# EFFICIENT BIT ALLOCATION FOR HIGH QUALITY SUBBAND CODING USING NON-SELECTIVE FILTER BANKS UNDER QUANTIZATION NOISE CONSTRAINTS


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## ABSTRACT

This paper addresses the problem of computing the optimum bit allocation in a subband coder with low selectivity filters, while maintaining the power of quantization noise below a given value. A low complexity strategy that takes into account the frequency responses of the synthesis filter bank, is proposed. The Lagrange multiplier method is used to obtain the optimum bit distribution. The number of bits to be allocated to a subband depends only on the frequency response of that subband, on the variance of the corresponding subband signal, on the maximum allowed noise power, and on the decimation factors of all the subbands.


## 1. INTRODUCTION

Dynamic bit allocation is a major concern in any coding scheme where a number of bits is adaptively assigned among multiple sources (e.g., subbands), so that the measured distortion is minimized. This problem was first addressed in [1], which provides a solution for M subbands, where the number of bits ($R_i$) allocated to a subband depends on the standard deviations of the subbands ($\sigma_i$), and on the available amount of bits ($R$), in the form:

$$R_i = \frac{R}{M} + log_2\left(\frac{\sigma_i}{\prod_{j=1}^{M}\sigma_j}\right) \quad (1)$$

This approach allows the number of bits to be any arbitrary real number, including negative values. An optimal bit allocation algorithm for nonnegative integer allocation was proposed in [2], based on marginal return analysis.

There are several references in the literature related to optimum bit allocation when low selectivity filter banks are used to decompose the input signal. This kind of filter banks cause quantization noise injected in a subband to spread over the rest of them. This problem is quite common in subband audio coding, where the use of low selectivity filter banks is useful to implement either low delay or low complexity audio coders. In [3] a low complexity strategy which takes into account the frequency responses of the synthesis filter bank is presented, ensuring that the overall distortion due to quantization noise will be always below the masking threshold. Unfortunately, this algorithm does not ensure the number of bits allocated is minimum.

In [4] an optimum bit allocation algorithm, applicable to either uniform and nonuniform frequency decompositions is presented, which considers the lack of selectivity of actual filters. It minimizes the noise to mask ratio under the constraint of the target average bit per input signal sample rate $R$. The noise power leakage from a given subband to the remaining is considered, but it does not deal with the potential appearance of negative solutions.

There are several applications where the objective is to maintain a target signal quality, often specified with a signal to noise ratio value, or some related global distortion measurement, such as the PRD (Percent Root-mean-square Difference). This approach is of particular interest in biomedical signal coding [5][6][7].

No bit allocation algorithms can be found in the literature which minimizes the number of bits to be allocated, with the constraint of maintaining the overall distortion below a maximum allowable value, and which takes into account the frequency responses of the actual filter which implements the subband decomposition. The solution to this problem is presented in this paper.

## 2. OPTIMUM BIT ALLOCATION WITH LOW SELECTIVITY FILTER BANKS

### 2.1 Preliminary concepts and definitions

A nonuniform critically sampled filter bank with M subbands is considered. The average number of bits per input signal sample is given by

$$R = \sum_{i=1}^{M} \mu_i R_i \tag{2}$$

where, $1/\mu_i$ is the decimation factor of the i-th subband, and $R_i$ is the number of bits used to code the corresponding subband samples. The quantization error variance $\sigma_{q_i}^2$ at the i-th subband quantizer is expressed in (3), where $\varepsilon_i$ is the quantizer performance factor [8]:

$$\sigma_{q_i}^2 = \varepsilon_i^2 2^{-2R_i} \sigma_i^2 \tag{3}$$

The power spectral density of the quantization noise is obtained taking into account the transfer function of the synthesis filters $H_i(\omega)$, $(i = 1, 2, ..., M)$:

$$N(\omega) = \sum_{i=1}^{M} \frac{\sigma_{q_i}^2}{\mu_i B} |H_i(\omega)|^2 \tag{4}$$

Integrating (4) on the input signal bandwidth ($B$) the overall noise power is obtained, $\sigma_q^2$:

$$\sigma_q^2 = \int_B N(\omega) d\omega \tag{5}$$

$$\sigma_q^2 = \int_B \sum_{i=1}^{M} \frac{\sigma_{q_i}^2}{\mu_i B} |H_i(\omega)|^2 d\omega \tag{6}$$

Substituting expression (3) in (6), $\sigma_q^2$ can also be written as:

$$\sigma_q^2 = \int_B \sum_{i=1}^{M} \frac{\varepsilon_i^2 2^{-2R_i} \sigma_i^2}{\mu_i B} |H_i(\omega)|^2 d\omega \tag{7}$$

Defining $c_i$ as in expression (8),

$$c_i = \frac{\varepsilon_i^2 \sigma_i^2}{\mu_i B} \int_B |H_i(\omega)|^2 d\omega \tag{8}$$

the overall noise power can also be expressed with (9):

$$\sigma_q^2 = \sum_{i=1}^{M} c_i 2^{-2R_i} \tag{9}$$

The expressions above indicate that the overall noise power at the output depends on the transfer function of the filters in the filter bank, which can differ from ideal in actual applications.

### 2.2 Problem formulation

The optimum bit allocation procedure is derived from the minimization of the average number of bits per input signal sample, under the constraint of maintaining the overall noise power below a given value. This is a constrained optimization problem, which can be solved using the Lagrange multipliers method [9].

The function to minimize is the number of bits to allocate to the subbands, and the constraints make the overall noise power to remain below the maximum allowable value ($\sigma_n^2$):

$$\min f(R_i) = \sum_{i=1}^{M} \mu_i R_i \tag{10}$$

Constraint:

$$\sum_{i=1}^{M} c_i 2^{-2R_i} - \sigma_n^2 \leq 0 \tag{11}$$

Another important constraint that must be considered is $R_i \geq 0$.

This problem can also be formulated using vector notation, considering $\mathbf{r} = [R_1, ..., R_M]^t$:

$$\min f(\mathbf{r}) = \sum_{i=1}^{M} \mu_i R_i \tag{12}$$

Constraint:

$$g(\mathbf{r}) = \sum_{i=1}^{M} c_i 2^{-2R_i} - \sigma_n^2 \leq 0 \tag{13}$$

The solution to the above formulated problem is obtained minimizing the following function:

$$L(\mathbf{r}, \lambda) = f(\mathbf{r}) + \lambda g(\mathbf{r}) \tag{14}$$

where $\lambda$ is the Lagrange multiplier. The vector $\mathbf{r}$ which minimizes (14) is obtained using the Kuhn-Tucker conditions [9], which state that a relative minimum point of $L(\mathbf{r}, \lambda)$ is obtained with $\lambda \geq 0$ solving the following system, which consists of the first-order necessary conditions:

$$\nabla f(\mathbf{r}) + \lambda \nabla g(\mathbf{r}) = 0 \tag{15}$$

$$g(\mathbf{r}) = 0 \tag{16}$$

### 2.3 Solution

The application of the Kuhn-Tucker conditions to the problem formulated in section 2.2, give rise to the following first-order necessary conditions:

$$\frac{\partial L(\mathbf{r}, \lambda)}{\partial R_i} = \mu_i - \lambda(2c_i 2^{-2R_i} \ln(2)) = 0, \quad i = 1, ..., M \tag{17}$$

$$\sum_{i=1}^{M} c_i 2^{-2R_i} - \sigma_n^2 = 0 \tag{18}$$

The value of the Lagrange multiplier can be found from (17):

$$\lambda = \frac{\mu_i}{2} \frac{2^{2R_i}}{c_i \ln(2)} \tag{19}$$

Equation (19) states that the value of $\lambda$ is non negative, as required by the Kuhn-Tucker conditions to find a solution to the problem. Furthermore, this expression can be combined for different values of $i$, to obtain a relation between the amount of bits allocated to different subbands:

$$R_i = R_j + \frac{1}{2} \log_2 \left( \frac{\mu_j c_i}{\mu_i c_j} \right) \tag{20}$$

The necessary conditions (18) can be used to obtain $R_j$. Substituting (20) in (18), the following expression is obtained:

$$\sum_{i=1}^{M} c_i 2^{-2(R_j + \frac{1}{2} \log_2((\mu_j c_i)/(\mu_i c_j)))} = \sigma_n^2 \tag{21}$$

This is equivalent to:

$$\sum_{i=1}^{M} c_i 2^{-2R_j} 2^{\frac{1}{2} \log_2((\mu_j c_i)/(\mu_i c_j))} = \sigma_n^2 \tag{22}$$

which allows to find $2^{-2R_j}$:

$$2^{-2R_j} = \frac{\sigma_n^2}{\sum_{i=1}^{M} c_i 2^{log_2((\mu_i c_j)/(\mu_j c_i))}} \tag{23}$$

The number of bits allocated to the j-th subband is finally found from expression (23):

$$R_j = \frac{1}{2} \log_2 \left( \frac{\sum_{i=1}^{M} c_j \frac{\mu_i}{\mu_j}}{\sigma_n^2} \right) \tag{24}$$

Substituting $c_j$ in expression (24) for expression (8), and taking into consideration that only integer values for $R_j$ are admissible, the number of bits allocated to the i-th subband is obtained with the following expression:

$$R_j = \left\lceil \frac{1}{2} \log_2 \left( \frac{\sum_{i=1}^{M} \mu_i \frac{\varepsilon_j^2 \sigma_j^2}{\mu_j^2 B} \int_B |H_j(\omega)|^2 d\omega}{\sigma_n^2} \right) \right\rceil \tag{25}$$

where $\lceil \quad \rceil$ represents the ceiling operator.

Obviously, expression (25) does not avoid the appearance of negative solutions. When a negative solution is obtained for a given subband, it must be replaced by the zero value. After that, expression (20) can be used to determine the number of bits allocated in the remaining subbands.

## 3. RESULTS

In this section, an example of the behavior of the proposed bit allocation algorithm is included. A low selectivity quadrature mirror filter bank ($M = 2$) is considered, whose frequency response is represented in figure 1. You can see that the two bands highly overlap, which makes the application of bit allocation algorithms that do not consider the frequency responses of the filters in the filter bank inappropriate.

For the experiment, white noise with uniform probability density function with zero mean and variance unity is applied to the filter bank. The subband signals are uniformly quantized using the number of bits per subband provided by the algorithm. The target signal to noise ratio (SNR) varies from 0 to 100dB. Figure 2 shows in dashed line the measured SNR obtained with the proposed algorithm, and in dotted line, the results obtained using one bit less. The former line is slightly above the ideal curve, which means that the obtained SNR is also slightly higher than the required one. Using only one bit less, the obtained curve is slightly below the ideal one.

## 4. CONCLUSIONS

In this paper, the problem of computing the optimum bit allocation in a subband coder with low selectivity filters, while maintaining the power of quantization noise below a given value has been addressed. A low complexity strategy that takes into account the frequency responses of the synthesis filter bank, is proposed. The Lagrange multiplier method is used to obtain the optimum bit distribution.

Until now, no bit allocation algorithms can be found in the literature with a solution to the problem this paper deals with. Several approaches can be found that minimize the overall distortion with a constrained number of bits, with or without considering the frequency responses of actual filters, but no effort has been made to minimize the number of bits maintaining the overall distortion below a given value.

The number of bits to be allocated to a subband depends only on the frequency response of the synthesis filter of that subband, on the variance of the corresponding subband signal, on the maximum allowed noise power, and on the decimation factors of all the subbands.

The proposed solution is suitable for applications where a given quality must be guaranteed, measured using some global distortion measurement such as the PRD (Percent Root-mean-square Difference). This approach is of particular interest in biomedical signal coding, for example, in electrocardiogram (ECG) signal compression.

## REFERENCES


[1] J.J. Huang, and P.M. Schultheiss, "Block quantization of correlated Gaussian random variables," *IEEE Trans. Commn. Sys.*, vol. CS-11, pp. 289–296, Sept. 1963.

[2] B. Fox, "Discrete optimization via marginal analysis," *Manage. Sci.*, vol. 13, no. 3, pp. 210–216, Nov. 1966.

[3] D. Martínez-Muoz, M. Rosa-Zurera, F. Cruz-Roldán, F. López-Ferreras, and N. Ruiz-Reyes, "Quantisation noise control in perceptual audio coding using low selectivity filter banks," *IEE Electronic Letters*, vol. 38, no. 16, pp. 932–933, August 2002.

[4] C. Caini, and A. Vanelli-Coralli, "Optimum bit allocation in subband coding with nonideal reconstruction filters," *IEEE Signal Processing Letters*, vol. 8, no. 6, pp. 157–159, June 2001.

[5] M.C. Aydin, A.E. Cetin, and H.Koymen, "ECG data compression by sub-band coding," *Electronics Letters*, vol. 27, no. 4, pp. 359–360, February 1991.

[6] S.M.S. Jalaleddine, C.G. Hutchens, R.D. Strattan, and W.A. Coberly, "ECG data compression techniques - A unified approach," *IEEE Trans. on Biomedical Engineering*, vol. 37, no. 4, Appril 1990.

[7] A.E. Cetin, H.Koymen, and M.C. Aydin, "Multichannel ECG data compression by multirate signal processing and transform domain coding techniques," *IEEE Trans. on Biomedical Engineering*, vol. 40, no. 5, pp. 495–499, February 1991.

[8] N.S. Jayant, and P. Noll, *Digital coding of waveforms*, Prentice-Hall, 1984.

[9] D.E. Luenberger, *Linear and non linear programming*. Addison-Wesley Inc., 1984.


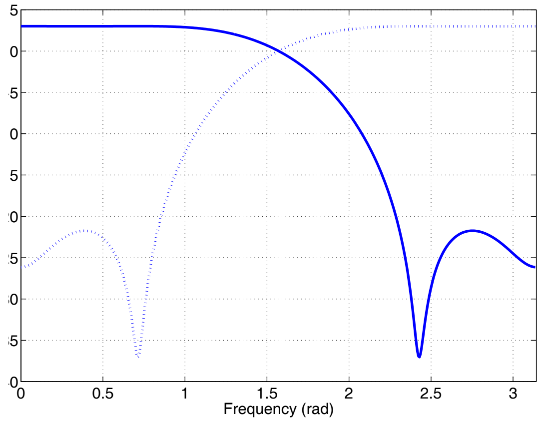


Figure 1: Frequency responses of the filters in the filter bank

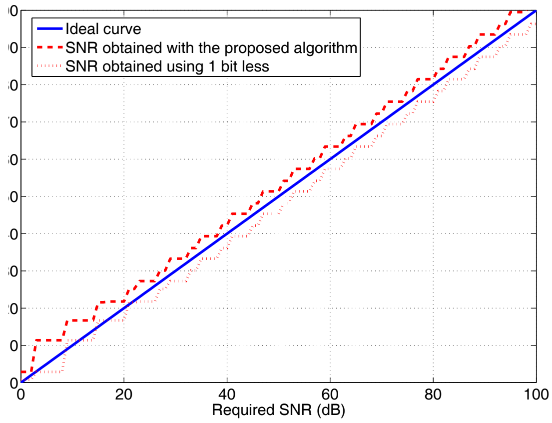


Figure 2: Measured SNR vs. required SNR at the output of the filter bank of figure 1 when the proposed bit allocation algorithm is applied