IMPROVED MULTIUSER DIVERSITY USING SMART ANTENNAS WITH LIMITED FEEDBACK

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ABSTRACT

Multiuser diversity and beamforming are two techniques that promise dramatically increased system throughput and spectral efficiency. In most systems it has however been considered infeasible to utilize spatial channel information, due to the increased feedback load.

In order to exploit the benefits of both multiuser diversity and beamforming, we show how to compute the second order channel statistics, conditioned on the norm of the current channel realization. The conditional channel statistics allow for elaborate scheduling and smart antenna techniques, with limited feedback.

Herein, the downlink of a single cell multiple-input single-output (MISO) system is considered. Only the current signal to noise ratio (SNR) is fed back from the mobile stations, whereas the second order channel statistics can be estimated at the base station from information collected in the uplink. A simple scheduling/eigenbeamforming scheme is proposed and shown to outperform opportunistic beamforming, which is a technique with similar feedback.

1. INTRODUCTION

Many techniques have been proposed to increase the downlink system throughput by means of efficient scheduling of users over time. The concept of multiuser diversity [1] has recently been given much attention. By always scheduling users in the time slots, in which they experience a particularly strong channel realization, a high throughput is ensured in each slot. Proportional fair scheduling [2] has been proposed as a good trade-off between fairness and system throughput.

Smart antenna techniques are a different approach to increase the system throughput and spectral efficiency. By employing multiple antennas at the base station, it is possible to utilize the spatial characteristics of the channel. By creating a beam to the targeted user, the received signal power is increased (without increasing the transmitted power), whereas the interference to others is decreased. The drawback is, apart from increased computational complexity, that the transmitter needs accurate channel state information (CSI). Unless a time division duplex (TDD) system is considered, the channel realizations can only be estimated at the receiver, and the CSI must be fed back to the transmitter. For a rapidly fading multiple-input single-output (MISO) channel, this is often prohibitive.

In the beamforming literature [3, 4] the feedback requirements are typically overcome by using the second order statistics of the vector channel, rather than the particular realization. This has the advantage that second order statistics of the downlink channel can be estimated directly at the transmitter from the uplink, even if the uplink and downlink are separated in frequency [3], and thus eliminating the need of feedback. The beamformer optimization will however only ensure that the *expected* value of the signal to interference and noise ratio (SINR) of each scheduled user will be above a chosen threshold. However, since the transmitter does not have any information about the current channel realization this scheme is unable to exploit the benefits of multiuser diversity, and further it is necessary to add a substantial fade margin to the target average SINR to ensure a reasonable outage probability.

Herein, it is shown how to combine the second order channel statistics with the *norm* of the current vector channel realization. The channel norm is obtained using fast feedback of an estimated instantaneous signal to interference and noise ratio (SNR). The channel norm provides a measure of the quality of the current channel realization, and is easily estimated from broadcasted pilot signaling at the mobile stations. The conditional second order statistics can be used for smart antenna beamforming and elaborate scheduling, which efficiently utilize the multiuser diversity. The conditional second order statistics further have several nice properties when the norm of the vector channel is particularly strong, which is the case for the users that are likely to be scheduled. These properties ensure reliable communication with relatively small fade margins.

We present a simple scheduling/beamforming scheme which utilizes the information provided by the conditional second order statistics. A single user is scheduled in each time slot, which is chosen according to the proportional fair criterion. The scheduled user exploits the spatial characteristics of the channel by means of eigenbeamforming [5]. With appropriate modifications, the concept of conditional statistics could however be used for more elaborate spatial-division multiple-access (SDMA) beamforming and scheduling techniques.

The proposed scheme should be put in contrast to opportunistic beamforming [2] which has a similar feedback rate. Opportunistic beamforming uses a randomized beamformer and relies on multiuser diversity to ensure that there is always a user in the main lobe. The scheduling scheme proposed herein is a smart antenna scheme, which actively steers the antenna array gain to any desired user, contrary to opportunistic beamforming. With such a scheme the throughput is dramatically increased and the delays are decreased, with the same low feedback rate.

2. SYSTEM MODEL

To benefit from multiuser diversity it is required that several users are present in the cell, hence a macro cell with an elevated base station is considered herein. The base station is equipped with an antenna array with M antennas and communicates with K mobile users with a single receive antenna. The mobile terminals are considered dumb, in the sense that no computationally demanding signal processing should be required of them.

A narrowband frequency flat channel is considered. The fading of the channel is modeled at two different time scales. The small scale fading, due to multipath propagation, is modeled as Rayleigh fading. The correlation matrix of the vector channel $\mathbf{h}_i \in \mathbb{C}^M$ of user i, is assumed to be of low rank and is denoted $\mathbf{R}_i$. The large scale fading, caused by macroscopic effects in the environment, varies much more slowly and affects $\mathbf{R}_i$. It is assumed that the base station is able to perfectly track the current $\mathbf{R}_i$ for all users using information collected from the uplink. This assumption is reasonable in most scenarios, even if the uplink and downlink streams are separated in frequency (using frequency transformation). For an overview of such estimation/transformation techniques, see [3].

The signal received by user i is given by

$$y_i(t) = \mathbf{h}_i^* \mathbf{u}\, x(t) + n_i(t),$$

where $\{\cdot\}^*$ denotes Hermitian complex transpose, $\mathbf{u} \in \mathbb{C}^M$ is the chosen unit norm beamformer and $n_i(t)$ is additive zero mean circular symmetric complex Gaussian noise with power σ_i^2. The signal intended for the scheduled user, $x(t)$, has an average power constraint $\mathrm{E}\left[|x(t)|^2\right] = \mathrm{P}_{\max}$.

3. CONDITIONING THE SECOND ORDER STATISTICS

In order to design a scheduler that utilizes the multiuser diversity, the base station must be provided with information about the current channel realization for all users. For simplicity all indexes denoting a particular user are dropped throughout this section. If the base station would have perfect channel knowledge and utilize maximum ratio combining beamforming, the received signal power at the desired user is given by $\|\mathbf{h}\|^2\,\mathrm{P}_{\max}$. This suggests that $\|\mathbf{h}\|$ is a good measure on the quality of the current channel realization, but does not provide any information about the spatial characteristics of the channel realization.

On the contrary, the correlation matrix, $\mathbf{R}$, provides information about the spatial characteristics, in particular if it is of low rank, but does not give any information about the quality of the current realization. The two quantities above thus complement each other and are therefore desirable to combine in order to give a better estimate of the power received at the users when applying different beamformers.

The minimum mean square error estimate of the received signal power for a user, given that $\|\mathbf{h}\| = r$ is given by

$$\begin{aligned} \hat{p}(r,\mathbf{u}) &= \mathrm{E}\left[|\mathbf{h}^*\mathbf{u}\, x(t)|^2 \middle| \|\mathbf{h}\| = r\right] \\ &= \mathbf{u}^*\widehat{\mathbf{R}}(r)\mathbf{u}\,\mathrm{P}_{\max}, \end{aligned}$$

where $\widehat{\mathbf{R}}(r) \triangleq \mathrm{E}\left[\mathbf{h}\mathbf{h}^* \middle| \|\mathbf{h}\| = r\right]$ is the channel covariance matrix conditioned on the given channel norm.

3.1 Computing the conditional covariance matrix

Theorem 1. *Let $\mathbf{R} = \mathbf{U}\Sigma\mathbf{U}^*$ be the eigenvalue decomposition of $\mathbf{R}$. Furthermore assume that the eigenvalues, λ_m, of $\mathbf{R}$ are distinct and strictly positive, then $\widehat{\mathbf{R}}(r) = \mathbf{U}\widehat{\Sigma}(r)\mathbf{U}^*$ where $\widehat{\Sigma}(r)$ is a diagonal matrix with diagonal elements $\widehat{\lambda}_m(r)$ given by*

$$\begin{aligned} \widehat{\lambda}_m(r) = \frac{2r}{f_{\|\mathbf{h}\|}(r)} \Bigg[& \frac{r^2 e^{-\frac{r^2}{\lambda_m}}}{\lambda_m \prod_{i\neq m}\left(1-\frac{\lambda_i}{\lambda_m}\right)} + \\ & + \sum_{k\neq m} \frac{e^{-\frac{r^2}{\lambda_m}} - e^{-\frac{r^2}{\lambda_k}}}{\left(1-\frac{\lambda_k}{\lambda_m}\right)\prod_{i\neq k}\left(1-\frac{\lambda_i}{\lambda_k}\right)} \Bigg], \end{aligned} \quad (1)$$

where $f_{\|\mathbf{h}\|}(r)$ is the distribution function of $\|\mathbf{h}\|$, given by

$$f_{\|\mathbf{h}\|}(r) = \sum_{k=1}^{n} \frac{2re^{-\frac{r^2}{\lambda_k}}}{\lambda_k \prod_{i\neq k}\left(1-\frac{\lambda_i}{\lambda_k}\right)}.$$

Proof. Not given here because of space limitations. □

The updated eigenvalues can thus be computed efficiently from the closed form formula (1). Note that the denominators are independent of r, and can be precomputed. Also note that $\sum_{m=1}^{M} \widehat{\lambda}_m = r^2$, by definition, so the normalization $2r/f_{\|\mathbf{h}\|}(r)$ can be computed without evaluating $f_{\|\mathbf{h}\|}(r)$ if all eigenvalues are computed.

The requirement that the eigenvalues λ_i of $\mathbf{R}$ should be distinct and strictly positive does not pose a problem since this is always the case in practice and typically true for any realistic stochastic MISO channel model.

3.2 Properties for strong channel norm

In order to get a better insight in the characteristics of the CSI available at the transmitter, when $\|\mathbf{h}_i\|$ is fed back from the terminals, it is useful to change the coordinates of the channel vector. Any zero-mean complex Gaussian vector $\mathbf{h}$ with covariance matrix $\mathbf{R}$ can be written as

$$\mathbf{h} = \tilde{h}_1\mathbf{u}_1 + \tilde{h}_2\mathbf{u}_2 + \cdots + \tilde{h}_M\mathbf{u}_M, \quad (2)$$

where $\tilde{h}_i \in \mathbb{C}$ are independent complex Gaussian variables with variance λ_i. $\mathbf{u}_i$ and λ_i are found as the eigenvectors and eigenvalues of $\mathbf{R}$, respectively, and are assumed ordered decreasingly ($\lambda_i > \lambda_j,\ j > i$). Furthermore, the current norm of the channel is given by $\|\mathbf{h}\|^2 = \sum_i |\tilde{h}_m|^2$ and the result of Theorem 1 translates into

$$\mathrm{E}\left[|\tilde{h}_m|^2 \middle| \|\mathbf{h}\| = r\right] = \widehat{\lambda}_m(r). \quad (3)$$

Several interesting observations can be made for realizations when $\|\mathbf{h}\|$ is particularly strong. Firstly, as $r = \|\mathbf{h}\|$ tends to infinity, the conditional correlation matrix tends to rank one

$$\lim_{r\to\infty} \frac{\widehat{\lambda}_m(r)}{\widehat{\lambda}_1(r)} = 0,\ \forall m > 1,$$

which follows directly from (1). This is an interesting observation since it states that the correlation matrices can be approximated as rank one with arbitrary (relative) accuracy when the channel norm is large enough. Hence, if the number of uses in the cell increases and the norm of the scheduled user's channel thereby becomes stronger, the associated correlation matrix will tend to rank 1 and eigenbeamforming along the principal eigenmode, i.e. $\mathbf{u} = \mathbf{u}_1$, will perform close to maximum ratio combining (MRC), which is optimal when only a single user is scheduled at a time.

Secondly, for a low rank model (as herein), with one or a few dominating eigenvalues, it is highly unlikely that $|\tilde{h}_1|^2 \ll \widehat{\lambda}_1(\|\mathbf{h}\|)$ when $\|\mathbf{h}\|$ is strong. For example, if there are two dominating eigenmodes, the event that $|\tilde{h}_1|^2$ is weak corresponds to $|\tilde{h}_2|^2 \gg \widehat{\lambda}_2(\|\mathbf{h}\|)$. Such an increase is highly unlikely for the Rayleigh distributed variable $|\tilde{h}_2|^2$. Figure 1 illustrates this effect for a scenario where the eigenvalues of $\mathbf{R}$ are distributed as $\lambda_i \propto e^{-\alpha i}$. The outage probability

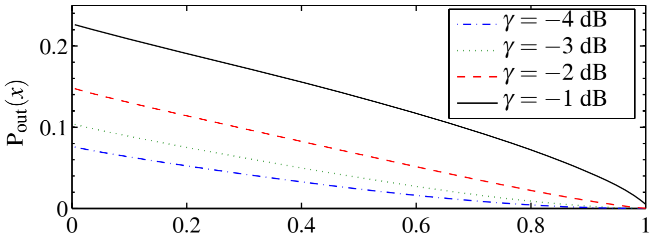
$$\mathrm{P}_{\mathrm{out}}(x) = \Pr\left(\frac{|\tilde{h}_1|^2}{\widehat{\lambda}_1(\|\mathbf{h}\|)} < \gamma \middle| \|\mathbf{h}\| > Q_x\right)$$

is plotted as a function of x for different values of α and γ. Q_x denotes the x-quantile, i.e. $\Pr(\|\mathbf{h}\| \leq Q_x) = x$. It is observed in the figure that the outage probability tends to zero as the channel realizations become stronger, which is explained by the correlation matrix tending to rank one.

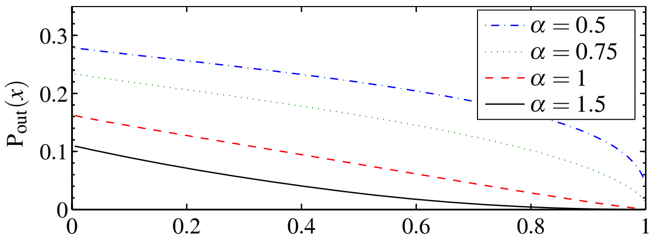
There is however limitations with the CSI provided by the conditional second order statistics. The most obvious is that no phase information about $\tilde{h}_i$ in (2) is available at the transmitter. Hence, when applying an arbitrary beamformer $\mathbf{u}$ it is not possible to reliably estimate the received signal power. To see this, it is useful to expand the received signal using (2),

$$r(t) = \left(\tilde{h}_1^*(\mathbf{u}_1^*\mathbf{u}) + \tilde{h}_2^*(\mathbf{u}_2^*\mathbf{u}) + \cdots + \tilde{h}_M^*(\mathbf{u}_M^*\mathbf{u})\right)s(t) + n(t). \quad (4)$$

Due to the lack of phase information, it is not possible to predict whether these terms will add constructively or destructively. The obvious solution is to beamform along the principal eigenmode (eigenbeamforming), $\mathbf{u} = \mathbf{u}_1$, which will cancel all terms, but the first. Such eigenbeamforming does however prevent an efficient implementation of SDMA, which requires more freedom in choosing the beamformer. Herein we focus on eigenbeamforming and scheduling of a single user at a time, but the concept of conditional second order statistics can be extended to be used with SDMA.



(a) Outage probability for different γ and fixed $\alpha = \frac{3}{2}$



(b) Outage probability for different α and fixed $\gamma = \frac{2}{3}$

Figure 1: The outage probability $P_{out}(x)$ is plotted against x for different thresholds γ and eigenvalue distributions α. In (a) it can be seen that for strong channel realizations the outage probability tends to zero for arbitrary $\gamma < 1$. In (b), the same is observed for arbitrary eigenvalue distributions, $\alpha > 0$.

4. EIGENBEAMFORMING WITH SNR FEEDBACK

In this section the proposed proportional fair scheduler, utilizing the second order statistics conditioned on the norm feedback, is presented in more detail. The system operates by alternating a pilot signaling interval for channel norm estimation, and a data transmission interval.

During the pilot signaling interval, the base station transmits orthogonal signals on all antennas, which allows all users to simultaneously estimate the norm of their current channel realization. This is done without any demanding signal processing at the user terminals, which just estimate the total received energy during the interval. Note that the pilot signals must be at least M symbols long in order to span all the spatial dimensions necessary to estimate the channel norm. The estimated SNRs, $\mathrm{SNR}_i = \mathrm{P}_{\max} \|\mathbf{h}_i\|^2 / \sigma_i^2$, are next fed back to the base station, from which the base station can obtain both $\|\mathbf{h}_i\|$ and σ_i^2. By taking the time average of SNR_i

$$\mathrm{E}[\mathrm{SNR}_i] = \mathrm{P}_{\max} \frac{\mathrm{E}\left[\|\mathbf{h}_i\|^2\right]}{\sigma_i^2} = \mathrm{P}_{\max} \frac{\mathrm{Tr}\{\mathbf{R}_i\}}{\sigma_i^2},$$

where $\mathrm{Tr}\{\cdot\}$ is the matrix trace operator, the base station obtains σ_i^2 as $\mathrm{P}_{\max} \frac{\mathrm{Tr}\{\mathbf{R}_i\}}{\mathrm{E}[\mathrm{SNR}_i]}$ and $\|\mathbf{h}_i\|^2 = \sigma_i^2 \frac{\mathrm{SNR}_i}{\mathrm{P}_{\max}}$.

The length of the data transmission interval is chosen to be comparable to the coherence time of the channel, and the SNRs of the different users are therefore assumed not to change during this interval. The data transmission interval is further divided into slots. In each slot a single user is scheduled to access the channel.

In line with Section 3.2, the beamformer $\mathbf{u}_i$ is always chosen as the principal eigenvector of $\mathbf{R}_i$, i.e. eigenbeamforming along the strongest eigenmode. Note the notational difference to Section 3; here $\mathbf{u}_i$ denotes the first eigenvector, for user i, not the i:th eigenvector.

The SNR experienced by user i, when beamforming along $\mathbf{u}_i$ is estimated, using (3), as

$$\widehat{\mathrm{SNR}}_i = \mathrm{E}\left[\mathrm{P}_{\max} \frac{|\mathbf{h}_i^* \mathbf{u}_i|^2}{\sigma_i^2} \,\middle|\, \|\mathbf{h}_i\|\right] \delta_i = \frac{\mathrm{P}_{\max} \widehat{\lambda}_1(\|\mathbf{h}_i\|)}{\sigma_i^2} \delta_i, \quad (5)$$

where $\widehat{\lambda}_1(\|\mathbf{h}_i\|)$, with slight abuse of notation, denotes the first (strongest) eigenvalue of $\widehat{\mathbf{R}}_i$, and $0 < \delta_i < 1$ is a fade margin to ensure that the SNR is not overestimated, which would cause an outage. As shown in Section 3.2, a relatively small fade margin typically ranging from -1 to -3 dB, is enough to guarantee a low outage probability.

In each slot, the user maximizing the proportional fair criterion [2] is scheduled. This criterion is given by

$$\text{user} = \arg\max_i \frac{\mathrm{rate}\left(\widehat{\mathrm{SNR}}_i\right)}{\overline{\mathrm{rate}}_i},$$

where $\mathrm{rate}(\mathrm{SNR})$ is a (non-decreasing) function, mapping the SNR to the maximum supported rate, and $\overline{\mathrm{rate}}_i$ is the average rate during some window of interest.

The average rate is typically updated using a first order low-pass filter,

$$\overline{\mathrm{rate}}_i(n+1) = \left(1 - \frac{1}{t_c}\right) \overline{\mathrm{rate}}_i(n) + \frac{1}{t_c} \mathrm{rate}_i,$$

where rate_i is the rate allocated to user i in the current slot, n, and t_c is the time scale of the scheduler, i.e. the time frame (in slots) in which to average over.

The proportional fair criterion favors users that experience a particularly strong channel realization and users that have been disfavored in previous slots. The proportional fair scheduler thereby provides a good trade-off between system throughput and fairness.

It should be noted that the proposed scheme would benefit if the mobile stations were to feedback the quantity

$$\mathrm{P}_{\max} \frac{|\mathbf{h}_i^* \mathbf{u}_i|^2}{\sigma_i^2},$$

which would eliminate the need for taking the expected value in (5), which in turn makes the fade margin δ_i superfluous. This gain does however come at the cost of computationally demanding signal processing at the mobile terminals, requiring the eigenvalue decomposition and estimation of $\mathbf{R}_i$, as well as adaptive tracking of $\mathbf{h}_i$.

5. PERFORMANCE EVALUATION

In this section the performance of the proposed scheduler is evaluated and compared to opportunistic beamforming, which requires the same level of feedback as the proposed scheme. The cumulative distribution functions (CDF) of the user rates, as well as the system throughput is computed by Monte Carlo simulations for different scenarios.

5.1 Simulation Parameters

The base station is equipped with an elevated uniform circular array (UCA) with half a wavelength antenna separation. A circular cell is considered, in which the users are distributed uniformly. Each user is surrounded by 1 to 3 scattering clusters (with equal probability). The scatterers within each scattering cluster are approximated as Gaussian distributed with an angular spread of 5°. The signal power received from each cluster is set to be proportional to $(r_1 r_2)^{-2}$, where r_1 is the distance from the base station to the scattering cluster, and r_2 is the distance from the scattering cluster to the mobile station. During the simulation, r_2 was drawn from a Rayleigh distribution, with second order moment $\mathrm{E}\left[r_2^2\right]$ set to 0.2 of the cell radius. (The scattering clusters are distributed circular symmetric around the mobile stations.)

The large scale shadow fading affects the scaling of $\mathbf{R}_i$ and was modeled as log normally distributed with 3dB standard deviation. The expected value of the diagonal elements of $\mathbf{R}_i$ is set to be proportional to $1/r^2$, where r is the distance to the mobile station. The noise power is set to be equal for all users, and the expected SNR (in dB) of a user at the cell border is 10 dB.

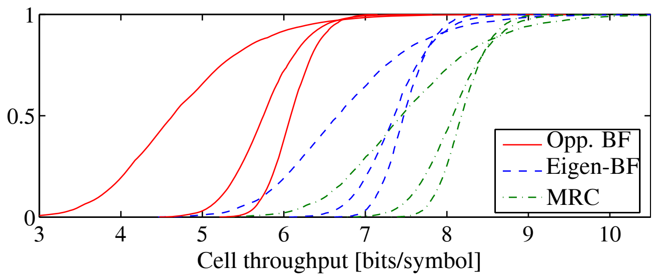


Figure 2: The CDF (over scenarios) of the total cell throughput for a system with 8 antennas. The throughputs of opportunistic beamforming is compared with the proposed eigenbeamforming with SNR feedback. The throughput of MRC is plotted as a reference. The CDFs are plotted for 4, 16 and 32 users (increasing performance).

The statistics in each simulation were collected during 3000 scenarios. Each scenario represents a macroscopic setup, i.e. a realization of the mobile terminal positions and correlation matrices $\mathbf{R}_i$. During each scenario the average performance in terms of throughputs of each user, as well as total system throughput, was evaluated.

The small scale (fast) fading of the channel was approximated as block fading. Each channel realization block fits the pilot signaling interval and 4 slots for data transmission. The performance in each scenario was evaluated during 650 channel realization blocks.

It is further assumed that the rate can be adapted continuously. The maximum supported rate for a given SNR is approximated by

$$\text{rate}(\text{SNR}) = \log_2\left(1 + \frac{\text{SNR}}{\text{Gap}}\right),$$

where the gap was set to 2 dB.

The randomized beamformer of the opportunistic beamforming was kept fix during each block of 4 slots and the SNR was only fed back once every block, like the proposed scheme.

Finally, the time scale of the scheduler, t_c, was set to 400 slots (100 blocks) and the margin δ_i in the SNR estimation was fixed to -2 dB. This resulted in an outage probability below 2.5% in all simulations. As the number of users in the cell increases this probability decreases to 0.2% in the case of 32 users.

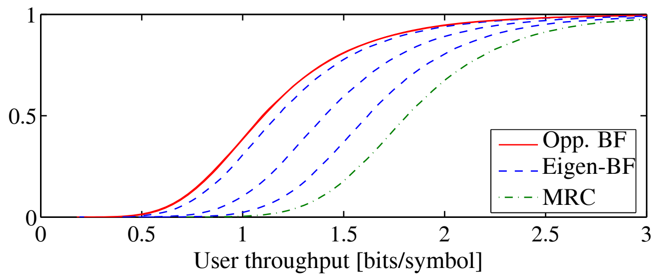
5.2 Simulation Results

The simulation results are summarized in Figures 2 and 3. In Figure 2 the cell throughput is considered. The cumulative distribution function (CDF) (over scenarios) of the average cell throughput is plotted for different numbers of users in a cell with 8 antennas. Opportunistic beamforming is compared to the proposed eigenbeamforming with SNR feedback. The performance of coherent beamforming (MRC) with perfect CSI is also given as reference. The proposed scheme has a significantly higher cell throughput in all cases, but the gap is reduced as the number of users increase.

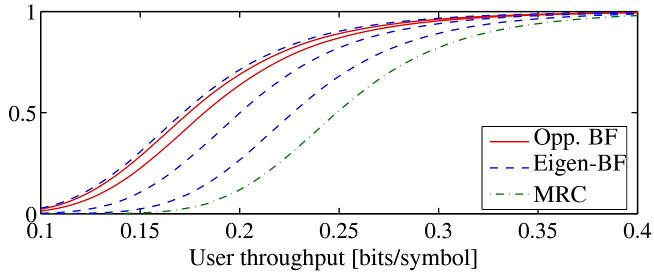
In Figure 3 the CDF (over users and scenarios) of the average throughput for a single user is plotted. A lightly loaded (4 users) and a heavily loaded (32 users) system is considered, and the user throughput CDF for different numbers of transmitting antennas is given. As a reference, the performance of MRC is given for the 8 antenna case.

It is observed that eigenbeamforming with SNR feedback outperforms opportunistic beamforming in most scenarios. Only for a heavily loaded system with few antennas is opportunistic beamforming marginally better. In such a scenario the probability that there are a user in the main lobe is high and opportunistic beamforming benefit by not needing any margins in the SNR estimates.

Opportunistic beamforming is however not able to utilize the beamforming gain provided when the number of antennas is increased, contrary to the proposed technique which operates close to coherent beamforming. The performance gain of the proposed scheduler over opportunistic beamforming is most significant for a moderate number of users.



(a) Lightly loaded (4 users)



(b) Heavily loaded (32 users)

Figure 3: The CDF (over users and scenarios) of the average user throughput for a cell with 4 and 32 users. For opportunistic beamforming the CDF for 2 and 8 antennas are plotted, but are only distinguishable for 32 users (8 antennas is better). The throughput for eigenbeamforming with SNR feedback is plotted for 2, 4 and 8 antennas (increasing performance). The performance of MRC for 8 antennas is plotted as a reference.

6. CONCLUSIONS

A closed form expression for the second order channel statistics, conditioned on the current norm of the channel, has been presented. The conditional statistics allow for elaborate scheduling and beamforming, which take advantage of the multiuser diversity and spatial characteristics of the channel.

A simple scheduling/eigenbeamforming technique for the downlink, utilizing SNR feedback from all users and the second order statistics of the channel, which is estimated at the transmitter, is proposed. The scheduler, based on the proportional fair criterion, is shown to outperform opportunistic beamforming, without increasing the computational load at the mobile terminals or the feedback rate. The difference in performance is particularly large for moderate numbers of users in the cell, due to the increased efficiency of utilizing the multiuser diversity.

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