JOINT OVERSAMPLING FDM DEMULTIPLEXING AND PERFECTLY RECONSTRUCTING SBC FILTER BANK FOR TWO CHANNELS[1]

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ABSTRACT

Within modern satellite communication scenarios, different users demand different bandwidths. Hence, digital processors should allow for flexible processing of FDM-signals for meeting these requirements. In that scope, two mutually exclusive processor tasks are distinguished: *i*) mere demultiplexing of independent FDM-signals and *ii*) demultiplexing of wideband channel signals with subsequent perfect reconstruction. In this paper, we present a matrix description of the well-known FDMUX filter bank [1] for two channels based on directional filter cells with reduced computational burden. Furthermore, we extend the improved approach to a novel joint SBC-FDFMUX filter bank structure, which combines both demanded tasks without extra computation compared to [1].

1. INTRODUCTION

Modern satellite communication systems demand highly efficient digital processors, which flexibly allow for allocating different channel bandwidths to different users. In case of a challenging FDM-scenario [4], for instance, a small number of ground-based gateways provides a large number of users with a ubiquitous internet access through a bent-pipe satellite.

Here, as for the digital satellite processor, the incoming FDM signal shall always be channelised to granularity level, i.e. the elementary channel size, before remultiplexing. To this end, in addition to a switch-function, on-board processing calls for digital filter banks with opposing features: *i*) The **FDFMUX filter bank**, where the FMUX solely performs remultiplexing of granules being independent of each other [1, 2]. In case of oversampling of the granules, suitable band limitation has to be foreseen between FDMUX and FMUX [3, 5]. *ii*) The **SBC filter bank**, where the FMUX is matched to the FDMUX for (almost) perfect reconstruction (PR) of those granules that belong to a single wideband channel signal allocated to one user [3, 5].

For both types of filter banks, efficient implementations are known. The efficiency of the FDFMUX is achieved by the use of halfband (or M-th band) filters where, roughly, every other (every M-th) coefficient is zero [1, 2, 3]. The PR property is obtained by the application of power complementary filters both for FDMUX and FMUX [3, 5]. Obviously, both requirements exclude each other.

The aim of this paper is to combine both of the above functionalities in a joint **SBC-FDFMUX filter bank** to the highest possible efficiency attainable. Subsequently, we restrict ourselves to the processing of two granules only, while the extension to a higher number of granules will be reported in forthcoming contributions. Furthermore, we apply oversampling by two according to [1, 2, 3, 4].

2. BASIC APPROACH

Our proposal is based on the Universal Directional Filter Cell (UNDIFICE) according to [1, 2], which applies oversampling by two to all signals involved. In its original form [1] it has the potential of

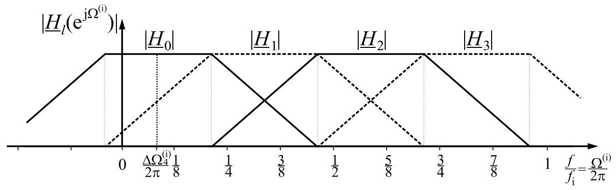

Figure 1: The four potential channel filter functions of the UNDIFICE with frequency offset $\Delta\Omega_4^{(i)}$.

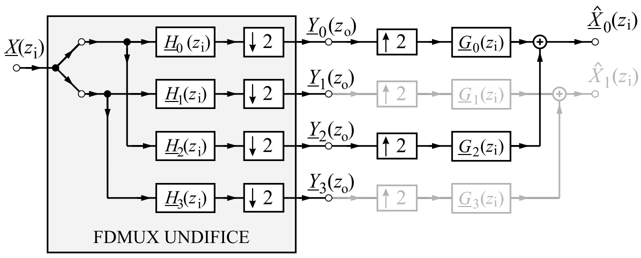

Figure 2: The SBC-FDFMUX filter bank in parallel structure.

four transfer functions, as depicted in figure 1 for the FDMUX case with an additional arbitrary frequency shift, from which only two are used for signal splitting, in general.

In order to construct an SBC-FDFMUX filter bank with small extra expenditure compared to an SBC filter bank, the full FDMUX UNDIFICE is split into two 2-channel FDMUX filter banks by assigning the even-numbered channels to one filter bank and the odd-numbered channels to the other one. Each of these filter banks is then separately complemented with a succeeding two channel FMUX filter bank to form an SBC filter bank, as shown in figure 2.

As a result, suitable filter design provided [3, 5], each output signal $\hat{\underline{X}}_0(z_i)$ and $\hat{\underline{X}}_1(z_i)$ represents an (almost) perfect reconstruction of the input signal $\underline{X}(z_i)$, whereas the signals $\underline{Y}_0(z_o)$ and $\underline{Y}_1(z_o)$ represent, for instance, the desired granules of the FDM input signal after spectral separation. Note that only one of the signals $\hat{\underline{X}}_0(z_i)$ and $\hat{\underline{X}}_1(z_i)$ must be calculated.

Subsequently, the UNDIFICE is recalled in matrix representation, and modified for getting maximum insight into the design problem and the reasons for efficiency in order to develop an efficient SBC-FDFMUX filter bank.

3. UNIVERSAL DIRECTIONAL FILTER CELL

A detailed time domain description of the UNDIFICE is given in [1] within the presentation of the *Hierarchical Multistage Method*: It is a 4-channel oversampling filter bank with a decimation factor of $M = 2$ applied for efficient demultiplexing, thus applying an

[1]This work was supported by Deutsche Forschungsgemeinschaft under contract GO849/3-1.

oversampling factor of 2 for all appearing signals.

This oversampling is exploited to design a real prototype FIR (halfband) filter with extended transition bands (cf. figure 1) and, hence, low order [1], from which the $I = 4$ channel filter functions of the UNDIFICE are derived by uniform frequency shifting by

$$\Omega_l^{(\mathrm{i})} = 2\pi \frac{f_l}{f_\mathrm{i}} = l \frac{2\pi}{I} + \Delta\Omega_I^{(\mathrm{i})}, \tag{1}$$

where $l \in \{0, 1, 2, 3\}$ is the channel index, f_i the input sampling frequency and $|\Delta\Omega_I^{(\mathrm{i})}| \in \left[0 \ldots \frac{\pi}{I} = \frac{\pi}{4}\right]$ an additional arbitrary frequency offset of the whole channel allocation scheme.

In the following, complex-valued signals and systems are indicated by underlining. The $R \times C$ matrix $\mathbf{M}$ with R rows and C columns is referred to by using $\mathbf{M} = [M(r,c)]_{\substack{r=0,1,\ldots,R-1;\\ c=0,1,\ldots,C-1.}}$, where $M(r,c)$ is the element of $\mathbf{M}$ in the $(r+1)th$ row and the $(c+1)th$ column. For a $Q \times Q$ diagonal matrix $\mathbf{D}$ the diagonal elements are denoted by $D(q,q) = D(q)$ and $\mathbf{D} = \mathrm{diag}\{D(q)\}_{q=0,\ldots,Q-1}$.

3.1 Alias Component Matrix Description of the UNDIFICE

As an oversampling 4-channel ($I = 4$) filter bank with a decimation factor of $M = 2$, the UNDIFICE is described in z-domain by its $M \times I$ rectangular alias component matrix [3]

$$\underline{\mathbf{H}}_I^{(\mathrm{ac})}(z_\mathrm{i}) = \underline{\mathbf{H}}_4^{(\mathrm{ac})}(z_\mathrm{i}) = \left[\underline{H}_l(z_\mathrm{i} W_2^k)\right]_{\substack{k=0,1;\\ l=0,\ldots,3.}}, \tag{2}$$

containing the channel filter functions and their alias components with $W_M = \mathrm{e}^{-j\frac{2\pi}{M}}$ and $M = 2$. The four potential output signals of the UNDIFICE are comprised in the $I \times 1$ vector

$$\underline{\mathbf{Y}}_I(z_\mathrm{i}^M) = \underline{\mathbf{Y}}_4(z_\mathrm{i}^2) = \left[\underline{Y}_l(z_\mathrm{i}^2)\right]_{\substack{l=0,\ldots,3;\\ c=0.}} \tag{3}$$

and given by [3]

$$\underline{\mathbf{Y}}_4(z_\mathrm{i}^2) = \frac{1}{2}\left[\underline{\mathbf{H}}_4^{(\mathrm{ac})}(z_\mathrm{i})\right]^T \cdot \underline{\mathbf{X}}_2^{(\mathrm{ac})}(z_\mathrm{i}), \tag{4}$$

where the $M \times 1$ vector

$$\underline{\mathbf{X}}_M^{(\mathrm{ac})}(z_\mathrm{i}) = \underline{\mathbf{X}}_2^{(\mathrm{ac})}(z_\mathrm{i}) = \left[\underline{X}(z_\mathrm{i} W_2^k)\right]_{\substack{k=0,1;\\ c=0.}} \tag{5}$$

contains the alias components of the input signal $\underline{X}(z_\mathrm{i})$. Since only two of the four output signals are of interest, (4) can be modified by considering $\underline{\mathbf{Y}}_2(z_\mathrm{i}^2) = \left[\underline{Y}_0(z_\mathrm{i}^2) \quad \underline{Y}_1(z_\mathrm{i}^2)\right]^T$ instead of $\underline{\mathbf{Y}}_4(z_\mathrm{i}^2)$ in conjunction with a proper matching of the right side of (4).

3.2 Efficient Realisation of the UNDIFICE

Uniform I-channel filter banks are efficiently realised as complex-modulated DFT polyphase filter banks [1]: the prototype filter is decomposed into its I polyphase components. The various channel filter functions are then obtained by a regular equidistant frequency shift by feeding the efficient IFFT algorithm and allowing for sharing many of the filter functions with all channels. For a more efficient realisation, the downsamplers at the system output are shifted to the system input using the noble identities [3, 5]. The resulting structure for a channel allocation scheme according to (1) with $\Delta\Omega_4^{(\mathrm{i})} = 0$ is well known and shown in figure 3 [3, 5].

Aiming at a matrix description of the efficient realisation of the UNDIFICE, we consider the relationship [3]

$$\left[\underline{\mathbf{H}}_4^{(\mathrm{ac})}(z_\mathrm{i})\right]^T = \underline{\mathbf{H}}^{(\mathrm{p1},4)}(z_\mathrm{i}^4) \underbrace{\begin{bmatrix} \mathbf{D}_2^{(\mathrm{p1})}(z_\mathrm{i}) & \mathbf{0}_2 \\ \mathbf{0}_2 & z_\mathrm{i}^{-2}\mathbf{D}_2^{(\mathrm{p1})}(z_\mathrm{i}) \end{bmatrix} \begin{bmatrix} \mathbf{W}_2^* \\ \mathbf{W}_2^* \end{bmatrix}}_{2\cdot \mathbf{M}_{\mathrm{SPI}}^{(\mathrm{p1})}(z_\mathrm{i})} \tag{6}$$

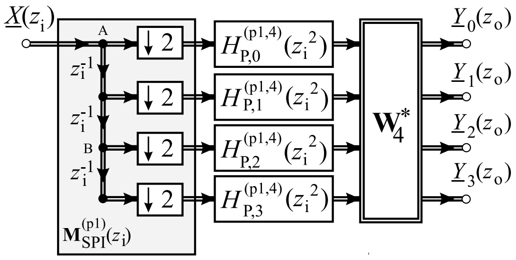

Figure 3: UNDIFICE as complex-modualted DFT polyphase filter bank with $\Delta\Omega_4^{(\mathrm{i})} = 0$.

between the alias component matrix $\underline{\mathbf{H}}_4^{(\mathrm{ac})}(z_\mathrm{i})$ and the polyphase matrix

$$\underline{\mathbf{H}}^{(\mathrm{p1},I)}(z_\mathrm{i}^4) = \underline{\mathbf{H}}^{(\mathrm{p1},4)}(z_\mathrm{i}^4) = \left[\underline{H}_{l,p}(z_\mathrm{i}^4)\right]_{\substack{l=0,\ldots,3;\\ p=0,\ldots,3.}} \tag{7}$$

that contains the $I = 4$ type-1 (p1) polyphase components (index p) of the channel filters (index l) according to [3, 5]

$$\underline{H}_l(z_\mathrm{i}) = \sum_{p=0}^{I-1=3} z_\mathrm{i}^{-p} \underline{H}_{l,p}^{(\mathrm{p1},4)}(z_\mathrm{i}^4). \tag{8}$$

In equation (6), $\mathbf{D}_M^{(\mathrm{p1})}(z_\mathrm{i}) = \mathrm{diag}\{z_\mathrm{i}^{-q}\}_{q=0,\ldots,M-1.}$ is an $M \times M$ delay matrix, and $\mathbf{W}_M^* = \left[W_M^{-pl}\right]_{\substack{l=0,\ldots,M-1;\\ p=0,\ldots,M-1.}}$ is the conjugated $M \times M$ DFT matrix, both for $M = 2$. The blocking matrix $\mathbf{M}_{\mathrm{SPI}}^{(\mathrm{p1})}(z_\mathrm{i})$ describes the serial-to-parallel interface in front of the filter operations [3], as shown in figure 3.

All filter functions $\underline{H}_l(z_\mathrm{i})$ are derived from the real prototype filter $H_\mathrm{P}(z_\mathrm{i})$ by complex modulation applying the frequency shifts according to (1). Hence, the polyphase decomposition (8) is expressed as

$$\begin{aligned} \underline{H}_l(z_\mathrm{i}) &= \underline{H}_\mathrm{P}\left(z_\mathrm{i} W_4^l \cdot \mathrm{e}^{-j\Delta\Omega_4^{(\mathrm{i})}}\right) && (9) \\ &= \sum_{p=0}^{3} z_\mathrm{i}^{-p} \underline{H}_{\mathrm{P},p}^{(\mathrm{p1},4)}\left(z_\mathrm{i}^4 \cdot \mathrm{e}^{-j4\Delta\Omega_4^{(\mathrm{i})}}\right) W_4^{-pl} \mathrm{e}^{jp\Delta\Omega_4^{(\mathrm{i})}}, && (10) \end{aligned}$$

leading, in conjunction with equation (7), to the polyphase matrix

$$\underline{\mathbf{H}}^{(\mathrm{p1},4)}\left(z_\mathrm{i}^4\right) = \left[W_4^{-pl} \cdot \mathrm{e}^{jp\Delta\Omega_4^{(\mathrm{i})}} \cdot \underline{H}_{\mathrm{P},p}\left(z_\mathrm{i}^4 \cdot \mathrm{e}^{-j4\Delta\Omega_4^{(\mathrm{i})}}\right)\right]_{\substack{l=0,\ldots,3;\\ p=0,\ldots,3.}},$$

which is decomposed into

$$\underline{\mathbf{H}}^{(\mathrm{p1},4)}\left(z_\mathrm{i}^4\right) = \mathbf{W}_4^* \, \underline{\mathbf{D}}_4^{(\mathrm{p1})}\left(\mathrm{e}^{-j\Delta\Omega_4^{(\mathrm{i})}}\right) \underline{\mathbf{E}}_4^{(\mathrm{p1})}\left(z_\mathrm{i}^4 \cdot \mathrm{e}^{-j4\Delta\Omega_4^{(\mathrm{i})}}\right). \tag{11}$$

The $I \times I$ diagonal matrix

$$\underline{\mathbf{E}}_4^{(\mathrm{p1})}\left(z_\mathrm{i}^4 \cdot \mathrm{e}^{-j4\Delta\Omega_4^{(\mathrm{i})}}\right) = \mathrm{diag}\left\{\underline{H}_{\mathrm{P},p}\left(z_\mathrm{i}^4 \cdot \mathrm{e}^{-j4\Delta\Omega_4^{(\mathrm{i})}}\right)\right\}_{p=0,\ldots,3.}$$

contains the polyphase components of the prototype filter shifted by the frequency offset $\Delta\Omega_4^{(\mathrm{i})}$ defined by (1).

With the equations (4), (6) and (11), the efficient decomposed UNDIFICE block structure is readily derived, as shown in figure 4. The first block, highlighted in grey, is an alternative realisation of the serial-to-parallel interface described by $\mathbf{M}_{\mathrm{SPI}}^{(\mathrm{p1})}(z_\mathrm{i})$ according to (6). In comparison to figure 3, this realisation requires only two

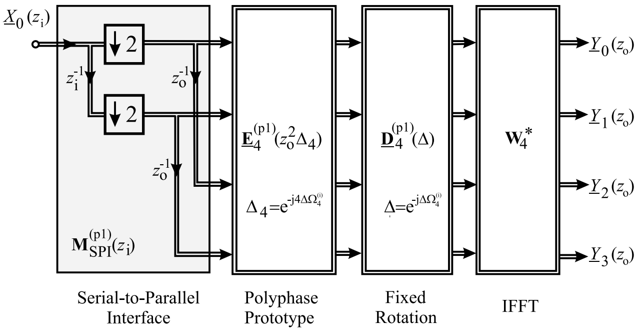

Figure 4: Decomposed efficient UNDIFICE block structure.

downsamplers instead of four, since it exploits the fact that the signals at the points A and B of figure 3 are identical except for a delay of z_i^{-2}. This extra delay is shifted across the downsamplers using the noble identities [3, 5], and, hence, the points A and B coincide.

Even and Odd Channel Allocation Scheme In [1], the even and odd channel allocation scheme are distinguished. Therefore, the matrix descriptions of the UNDIFICE are explicitly stated for both cases in the following. For the even channel allocation scheme with $\Delta\Omega_4^{(\mathrm{i})}=0$, the channel with index $l=0$ is centred at zero frequency. As a result, the rotation matrix $\underline{\mathbf{D}}_4^{(\mathrm{p1})}\left(\mathrm{e}^{-j\Delta\Omega_4^{(\mathrm{i})}}\right)=\underline{\mathbf{D}}_4^{(\mathrm{p1})}(1)$ becomes the identity matrix, yielding

$$\underline{\mathbf{Y}}_4(z_\mathrm{i}^2)=\mathbf{W}_4^*\,\mathbf{E}_4^{(\mathrm{p1})}(z_\mathrm{i}^4)\,\mathbf{M}_{\mathrm{SPI}}^{(\mathrm{p1})}(z_\mathrm{i})\cdot\underline{\mathbf{X}}_2^{(\mathrm{ac})}(z_\mathrm{i}). \tag{12}$$

The rotation block in figure 4 is replaced with a through connection. As for the odd channel allocation scheme with $\Delta\Omega_4^{(\mathrm{i})}=\frac{\pi}{4}$, the output signals of the UNDIFICE are given by

$$\underline{\mathbf{Y}}_4(z_\mathrm{i}^2)=\mathbf{W}_4^*\,\underline{\mathbf{D}}_4^{(\mathrm{p1})}\left(\mathrm{e}^{-j\frac{\pi}{4}}\right)\,\mathbf{E}_4^{(\mathrm{p1})}(-z_\mathrm{i}^4)\cdot\mathbf{M}_{\mathrm{SPI}}^{(\mathrm{p1})}(z_\mathrm{i})\cdot\underline{\mathbf{X}}_2^{(\mathrm{ac})}(z_\mathrm{i}). \tag{13}$$

It should be noted that in both cases $\mathbf{E}_4^{(\mathrm{p1})}(z_\mathrm{i}^4)$ and $\mathbf{E}_4^{(\mathrm{p1})}(-z_\mathrm{i}^4)$, respectively, represent real-valued systems.

3.3 Computational Load

In the scope of this paper, the computational load is measured by the multiplication rate, i.e. the number of real multiplications per time unit. Due to oversampling by two, the filter transition bands are as wide as their passbands resulting in FIR filters of short length N. As for the UNDIFICE, halfband filters are applied [1], where about half of the number of coefficients are zero leading to an additional reduction of the amount of multiplications. Assuming a halfband prototype filter with N_eff coefficients unequal zero and a complex input signal, the multiplication rate of the UNDIFICE in parallel structure according to figure 2 results in

$$A_\mathrm{PS}=4\cdot I\cdot N_\mathrm{eff}\cdot f_\mathrm{i}=16\cdot N_\mathrm{eff}\cdot f_\mathrm{i}.$$

In contrast, the multiplication rate of the efficient complex-modulated DFT polyphase UNDIFICE is given by

$$A_\mathrm{CM}=A_\mathrm{E}+A_\mathrm{D}=4\cdot[N_\mathrm{eff}+I]\cdot\frac{f_\mathrm{i}}{M}=[2N_\mathrm{eff}+8]\cdot f_\mathrm{i}.$$

Since a 4-point IFFT requires only addition, it does not contribute to the multiplication rate. In comparison to the parallel structure, the contribution of the filter operations to the multiplication rate of the efficient structure is reduced by a factor of $I\cdot M$.

For the even and odd channel allocation scheme the contribution to the multiplication rate due to filtering A_E is reduced by a factor of two, since the polyphase components are real-valued, as stated in (12) and (13). Finally, for the even channel allocation scheme, their is no contribution due to the fixed rotation , i.e. A_D=0. Hence, in this particular case, the multiplication rate results in

$$A_\mathrm{CM}^{(1)}=N_\mathrm{eff}\cdot f_\mathrm{i}.$$

4. THE SBC-FDFMUX FILTER BANK

In the following, investigations are extended to the complete structure of figure 2, i.e. the SBC-FDFMUX filter bank. It is derived by splitting the FDMUX UNDIFICE into two 2-channel FDMUX filter banks assigning the even-numbered channels ($l=0,2$) to one filter bank and the odd-numbered channels ($l=1,3$) to the other one. Hence, the two FDMUX filter banks are independent of each other, but (9) is still valid. The filter bank with the even channel is then complemented with a succeeding 2-channel FMUX filter bank to form an SBC filter bank. Note that the SBC filter bank is maximally decimating since $I=M=2$. Subsequently, we restrict our considerations on a 4-channel SBC-FDFMUX filter bank with an even channel allocation scheme, i.e. $\Delta\Omega_4^{(\mathrm{i})}=0$.

4.1 Alias Component Matrix Description of an SBC filter bank

The FDMUX of a maximally decimating 2-channel (I=2) SBC filter bank is described according to (2)–(4) by

$$\underline{\mathbf{Y}}_2(z_\mathrm{i}^2)=\frac{1}{2}\left[\underline{\mathbf{H}}_2^{(\mathrm{ac})}(z_\mathrm{i})\right]^T\cdot\underline{\mathbf{X}}_2^{(\mathrm{ac})}(z_\mathrm{i}), \tag{14}$$

where $\underline{\mathbf{H}}_2^{(\mathrm{ac})}(z_\mathrm{i})$ is the 2×2 FDMUX alias component matrix [3, 5]. Its FMUX is described by

$$\hat{\underline{\mathbf{X}}}_2^{(\mathrm{ac})}(z_\mathrm{i})=\underline{\mathbf{G}}_2^{(\mathrm{ac})}(z_\mathrm{i})\cdot\underline{\mathbf{Y}}_2(z_\mathrm{i}^2), \tag{15}$$

where the 2×2 vector $\hat{\underline{\mathbf{X}}}_2^{(\mathrm{ac})}(z_\mathrm{i})$ contains the alias components of the output signal and $\underline{\mathbf{G}}_2^{(\mathrm{ac})}(z_\mathrm{i})$ is the 2×2 alias component matrix of the FMUX. The whole SBC filter bank is described by [3, 5]

$$\hat{\underline{\mathbf{X}}}_2^{(\mathrm{ac})}(z_\mathrm{i})=\frac{1}{2}\underline{\mathbf{G}}_2^{(\mathrm{ac})}(z_\mathrm{i})\left[\underline{\mathbf{H}}_2^{(\mathrm{ac})}(z_\mathrm{i})\right]^T\cdot\underline{\mathbf{X}}_2^{(\mathrm{ac})}(z_\mathrm{i}).$$

4.2 The FDMUX of the SBC-FDFMUX Filter bank

Firstly, we will separately describe the FDMUX of the SBC-FDFMUX filter bank and its efficient realisation to allow for a better comparison with the UNDIFICE.

4.2.1 Equivalence to the FDMUX UNDIFICE

Retaining the channel numbering of figure 2, the two 2-channel FDMUX filter banks are described according to (14) in one system of equations by

$$\begin{bmatrix}\check{\underline{\mathbf{Y}}}_2(z_\mathrm{i}^2)\\ \tilde{\underline{\mathbf{Y}}}_2(z_\mathrm{i}^2)\end{bmatrix}=\frac{1}{2}\begin{bmatrix}\check{\underline{\mathbf{H}}}_2^{(\mathrm{ac})}(z_\mathrm{i}) & \mathbf{0}_{2\times2}\\ \mathbf{0}_{2\times2} & \tilde{\underline{\mathbf{H}}}_2^{(\mathrm{ac})}(z_\mathrm{i})\end{bmatrix}^T\begin{bmatrix}\underline{\mathbf{X}}_2^{(\mathrm{ac})}(z_\mathrm{i})\\ \underline{\mathbf{X}}_2^{(\mathrm{ac})}(z_\mathrm{i})\end{bmatrix}.$$

Rearranging the right side of (16) leads to

$$\begin{bmatrix}\check{\underline{\mathbf{Y}}}_2(z_\mathrm{i}^2)\\ \tilde{\underline{\mathbf{Y}}}_2(z_\mathrm{i}^2)\end{bmatrix}=\frac{1}{2}\begin{bmatrix}\check{\underline{\mathbf{H}}}_2^{(\mathrm{ac})}(z_\mathrm{i})\\ \tilde{\underline{\mathbf{H}}}_2^{(\mathrm{ac})}(z_\mathrm{i})\end{bmatrix}^T\cdot\underline{\mathbf{X}}_2^{(\mathrm{ac})}(z_\mathrm{i})\;. \tag{16}$$

Formally, (16) describes a 4-channel oversampling FDMUX with $M=2$, i.e. the FDMUX UNDIFICE, even though the two 2-channel filter banks are maximally decimating. Obviously, (16) differs from (4) just by a permutation of the second and third line of the respective system of equations.

Here, it should be noted that a similar relationship between the FMUX UNDIFICE and the FMUX of the PR SBC-FDFMUX filter bank according to figure 2 exists, with the exception of a scaling factor of 2.

4.2.2 Efficient Realisation

The FDMUX of the SBC-FDFMUX filter bank is developed according to section 3.2. Note that due to the splitting of the UNDIFICE, the filter bank for the even-numbered channels has an even 2-channel allocation scheme with $\Delta\Omega_2^{(\mathrm{i})} = 0$, while the filter bank with the odd-numbered channels has an odd 2-channel allocation scheme with $\Delta\Omega_2^{(\mathrm{i})} = \frac{\pi}{I} = \frac{\pi}{2}$. With (14) and

$$\left[\underline{\mathbf{H}}_2^{(\mathrm{ac})}(z_\mathrm{i})\right]^T = \underline{\mathbf{H}}^{(\mathrm{p1,2})}(z_\mathrm{i}^2)\mathbf{D}_2^{(\mathrm{p1})}(z_\mathrm{i})\mathbf{W}_2^*,$$

compliant with (6), and the polyphase decomposition in $I = 2$ type-1 polyphase components

$$\begin{aligned}\underline{H}_l(z_\mathrm{i}) &= \underline{H}_\mathrm{P}\left(z_\mathrm{i} W_2^l \cdot \mathrm{e}^{-j\Delta\Omega_2^{(\mathrm{i})}}\right) \\ &= \sum_{p=0}^{1} z_\mathrm{i}^{-p}\underline{H}_{\mathrm{P},p}^{(\mathrm{p1,2})}\left(z_\mathrm{i}^2 \cdot \mathrm{e}^{-j2\Delta\Omega_2^{(\mathrm{i})}}\right) W_2^{-pl}\mathrm{e}^{jp\Delta\Omega_2^{(\mathrm{i})}}\end{aligned}$$

we finally get

$$\underline{\mathbf{H}}_2^{(\mathrm{ac})}(z_\mathrm{i}) = \mathbf{W}_2^*\underline{\mathbf{D}}_2^{(\mathrm{p1})}(\mathrm{e}^{-j\Delta\Omega_2^{(\mathrm{i})}})\mathbf{E}_2^{(\mathrm{p1})}(z_\mathrm{i}^2 \cdot \mathrm{e}^{-j\Delta\Omega_2^{(\mathrm{i})}})\mathbf{D}_2^{(\mathrm{p1})}(z_\mathrm{i})\mathbf{W}_2^*,$$

yielding

$$\begin{aligned}\check{\mathbf{H}}_2^{(\mathrm{ac})}(z_\mathrm{i}) &= \mathbf{W}_2^*\mathbf{E}_2^{(\mathrm{p1})}(z_\mathrm{i}^2)\mathbf{D}_2^{(\mathrm{p1})}(z_\mathrm{i})\mathbf{W}_2^* && (17)\\ \tilde{\underline{\mathbf{H}}}_2^{(\mathrm{ac})}(z_\mathrm{i}) &= \mathbf{W}_2^*\underline{\mathbf{D}}_2^{(\mathrm{p1})}(\mathrm{e}^{-j\frac{\pi}{2}})\mathbf{E}_2^{(\mathrm{p1})}(-z_\mathrm{i}^2)\mathbf{D}_2^{(\mathrm{p1})}(z_\mathrm{i})\mathbf{W}_2^* && (18)\end{aligned}$$

for the two FDMUX of the SBC-FDFMUX filter bank. Since the real matrices $\mathbf{E}_2^{(\mathrm{p1})}(z_\mathrm{i}^2)$ and $\mathbf{E}_2^{(\mathrm{p1})}(-z_\mathrm{i}^2)$ just differ by a sign in the argument, the polyphase component block given by $\mathbf{E}_2^{(\mathrm{p1})}(z_\mathrm{i}^2)$ is shared by both filter banks applying an additional sign alternation block for the second FDMUX. The resulting novel structure is shown in the left part in figure 5 using an input commutator [3, 5] for serial-to-parallel conversion.

4.2.3 Computational Load

If halfband filters are applied, while disregarding the SBC function, a reduction of the multiplication rate is achieved w.r.t. the original UNDIFICE in figure (4), since the prototype filter is decomposed in $I = 2$ polyphase components, permitting the exploitation of the symmetry of coefficients for a higher efficiency. In this case and with $\Delta\Omega_4^{(\mathrm{i})} = 0$ the multiplication rate of the novel FDMUX structure according to figure 5 results in

$$A_\mathrm{CM}^{(2)} \approx \frac{A_\mathrm{CM}^{(1)}}{2} = \frac{N_\mathrm{eff} \cdot f_\mathrm{i}}{2}.$$

4.3 The SBC of the SBC-FDFMUX Filter bank

According to figure 2 we feed the FMUX with the even-numbered channel signals. Due to the duality [3, 5] between an FDMUX and an FMUX, the efficient realisation of the FMUX of the SBC-FDFMUX filter bank is developed according to section 4.2.1, but by decomposing the filters into their type-2 polyphase components [3, 5] leading to

$$\left[\mathbf{G}^{(\mathrm{p2,2})}(z_\mathrm{i}^2)\right]^T = \mathbf{R}_2^{(\mathrm{p2})}(z_\mathrm{i}^2)\mathbf{W}_2,$$

which, with

$$\underline{\mathbf{G}}_2^{(\mathrm{ac})}(z_\mathrm{i}) = \mathbf{W}_2^*\mathbf{D}_2^{(\mathrm{p2})}(z_\mathrm{i})\left[\mathbf{G}^{(\mathrm{p2,2})}(z_\mathrm{i}^2)\right]^T.$$

and (15), yields

$$\hat{\underline{\mathbf{X}}}_2^{(\mathrm{ac})}(z_\mathrm{i}) = \mathbf{W}_2^*\mathbf{D}_2^{(\mathrm{p2})}(z_\mathrm{i})\mathbf{R}_2^{(\mathrm{p2,2})}(z_\mathrm{i}^2)\mathbf{W}_2\underline{\mathbf{Y}}_2(z_\mathrm{i}^2).$$

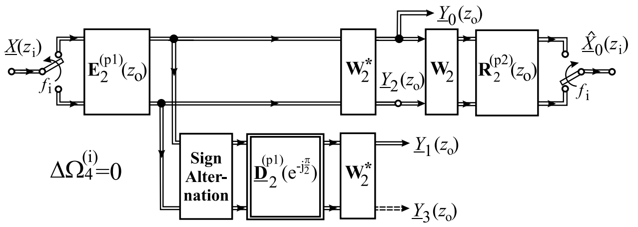

Figure 5: Efficient decomposed SBC-FDFMUX filter bank.

Figure 5 depicts the complete efficient decomposed block structure of the SBC-FDFMUX filter bank for $\Delta\Omega_4^{(\mathrm{i})} = 0$. The type-2 polyphase matrices $\mathbf{G}^{(\mathrm{p2,2})}(z_\mathrm{i}^2)$, $\mathbf{D}_2^{(\mathrm{p2})}(z_\mathrm{i})$ and $\mathbf{R}_2^{(\mathrm{p2,2})}(z_\mathrm{i}^2)$ emerge from $\mathbf{G}^{(\mathrm{p1,2})}(z_\mathrm{i}^2)$, $\mathbf{D}_2^{(\mathrm{p1})}(z_\mathrm{i})$ and $\mathbf{R}_2^{(\mathrm{p1,2})}(z_\mathrm{i}^2)$, respectively, by substituting $p := M - 1 - p$ for $p = 0, 1$ with $M = 2$. Figure 5 shows the efficient decomposed SBC-FDFMUX filter bank block structure.

By applying the SBC-FDFMUX filter bank in conjunction with perfect reconstruction, it has to fulfil the PR condition for a 2-channel SBC filter bank [3, 5]

$$\mathbf{F}_2^{(\mathrm{ac})}(z_\mathrm{i}) = \frac{1}{2}\underline{\mathbf{G}}_2^{(\mathrm{ac})}(z_\mathrm{i})\left[\underline{\mathbf{H}}_2^{(\mathrm{ac})}(z_\mathrm{i})\right]^T = c\begin{bmatrix} z_\mathrm{i}^{-k} & 0 \\ 0 & (z_\mathrm{i}W_2)^{-k}\end{bmatrix} \quad (19)$$

with $k \in \mathbb{N}_0$ and the constant c. With the *Standard QMF* approach linear-phase filters are designed that nearly fulfil the conditions of (19), thus leading to almost PR [3, 5]. All requirements of (19) are exactly met by filters based on the *Conjugated QMF* design approach, which delivers minimum or mixed-phase filters [3, 5].

For SQMF designs, the coefficient symmetry of the FDMUX with the potential of PR can again be exploited calling for a multiplication rate of

$$A_\mathrm{CM}^{(3)} = \frac{N}{2} \cdot f_\mathrm{i} \approx N_\mathrm{eff} \cdot f_\mathrm{i} \approx 2 \cdot A_\mathrm{CM}^{(2)} \approx A_\mathrm{CM}^{(1)}.$$

Obviously, the multiplication rates of the original FDMUX applying halfband filters and that of the novel SQMF FDMUX with the potential of PR are essentially identical.

5. CONCLUSION

In this paper we recall the efficient UNDIFICE in a general matrix description and extend it both to a novel efficient and joint SBC-FDFMUX filter bank for two channels combing the features of the UNDIFICE with the property of a PR SBC filter bank in one single structure. In the first case a reduction of the computational load by a factor of two is achieved, whereas in the latter case the additional SBC functionality is possible with no extra expenditure in comparison to the original UNDIFICE.

REFERENCES

[1] H. G. Göckler and H. Eyssele, “Study of On-Board Digital FDM-Demultiplexing for Mobile SCPC Satellite Communications,” *Europ. Trans. Telec.*, vol. 3, pp. 7–30, Jan.-Feb. 1992.

[2] H. G. Göckler and T. Gebauer, “A digital FDM-demultiplexer for beamforming environment,” *Space Communications*, vol. 10, pp. 197–205, 1992.

[3] H. G. Göckler and A. Groth, *Multiratensysteme*. Wilburgstetten, Germany: J. Schlembach Fachverlag, 2004.

[4] X. Maufroid, et al., “High Throughput Bent-Pipe Processor for Future Broadband Satellite Access Networks,” in *8th Int. Workshop on Signal Proc. for Space Comm.*, Catania, Italy, Sept. 2003, pp. 277–292.

[5] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*. New Jersey: Prentice Hall Signal Processing Series, 1993.