# DIGITAL WATERMARKING TECHNIQUE FOR PROJECTIVE-DISTORTED IMAGES USING COLLINEAR POINTS

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## ABSTRACT

Detection of a digital watermark in images distorted by geometric transforms is made difficult by the so-called synchronization problem. Many watermarking methods have been developed to be robust against an affine transform. Based on such assumption, those methods fail under a more general projective transform. In this paper, a method that is invariant to planar projective transform is presented. The method is based on a projective invariance property of the cross-ratio of four collinear points. Based on four coplanar points as obtained by extracting feature points from the host image, two sets of three collinear points are obtained. From each set of three collinear points, each watermark embedding location can be obtained through its cross-ratio relationship with those aforementioned three collinear points. A method to increase the number of embedding points in addition to those primarily derived from these sets of collinear points is also proposed. An algorithm to form a set of four coplanar points from feature points is also described.

## 1. INTRODUCTION

In recent works, many digital watermarking schemes have been developed to be robust against an affine transform [1-7]. Depending on their approaches, each of these methods falls roughly under one of the following categories: a normalization-based watermarking [1], invariant domain-based watermarking [2], template matching-based watermarking [3] and feature-based watermarking [4, 7].

These methods have been developed under the assumption that the watermarked image is geometrically distorted by an affine transform. They are thus not robust against projective distortion. In this paper, a watermarking method is developed to be robust against such distortion. The proposed scheme makes use of the projective invariant property of four collinear points. Based on a set of four coplanar feature points, watermark embedding locations are computed through the cross ratio relationship of each embedding point with three collinear points, which are in turn derived from the feature points. Perhaps use of the cross-ratio of collinear points under the context of digital watermarking was first investigated in [5]. However, there the theory is applied in the frequency domain, making it inapplicable for spatially distorted images (because a projective transform is non-linear). Use of the cross-ratio theory for projective-invariant watermarking was first described in [8]. The method in [8] is, however, not robust against image cropping.

This paper is organized as follow. In section 2, a theory of four collinear points and their invariance against planar projection is briefly described. In section 3, detail of the proposed perspective-invariant digital watermarking scheme using four collinear points is described. In section 4, a method for constructing a set of collinear points from extracted image features is provided. Section 5 reports some experimental results. Conclusion is given in section 6.

## 2. INVARIANCE THEORY OF FOUR COLLINEAR POINTS

The cross-ratio theory of four collinear points is briefly explained in this section. From Fig. 1, A, B, C, and D are points that lie along a straight line. These points are said to be collinear points. The cross-ratio of these points are given by

$$r_{CR} = \frac{\overline{CA} \cdot \overline{DB}}{\overline{CB} \cdot \overline{DA}} \tag{1}$$

where $\overline{ab}$ is the signed Euclidean distance of the line segment connecting two arbitrary points $a$ and $b$. For example, in this particular point arrangement, with $A$ as a reference point, $\overline{CA}$ is positive while $\overline{BD}$ is negative.

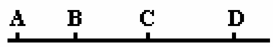


Figure 1: Example of four collinear points

## 3. DIGITAL WATERMARKING TECHNIQUE BASED ON THE CROSS-RATIO THEORY

Based on the cross-ratio theory as outlined in Section 2, an algorithm for embedding and detecting a watermark is described below. The algorithm assumes that four coplanar feature points on the image are given. This means no three of those points are collinear. How these points are obtained is deferred until Section 4.

### 3.1 Embedding Algorithm

1. Select a set of predefined cross-ratio values, to be used in subsequent steps.

2. Based on four coplanar feature points $P_i$ , $i = 1,...4$ , (see Fig. 2 where the four image corners are chosen as $P_i$ ), compute the intersection point of two lines which are constructed from these coplanar points. Denote the intersected point as $P_c$ .
3. By using the predefined cross-ratios, and two sets of three collinear points, Eq. (1) is applied to calculate the primary watermark embedding points along each of the two lines in Step 2. For example, in Fig. 2 (a), points along the line connecting $P_1$ and $P_3$ are calculated using $P_1$, $P_3$ and $P_c$ . Then, with $P_c$ , the two intersected lines are divided into four line segments. This in turn, divides these primary embedding points into four sets, denoted by $S_{tr}$, $S_{tl}$, $S_{br}$, and $S_{bl}$ (see Fig. 2 (a) ).
4. Compute the secondary watermark embedding points in the 'left' image segment as defined by the triangle $P_1 P_c P_4$ . This is achieved by first constructing lines joining each point in $S_{tl}$ with $P_4$ . Then construct similar lines which join each point in $S_{bl}$ with $P_1$ . The secondary watermark embedding points are obtained as the intersection points between these lines (see Fig. 2 (b)).
5. Repeat Step 4 to calculate secondary watermark embedding points on each of the other three image segments ('right', 'top', and 'bottom') as defined by the triangles $P_2 P_3 P_c$ , $P_1 P_c P_2$ , and $P_3 P_4 P_c$ . Fig 2 illustrates 1600 watermark embedding locations obtained using 40 primary-level watermark embedding points.
6. Embed a watermark pattern onto those embedding points obtained from Steps 4-5. Here a spread-spectrum based spatial watermarking method is assumed. Let $p_i^e$ , $i = 1, 2, \ldots, N$, be $N$ watermark embedding points. Given a watermarking pattern bits $w_i \in \{-1, 1\}$ , $i = 1, 2, \ldots, N$, a watermarked image is obtained by using the following equation [6]

$$I_e(x_i, y_i) = I(x_i, y_i) + (\alpha \cdot w_i) \tag{2}$$

where $I(x_i, y_i)$ is the original image, $(x_i, y_i)$ is the coordinate of $p_i^e$ , and $\alpha$ is the modulation factor.

### 3.2 Detecting Algorithm

To detect the watermark from a possibly distorted image $I_e^{'}$ , watermark embedding locations must first be recovered. Those points are obtained by applying Steps 1-5 of the embedding algorithm to $I_e^{'}$ . By using a correlator as a watermark detector, detection is performed by computing the following correlation coefficient.

$$C(I_e^{'}, w) = \frac{\frac{1}{N}\sum_{i=1}^{N}(\widetilde{I}_e^{'}(\widetilde{x}_i, \widetilde{y}_i) \cdot \widetilde{w}(i))}{\sigma_{I_e^{'}} \cdot \sigma_w} \tag{3}$$

where $(\widetilde{x}_i, \widetilde{y}_i)$ are watermark embedding points estimated from $I_e^{'}$, $\sigma_{I_e^{'}}$ and $\sigma_w$ are the standard deviations of $I_e^{'}(\widetilde{x}_i, \widetilde{y}_i)$ and the watermark pattern, respectively. In addition, $\widetilde{I}_e^{'}(\widetilde{x}_i, \widetilde{y}_i)$ and $\widetilde{w}(i)$ are zero-mean versions of $I_e^{'}(\widetilde{x}_i, \widetilde{y}_i)$ and $w(i)$, respectively. The watermark is detected if the correlation value is above a predefined threshold. The threshold may be defined, for example, by using certain statistical criteria [6].

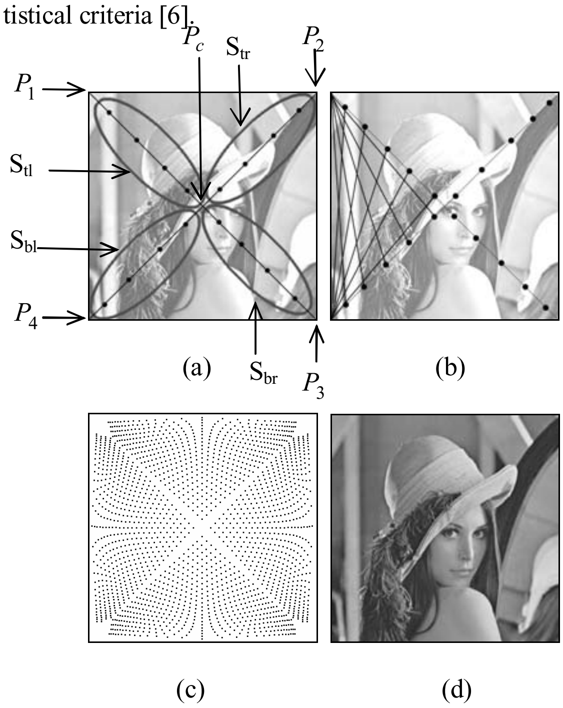


Figure 2: (a) Sets of primary watermark embedding points, (b) Secondary watermark embedding points obtained through line intersection, (c) Watermark embedding locations and (d) Watermarked image.

## 4. EXTRACTING COLLINEAR POINTS FROM IMAGE CONTENT

In the previous section, it is assumed that the four coplanar points are available. These points may be obtained, for example, by using four image corners. This method, however, is not applicable to a cropped image. In [7], image feature points are applied for normalization of an affine-transformed image. The method in [7] uses Delaunay tessellation to create triangles from a set of feature points. For each of the triangles formed in this manner, its three vertex points are used for image normalization, and thus watermark synchronization. Although for a particular set of points, Delaunay triangulation is unique, change in members of the point set as well as their coordinates may result in different set of triangles. Therefore, Delaunay triangulation does not guarantee that triangles formed during the embedding stage will be topologically the same as those computed during the detection stage. A set of feature points obtained from the possibly watermarked image may differ from that of the original one due to the following reasons.

1. A feature detector may produce different sets of feature points, when applied to the original and distorted images.
2. Change in relative locations of the feature points due to geometric distortion may result in different ways of triangle tessellation, to preserve the non co-circular rule of Delaunay tessellation. This problem arises due to the fact that the projective transform does not preserve length and angle.

To avoid these problems, a quadrilateral tessellation method is developed for our problem. Outline of the algorithms for the embedding and detecting stages is given below.

### 4.1 Feature extraction for the embedding stage

1. Use a detector of choice to detect a set of feature points. Let's $K$ be the number of chosen points which represent salient features in the image. For instance, a 'score' obtained from a detector for each point may be used as a criterion for the selection.
2. Choose the center points among the points with highest feature scores, to be later used for quadrilateral tessellation. There should be more than one of such points to avoid the cropping attack. The points should also be well distributed and cover most of the image area. Denote these points as $p_k^c$, $k = 1, \ldots, L$.
3. For each of $p_k^c$, form a quadrilateral with three other surrounding feature points. There can be many of such quadrilaterals. However, those with a large internal angle on any of its vertices are rejected, because they results in a small and dense embedding area.

Examples of quadrilaterals formed by the method is shown in Fig. 3 (a-b). In Fig 3(b), 3 sets of quadrilaterals are formed with center points located in different image parts. Such formation may be used to cope with image cropping.

### 4.2 Search algorithm for the detecting stage

The feature points extracted from the distorted image may differ from those of the original one. Some feature points appeared in the original image may not be detected, and vice versa. Exhaustive search is necessary to find a set of feature points corresponding to the watermark embedded area. To reduce computation required for direct search, a systematic algorithm is developed to search for a watermark. It is described below.

1. Select feature points with high scores as center point candidates. The number of candidate center points chosen should be slightly higher than the ones used at the embedding stage.
2. For each $p_k^c$, form a list of the surround feature points by sorting them according to their angular relationship with respect to $p_k^c$ (see Fig. 3 (c) for the counter-clockwise arrangement).
3. Choose each of the points in the list as a starting point, denoted by $p_k^s$. From such point form a set of $Q$ consecutive points in the sorted list. For example, in Fig. 3(b), the point $p_k^1$ is chosen as $p_k^s$, and along with $p_k^1$ to $p_k^5$, they form a set of $Q = 5$ consecutive points.
4. Use $p_k^c$ and $p_k^s$ to form all possible quadrilaterals with two of the other $Q$ - 1 points. For each quadrilateral formed, a watermark detection algorithm is applied (see Fig. 3(c), for example).
5. If in Step 4 a watermark is found, a detector is now 'synchronized' and the third point of the quadrilateral containing a watermark is used as a starting point ($p_k^s$) of the next block. Step 4 of the algorithm is then repeated.
6. If in Step 4 a watermark is not found, from the list obtained by Step 2, choose the point next to the current $p_k^s$ as a new starting point. Then repeat Step 4.

The choice of the search window size $Q$ depends on how different the feature points of the original and distorted images are likely to be. If a large difference is likely to be, $Q$ should be a high value. Based on empirical study, the choice of $Q$ between 4 to 7 is sufficient.

(a) (b) (c)

Figure 3: (a) Typical quadrilaterals formed by each reference point, (b) Using multiple center points to cope with image cropping, (c) Quadrilaterals formed at the detecting process.

## 5. EXPERIMENTAL RESULT

Experiment was performed to evaluate the performance of the proposed method. Six grayscale images of size $512 \times 512$ pixels were used in the experiment. Harris corner detector was employed to extract image features. Before applying the detector to extract feature points, a low-pass filter operator was first applied to each image.

In the first experiment, an affine-based method was tested to verify that a method based on the affine transform assumption fails under the projective transform. Four image corners were chosen as registration points, to estimate the affine transform parameters. Fifty different watermarks of length $N$ = 1,600 each were embedded over the image area by the proposed method with three center points. The fixed modulation factor $\alpha$ = 3 was used in all experiments. To reduce a problem due to local non-linear distortion, each watermark bit was embedded over an image area of size $3 \times 3$. The watermarked images were distorted by planar projection in the manners as shown in Fig. 5, with angle of projection changing from 5°, 10° and 15°. The distance between the center of projection and the image object is two times the object size. In Fig. 4, the resulting normalized correlation coefficients

($=C(I_e', w)\cdot\sqrt{N}$) are shown for the proposed method, along with the values obtained by using the affine-based method. From the figure, deficiency of an affine-based method is evident.

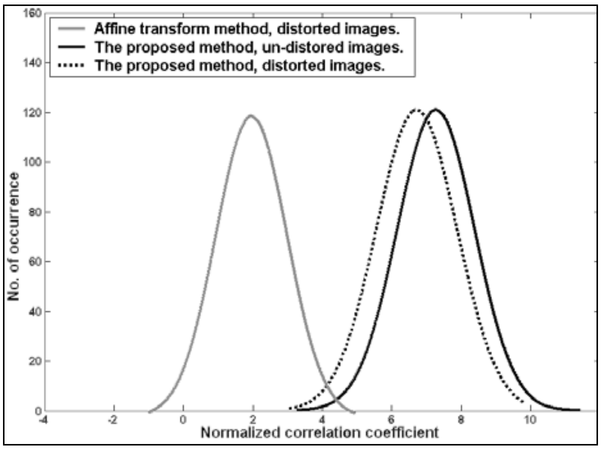


Figure 4: Distributions of the normalized correlation coefficients, obtained by an affine-based method and the proposed method.

By using the proposed detection scheme with the detection threshold of 4, the overall false positive detection rate was 0.11%, while the overall false negative rate was 1.21 %. Percentages of correct detection for each projection direction are provided Table 1. In all cases, correct detection rate is above 98.00 %.

In the second experiment, all embedded images were cropped at 80% and 90% of the original image area. In a separated experiment, they were scaled down by 80% and 90% of the original size. After cropping (scaling), images were projective distorted with the projection angle of 15$^o$, both horizontally and vertically. By using the proposed method with the detection threshold of 4, the overall false positive detection rate was 0.15%, while the overall false negative rate was 2.15%. In all cases, correct detection rate is above 97.00 %. See Table 2 for detailed results.

(a) (b) (c)

Figure 5: (a) Angle of projection = 5$^o$, (b) Angle of projection = 10$^o$, (c) Angle of projection = 15$^o$.

## 6. CONCLUSION

In this paper, the watermarking method that is invariant to planar projective transform has been presented. The method makes use of the invariance property of the cross-ratio of four collinear points. Watermark embedding locations are calculated from four co-planar feature points using a predefined set of cross ratios. A window-sliding algorithm has been provided to reduce watermark searching complexity.

| **Images** | **Angle of projection** | | |
|---|---|---|---|
| | **5$^o$** | **10$^o$** | **15$^o$** |
| Lena | 99.12 | 98.95 | 98.64 |
| Pepper | 99.21 | 98.83 | 98.59 |
| Baboon | 98.45 | 98.15 | 97.94 |
| Cameraman | 99.14 | 98.94 | 98.73 |
| Plane | 99.25 | 98.96 | 98.79 |
| Boat | 99.04 | 98.81 | 98.62 |
| **Total** | **99.04** | **98.77** | **98.56** |

Table 1: Experimental result for the first experiment

| **Images** | **Cropping** | | **Scaling down** | |
|---|---|---|---|---|
| | **90%** | **80%** | **90%** | **80%** |
| Lena | 98.25 | 97.96 | 98.01 | 97.85 |
| Pepper | 97.94 | 97.63 | 97.94 | 97.67 |
| Baboon | 97.59 | 97.16 | 97.21 | 97.02 |
| Cameraman | 98.14 | 97.92 | 98.12 | 97.87 |
| Plane | 98.34 | 98.04 | 97.98 | 97.61 |
| Boat | 98.24 | 97.98 | 98.04 | 97.85 |
| **Total** | **98.08** | **97.10** | **97.88** | **97.65** |

Table 2: Experimental result for the second experiment

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