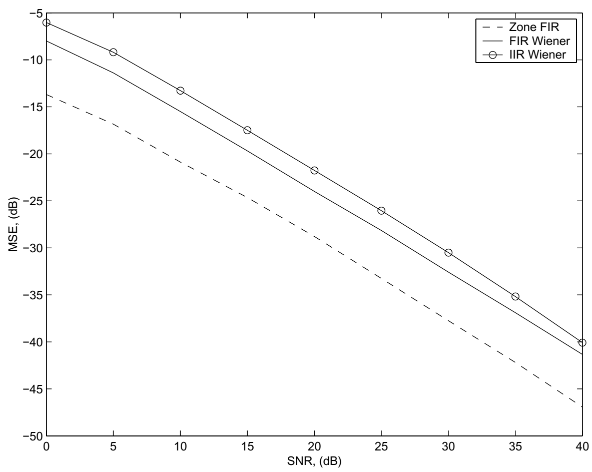

Figure 6: MSE performances for a chirp signal.