GPS/GNSS RESIDUAL ANALYSIS VIA COMPETITIVE-GROWTH MODELING OF IONOSPHERE DYNAMICS

Kohji Kamejima
Faculty of Information Science and Technology, Osaka Institute of Technology
1-79-1 Kitayama, Hirakata 573-0196 JAPAN
phone:+81(Japan)-72-866-5406, fax: +81(Japan)-72-866-8499, email: kamejima@is.oit.ac.jp
web:http://www.is.oit.ac.jp/~kamejima

ABSTRACT

A method is presented for estimating dynamic behavior of clock bias of GPS/GNSS signals. By balancing charge-carrier populations in solar-terrestrial system, a competitive growth model is introduced to formulate ionosphere dynamics. The model is adapted under the constraint of the Volterra's principle to restore the positioning residual.

1 INTRODUCTORY REMARKS

Self-locating is the first step for both autonomous guidance and interactive driver support of vehicle systems. Normally, the location of vehicles can be confined within roadway area. In practical situations, the location is 'absolutely' measured with respect to a coordinate system then mapped on a properly selected roadway. Thus, intelligent vehicles, either autonomously navigated or controlled by human drivers, are supported by well-structured map and precise positioning devices with a coordinate system.

The concept of map based self-location has been introduced as an essential part of autonomous mobile robots and extended to various vehicle navigation systems. Latest versions of the location devices often invoke GPS/GNSS signal as coordinate estimate. Through the matching with 'digital map', navigation systems provide well-structured prediction for the scene to be analyzed by human drivers and/or computer vision.

For stable maneuvering, the prediction error should be bounded within the scope of online adaptation processes. Available GPS/GNSS systems provide the estimation of relative distance between satellites and the vehicles within typically 3m error as shown in Fig. 1. The accuracy is sufficient for selecting a roadway on an exact map. However, aforementioned positioning error may yield serious distortion in predicted scene. Due to the error of practical maps, furthermore, selected lane maybe directed to wrong roadways. For stable maneuvering, final positioning error should be reduced to one tenth of current value. In this paper, the estimation of GPS/GNSS residual is considered based on solar-terrestrial dynamics.

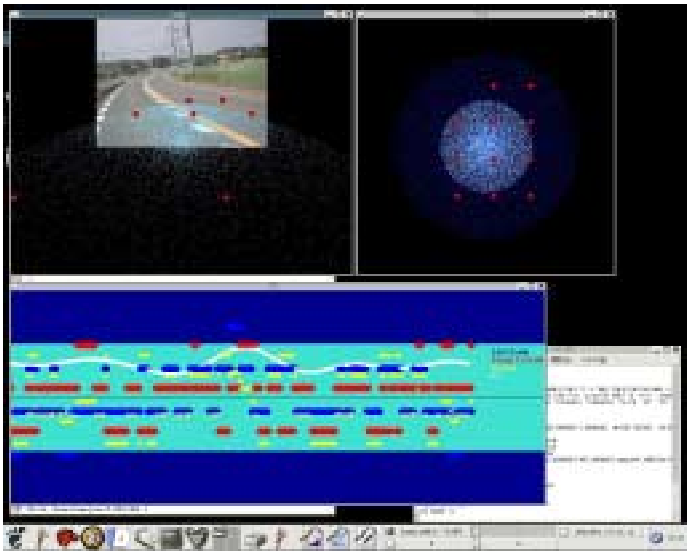


Figure 1: Positioning and Mapping by GPS/GNSS

Measurements of longitude-latitude bias are ploted in a circle with 0.1 sec. radius on the horizontal plane (upper right) and mapped on a scene (upper left). The bias level exhibits periodic behavior (lower window).

2 GPS/GNSS RESIDUAL

By detecting a set of satellite located at p_i^*, $i = 1, 2, \ldots, n$, with respect to a coordinate, positioning systems computes 3D position of a receiver p based on the following relative distance

$$\tilde{\rho}_i \quad = \quad \rho_i + \tau + r_i, \tag{1}$$

where $\rho_i = |p_i^* - p|$ and r_i is measurement error. In Eq. (1), geometrically defined distance ρ_i is shifted by τ denoting equivalent distance associated with the clock bias. By identifying the bias with a 'constant-on-average', i.e.,

$$\dot{\tau} \quad = \quad \dot{v}_t, \tag{2}$$

where $\dot{v}_t$ denotes white Gaussian process, the extended Kalman filter has been applied to estimate the posi-

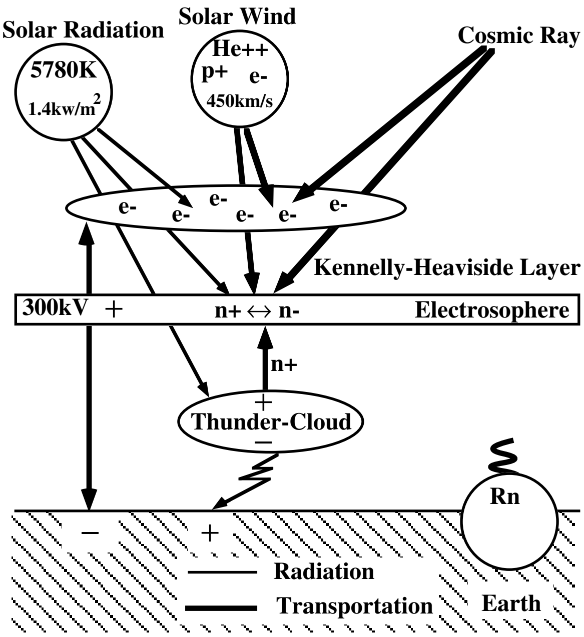


Figure 2: Solar-Terrestrial Dynamics

tion r based on noisy information $\{\tilde{\rho}_i\}$ [1]. However, the assumption (2) often fails due to periodic bias. For instance, Fig. 1 demonstrates that the residual indicates the variation of 1 – 3m amplitude (white line) with approximately 10min period.

In what follows, we consider dynamic estimation of the positioning bias τ within the framework of solar-terrestrial system.

3 DYNAMIC IONOSPHERE MODEL

Excited by mainly solar energy, the atmosphere exhibits polarized structure with positive electrosphere sandwiched between upper free electrons and normally negative ground as shown in Fig. 2. The ionization process in Kenneley-Heaviside layer is activated by energy transmission via solar radiation, solar wind and cosmic ray. Excited atmosphere is expanded to 'inflate' the ionization process. Simultaneously, positive particles of molecular or larger scale are generated in the lower layers. Since the charge-carrier separation distance corresponds to only a few diameters of the particles in the upper layer of ionosphere [2], generated ions are considered to be diffused as the "predator" of free electrons. Thus, the population of free electron in the upper layer tends to an equilibrium via the following dynamics.

$$\frac{1}{e^-}\frac{de^-}{dt} \quad = \quad a(1-k_+n^+). \qquad (3a)$$

Solar radiation reached at the surface of the earth evokes ascending current. The vertical temperature distribution is maintained by the radiation and relaxed through the nocturnal radiation. The humidity in the ascending air is condensed to yield thunder cloud: a "generator" of positive ions to the bottom layer of the ionosphere. Despite downward leakage, thus, the ionization level of the lower electrosphere is maintained by upward ion supply. In other wards, net downward leakage must be reduced by the ionization loss to balance charge-carrier populations. Hence, we have the following population dynamics for the predator ions.

$$\frac{1}{n^+}\frac{dn^+}{dt} \quad = \quad -b(1-k_-e^-). \qquad (3b)$$

4 VOLTERRA'S PRINCIPLE

Under the Predator-Prey scheme (3), population parameters (e^-, n^+) jointly maintain 'averaging constant' state exactly. Let t_0 and t_1 be start and terminal times satisfying $n^+(t_1) = n^+(t_0)$. Since

$$\int_{t_0}^{t_1} \frac{d}{dt}(\log n^+)dt + \int_{t_0}^{t_1} b(1-k_-e^-)dt \quad = \quad 0,$$

by the definition of t_0 and t_1, we have the Volterra's principle stated as follows.

$$1 - k_-\overline{e^-} = 0, \qquad (4a)$$

$$\overline{e^-} = \frac{1}{t_1 - t_0}\int_{t_0}^{t_1} e^-(t)dt. \qquad (4b)$$

This implies that the level of positioning bias $\tau \sim e^-$ can be predicted in terms of the system parameter k_- governing the predator dynamics (3b).

For identifying dynamic behavior of inaccessible n^+, let $\mathcal{F}_t$ be the Borel field generated by the information $\{(e_s^-, n_s^+), s \le t\}$ and define the following stochastic differentials with drift terms $\alpha(n^+) = a(1-k_+n^+)$ and $\beta(e^-) = b(1-k_-e^-)$.

$$d\xi = -\frac{dn^+}{n^+} \quad = \quad \beta(e^-)dt + Gdw, \qquad (5a)$$

$$d\lambda = \frac{de^-}{e^-} \quad = \quad \alpha(n^+)dt + Rdv, \qquad (5b)$$

where dw and dv are the increments of mutually independent Wiener processes. Noticing the combination of Eqs. (5a) with (4) implies

$$\xi_{t_1} - \xi_{t_0} \quad = \quad -\int_{t_0}^{t_1} Gdw_s,$$

we have a stochastic version of the Volterra's principle as a 'periodic' martingale $\{\xi_t, \mathcal{F}_t, 0 \le t\}$ satisfying

$$\mathcal{E}\{\xi_{s+m(t_1-t_0)}|\mathcal{F}_s\} \quad = \quad \xi_s \qquad m = 0, 1, 2, \ldots, \quad (6)$$

with probability one. Furthermore, noticing the association

$$\lambda \sim e^- \sim \beta(e^-) \sim n^+ \sim \xi,$$

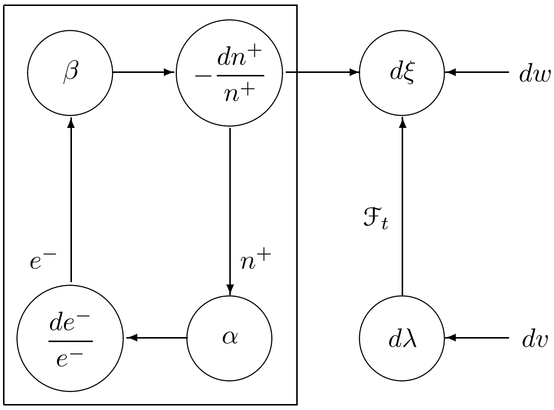


Figure 3: Intrinsically Stable Stochastic System
Mutually constrained (e^-, n^+) populations generates increasing information $\mathcal{F}_t$ to restore ionosphere dynamics.

we can exploit the random variable λ as an observation of ξ (Fig. 3). Thus for the control parameter of the ionosophere dynamics, ξ, we have the following

Proposition 1 *For the the Gaussian random variable ξ under Gaussian observation λ, the expectation and covariance are given by*

$$\begin{aligned} \mathcal{E}\Big\{d\xi \mid \mathcal{F}_t, d\lambda\Big\} - \beta dt &= \sigma_{\xi\lambda}\sigma_{\lambda\lambda}^{-1}(d\lambda - \alpha dt), \\ \mathcal{E}\Big\{(d\xi - \beta dt)^2 \mid \mathcal{F}_t, d\lambda\Big\} &= (\sigma_{\xi\xi} - \sigma_{\xi\lambda}\sigma_{\lambda\lambda}^{-1}\sigma_{\lambda\xi})dt, \end{aligned} \tag{7}$$

where $\sigma_{(\cdot)(\cdot)}$ denotes the covariance of $d(\cdot)d(\cdot)$.

In what follows, the periodic martingele (6) and update rule (7) are applied to parameter estimation and Kalman filtering, respectively.

5 STOCHASTIC GROWTH MODEL

Consider online estimation of inaccessible predator n^+ based on the observation e^- available as the positioning residual $\tilde{\tau}$. For this purpose, let the population of the prey electron e^- be observed via sampling of positioning bias τ. Since the population of the predator n^+ depends on unknown level $\overline{e^-}$, the evaluation of e^- results in the estimation of n^+ based on a complex predator-prey interaction. In the nondeterministic predator-prey system (5) define

$$\begin{aligned} dz_t &= dn^+, \\ dy_t &= adt - \frac{de^-}{e^-}. \end{aligned}$$

Then we have the linear stochastic system consisting of dynamics

$$dz_t = A_t z_t dt + G_t dw_t, \tag{8a}$$

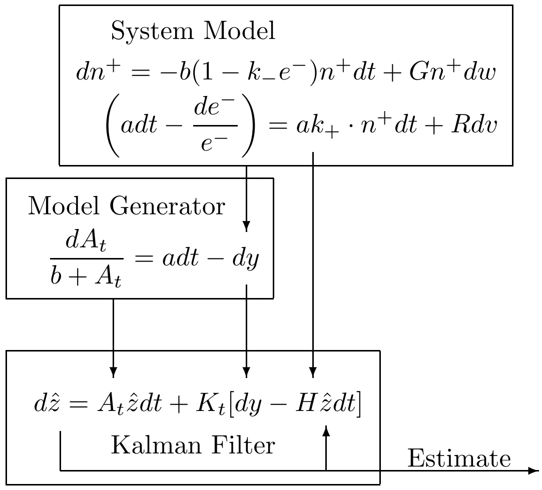


Figure 4: Kalman Filter for Nonlinear Growth Model

with the observation channel

$$dy_t = Hz_t dt + Rdv_t, \tag{8b}$$

and the generator of *as-is* model.

$$\frac{dA_t}{b + A_t} = adt - dy_t. \tag{8c}$$

In Eq. (8a), the intensity of the disturbance to z_t is magnified in accordance with the population n^+.

$$G_t = Gn^+.$$

This implies that the linear system (8) is subjected to state dependent noise.

6 FILTERING OF SOLUTION PROCESSES

Consider the scheme for estimating the population of the predator n^+ based on noisy observation of e^-. The problem is to design a dynamical system to generate the minimum variance estimate of z_t driven by the observation dy_t as illustrated in Fig. 4. By combining the stochastic model (8) with update rule (7), we have

Proposition 2 *Let G_t be a positive stochastic process adapt to $\mathcal{F}_t$. Then the state estimate $\hat{z}_t$ for the nonlinear growth system (8) is generated by the following dynamics*

$$d\hat{z}_t = A_t\hat{z}_t dt + K_t d\nu_t, \tag{9a}$$

$$d\nu_t = dy_t - H\hat{z}_t, \tag{9b}$$

with the filter gain given by

$$K_t = \frac{p_t H}{R^2},$$

where p_t is the solution to the following Riccati equation

$$\frac{dp_t}{dt} = 2A_t p_t + G_t^2 - \frac{H^2}{R^2}p_t^2,$$

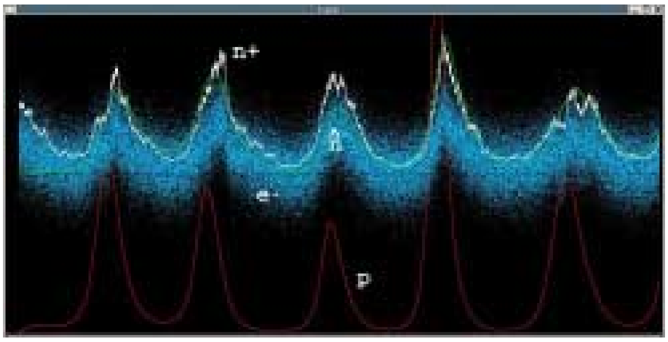


Figure 5: Filtering Results: Simulation

and ν_t is a version of Wiener process, designated by innovation process. By the evaluation $G_t = G\hat{z}_t$, resulted estimate satisfies $\hat{z}_t > 0$.

Figure 5 illustrates the behavior of the estimate $\hat{z}_t \sim n^+$ based on noisy observation $y_t \sim e^-$. As shown in this figure, the mechanism (9) generates an acceptable version of stochastic population process (e^-, n^+) even through considerably corrupted channel successfully.

7 ADAPTATION PROCESSES

Following the Volterra's principle (4), the system parameter k_- and observation sensitivity k_+ should satisfy the following constraints.

$$\begin{aligned} k_- e^- &\to 1, \\ k_+ n^+ &\to 1. \end{aligned}$$

By adapting (k_-, k_+)-parameter, the population dynamics (3) is updated by the observation.

Proposition 3 *By updating the parameter (k_+, k_-) following*

$$\frac{d\hat{k}_-}{dt} = \kappa_- \tilde{e}^- (1 - \hat{k}_- \tilde{e}^-), \tag{10a}$$

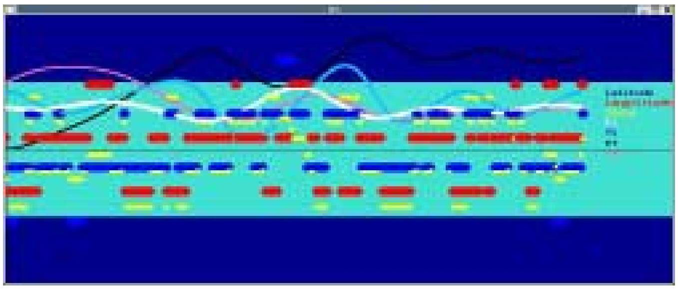
$$\frac{d\hat{k}_+}{dt} = \kappa_+ \hat{z}_t (1 - \hat{k}_+ \hat{z}_t), \tag{10b}$$

with positive gain (κ_-, κ_+), the Kalman filter (9) is adapted to observation $dy_t \sim \tau_t$. In the scheme (10), $\tilde{e}^-$ is simulated via the following observer.

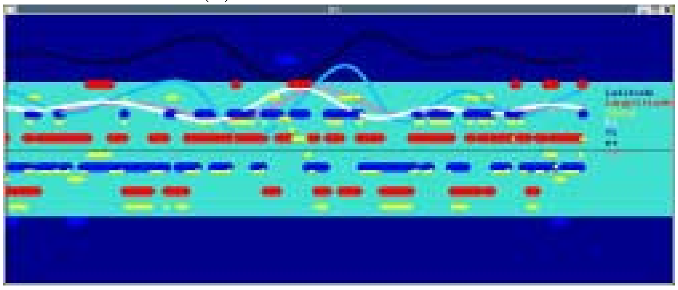
$$\frac{d\tilde{e}^-}{dt} = \alpha(1 - \hat{k}_+ \tilde{n}^+)\tilde{e}^- + \kappa_e(\tau_t - \tilde{e}^-), \tag{11a}$$

$$\frac{d\tilde{n}^+}{dt} = -\beta(1 - \hat{k}_- \tilde{e}^-)\tilde{n}^+ + \kappa_n(\hat{z}_t - \tilde{n}^+), \tag{11b}$$

based on the record of clock bias τ_t.



(a) Initial Estimation



(b) Updated Estimation

Figure 6: Filtering Results of a GPS/GNSS Data Based on themagnitude of positioning error (y_t, white), unknown population n^+ is estimated ($\hat{z}_t$, black). Through the adaptation of interaction parameters $(\hat{k}_-, \hat{k}_+)$, the estimate of the clock bias (pink), converges to the measurement (b).

8 EXPERIMENTS

The GPS/GNSS residual estimation system was implemented based on Eqs. (9), (10) and (11) with $a = b \sim 1/(t_1 - t_0)$ where t_1 and t_0 are determined to satisfy a stochastic constraint Eq. (6). Examples of estimation results are displayed in Fig. 6 where τ is assumed to be observed as the residual indicated in Fig. 1. In these experiment, 1446 samples of positioning data were applied iteratively to yield initial results (a) and the results by the second iteration (b). As shown in Fig. 6, the estimation system restore GPS/GNSS bias dynamically. In addition, it is shown that a couple of $t_1 - t_0$ period is sufficient for adapting the estimator to ionosphere state.

9 CONCLUDING REMARKS

A charge-carrier balancing model in ionosphere was applied to the estimation of GPS/GNSS residual. The model is adapted under the constraint of the Volterra's principle to restore the positioning error. Estimation system was verified to restore positioning bias through experiments.

References

[1] J. L. Crassidis and J. L. Junkins. *Optimal Estimation of Dynamical Systems*, Chapman & Hall, 2004.

[2] G. W. Prölss. *Physics of the Earth's Space Environment: An Introduction*, Springer-Verlag, 2004.