MULTICHANNEL BLIND DECONVOLUTION OF IMPULSIVE SIGNALS

Pablo Lecumberri [†], *Marisol Gómez* [†], *and Alfonso Carlosena* [‡]

[†]Mathematics and Computer Science Department, [‡]Electrical and Electronic Engineering Department
Universidad Pública de Navarra
Campus Arrosadía s/n, 31006, Pamplona, Spain
phone: + (34) 948168055, email: pablo.lecumberri@unavarra.es

ABSTRACT

In this communication, the problem of blind deconvolution of transient, impulsive signals in a multichannel environment is addressed. This kind of signals arise naturally, or are used as external excitation, in many mechanical and acoustical systems and can only be observed indirectly, after propagation through the medium. Blind deconvolution or identification methods published to date are not suitable for recovering these sources or the system response, as identifiability conditions are not met. We fully develop here a deterministic subspace method for the blind deconvolution in a multichannel environment which does not impose any restrictions on the excitation signals or on the impulse response of propagation channels, apart from finite length and channel diversity. The method is also extended to cope with signals in noisy environments.

1. INTRODUCTION

In recent years there has been a growing interest in the blind identification / deconvolution problem, aiming at compensating the linear distortion introduced by a system, which usually models the effects of propagation through the medium and signal acquisition distortion. The deconvolution process directly equalizes the signal, whereas in the case of identification, the system response is first estimated.

Being a classical problem, current research efforts in this area are mainly motivated by the explosive growth experienced in the field of wireless communications, and the necessity to compensate the distortion effects inherent to radio signal propagation. As a result, most blind deconvolution/identification methods published to date have been developed having typical digital communication signals in mind, exploiting their repetitive characteristics in order to estimate either the transmitted signal or the response of the system.

In many applications, recovering an impact signal in a mechanical system (or the system response) is of great importance. These applications include signature analysis, coin-tap tests, seismic exploration and modal analysis, just to name a few. Usually, we do not have access to a previous estimation of the source or the system response in order to apply classical identification or deconvolution approaches, so blind estimation techniques are mandatory.

In [1], Tong and Perreau review and classify multichannel blind identification methods. In a whole, these methods fall into one of two broad categories: Statistical methods - which take some statistical properties of the input for granted - and deterministic methods - which do not use this information, even though signals involved may be random. This classification can also be applied to the dual problem of blind deconvolution. Statistical methods are not suitable for non-repetitive, impulsive excitation signals, as they do not show stationarity and hence no relevant statistical information can be obtained by means of sample averages. This characteristic also impairs the aplicability of deterministic subspace methods which rely on second-order statistics to recover signal and noise subspaces [2]. This work is based on other deterministic subspace techniques [3] and avoids the troublesome estimation of the length of the different channels, required by these methods.

This paper is organized as follows. In the next section the data model and the notation employed throughout this work is introduced. The propposed deconvolution algorithm is disclosed in section 3, starting from matrix theory and linear prediction principles. All statements upon which the deconvolution algorithm is built are given without proofs. A more thorough development with formal demonstrations can be found in [4]. Section 4 is devoted to results from synthetic and real signals.

2. DATA MODEL

This work deals with the outputs of a system composed of several (Q) linear, time-invariant (LTI) channels, all of them excited by a single input. This is known as a single-input, multi-output (SIMO) system. LTI systems are common and mathematically tractable models for the effects of propagation and acquisition of signals. The well-known convolution sum (1) relates the input and output of such systems. For the time being, we will ignore the effects of noise in the output signals. We will assume that the length of the input signal ($P+1$) and the length of the impulse response ($L+1$) are finite, and hence, so is the length of the output signal ($M+1 = P+L+1$).

$$x_q[n] = b[n] * h_q[n] = \sum_{k=0}^{P} b[k] \cdot h_q[n-k] \qquad q = 1, \ldots, Q \tag{1}$$

We may express the convolution in matrix form (2):

$$\begin{bmatrix} x_q[0] \\ x_q[1] \\ \vdots \\ x_q[M] \end{bmatrix} = \begin{bmatrix} b[0] & 0 & 0 & \dots & 0 \\ b[1] & b[0] & 0 & \dots & 0 \\ b[2] & b[1] & b[0] & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \end{bmatrix} \cdot \begin{bmatrix} h_q[0] \\ h_q[1] \\ \vdots \\ h_q[L] \end{bmatrix}$$

$$\mathbf{x}_q = \mathbf{B} \cdot \mathbf{h}_q \tag{2}$$

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$\mathbf{x}_q \in \mathbb{R}^{(M+1)\times 1}$ is a vector whose elements are samples of the q-th channel output signal, $\mathbf{B} \in \mathbb{R}^{(M+1)\times(L+1)}$ is a Toeplitz matrix made up with samples of the input signal and $\mathbf{h}_q \in \mathbb{R}^{(L+1)\times 1}$ is a column vector whose elements are samples of the q-th channel impulse response.

Lower-case, bold-face letters, such as $\mathbf{x}$, denote column vectors, whereas capital, bold-face letters, such as $\mathbf{B}$, denote matrices. T stands for the matrix transpose operation. $x(z)$ is a polynomial whose coeficients are taken from the elements of $\mathbf{x}$: $x(z) = x[0] + x[1]z + \cdots + x[M]z^M$.

$\left[\mathbf{A}^j\right]$ and $\left\langle \mathbf{A}^j \right\rangle$ denote respectively the set of columns of matrix $\mathbf{A}$ and the subspace spanned by those columns.

3. MULTICHANNEL BLIND DECONVOLUTION

3.1 Subspace approach

In [3, Lemma 2] it is shown that the standard form of the null space of a rank deficient Hankel matrix possess the structure of matrix $\mathbf{B}$ in (2). Consequently, we may take the columns of $\mathbf{B}$ as the null space of some Hankel matrix $\mathbf{S}(M+1)$ generated by a sequence $s[n]$ as in (3). Moreover, the rank of this Hankel matrix is shown in [3] to be P.

$$\mathbf{S}(r) = \begin{bmatrix} s[0] & s[1] & \cdots & s[r-1] \\ s[1] & s[2] & \cdots & s[r] \\ \vdots & \vdots & \iddots & \vdots \\ s[N-r+1] & s[N-r+2] & \cdots & s[N] \end{bmatrix} \tag{3}$$

It is a well-known result from Linear Prediction theory that the maximum rank of the Hankel matrices generated by a sequence composed of a finite number P of modes is equal to P. A mode $m_i[n]$ is a sequence defined as

$$m_i[n] = c_i z_i^n \qquad n = 0, \ldots, N$$

for some amplitude factor c_i and root z_i (possibly complex).

It can be shown that the roots of those modes are the roots of the polynomial $b(z)$ which appears in the standard form of the null space of the rank deficient Hankel matrices. In fact, the linear prediction polynomial of sequence $\{s[n]\}$ is:

$$A(z) = 1 + a_1 \cdot z^{-1} + \ldots + a_P \cdot z^{-P}$$

with $a_i = \frac{b[P-i]}{b[0]}$, $i = 0, \ldots, P$.

Vectors $\mathbf{x}_q$ are linear combinations of the columns of $\mathbf{B}$, so they also lie in the null space of $\mathbf{S}(M+1)$:

$$\mathbf{S}(r) \cdot \mathbf{x}_q = 0 \qquad q = 1, \ldots, Q \tag{4}$$

The Hankel structure of $\mathbf{S}(M+1)$ allow us to rewrite equation (4) as follows:

$$\mathbf{X}_q \cdot \mathbf{s} = 0 \qquad q = 1, \ldots, Q$$

where $\mathbf{X}_q \in \mathbb{R}^{(M+1)\times N}$ is a Toeplitz matrix as defined in (5) and $\mathbf{s} = \begin{bmatrix} s[0] & s[1] & \cdots & s[M] \end{bmatrix}^T$.

$$\mathbf{X}_q = \begin{bmatrix} x_q[0] & x_q[1] & \cdots & x_q[M] & 0 & \cdots & 0 \\ 0 & x_q[0] & \cdots & x_q[M-1] & x_q[M] & \cdots & 0 \\ \vdots & \vdots & \ddots & & & \ddots & \vdots \\ 0 & 0 & \cdots & x_q[0] & x_q[1] & \cdots & x_q[M] \end{bmatrix} \tag{5}$$

Piling up all matrices $\mathbf{X}_q$ for $q = 1, \ldots, Q$ we get a block Toeplitz matrix $\mathbf{X}_{GS} = \begin{bmatrix} \mathbf{X}_1^T & \mathbf{X}_2^T & \cdots & \mathbf{X}_Q^T \end{bmatrix}^T \in \mathbb{R}^{Q(M+1)\times N}$. This matrix is know as Generalized Sylvester Resultant and was studied in [5] in order to obtain the greatest common divisors of matrix polynomials.

$$\mathbf{X}_{GS} \cdot \mathbf{s} = 0 \tag{6}$$

The results of [5] state that the dimension of the null space of $\mathbf{X}_{GS}$ is equal to the number of roots common to all polynomials $x_i(z)$, $i = 1, \ldots, Q$. Imposing the condition of channel diversity, that is, that no zero is shared by all channels, the only roots common to all $x_i(z)$, $i = 1, \ldots, Q$, are the P zeros of the input sequence.

Since all sequences with $A(z)$ as their linear prediction polynomial lie in a P-dimensional subspace and they satisfy equation (6), the null space of $\mathbf{X}_{GS}$ is exactly that subspace.

3.2 Krylov subspace estimation

$s[n]$ was defined as a sequence with linear prediction polynomial $A(z)$. The objective of the method developed below is to find $A(z)$, which will yield a scaled copy of the source signal $b[n]$. Equation (6) is equivalent to the following set of equations:

$$\begin{aligned} &\mathbf{X}^T \cdot \mathbf{s}_0 = 0 \\ &\mathbf{X}^T \cdot \mathbf{C} \cdot \mathbf{s}_0 = 0 \\ &\cdots \\ &\mathbf{X}^T \cdot \mathbf{C}^{N-M+1} \cdot \mathbf{s}_0 = 0 \end{aligned}$$

where $\mathbf{s}_0 \in \mathbb{R}^{(M+1)\times 1}$ is determined by P initial values and linear prediction polynomial $A(z)$, $\mathbf{X} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_Q \end{bmatrix} \in \mathbb{R}^{(M+1)\times Q}$ and $\mathbf{C} \in \mathbb{R}^{(M+1)\times(M+1)}$ is defined as follows:

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & & & & & \\ & 0 & \ddots & & & & \\ & & \ddots & 1 & & & \\ & & & 0 & \ddots & & \\ & & & & \ddots & 1 & \\ & & & & & 0 & 1 \\ 0 & 0 & \cdots & -a_P & \cdots & -a_2 & -a_1 \end{bmatrix} \tag{7}$$

In order to estimate $b[n]$, the unique Krylov subspace $\mathbf{K}_{N-M+1}(\mathbf{C}, \mathbf{s}_0) = \left\langle \mathbf{s}_0, \mathbf{C}\cdot\mathbf{s}_0, \ldots, \mathbf{C}^{N-M}\cdot\mathbf{s}_0 \right\rangle$ orthogonal to the range of $\mathbf{X}$ must be identified. It's trivial to prove that when $\mathbf{s}_0$ is generated by the linear prediction polynomial that appears in $\mathbf{C}$, the dimension of $\mathbf{K}_{N-M+1}(\mathbf{C}, \mathbf{s}_0)$ is P, so $\mathbf{K}_{N-M+1}(\mathbf{C}, \mathbf{s}_0) = \mathbf{K}_P(\mathbf{C}, \mathbf{s}_0)$.

Let $\mathbf{C}_\mathbf{R}$ be the matrix formed with the last P columns of $\mathbf{C}^{M-P}$. It is easy to show that $\left[\mathbf{C}_\mathbf{R}{}^j\right]$, $j = 1, \ldots, P$, is a base for $\mathbf{K}_P(\mathbf{C}, \mathbf{s}_0)$ and that its top P rows are taken from a $P \times P$ identity matrix. The coefficients of linear prediction polynomial $A(z)$ appear in its $(P+1)$th row.

Let $\mathbf{G}$ be an orthogonal matrix whose columns form a basis for the orthogonal complement of the range of $\mathbf{X}$. Let $\mathbf{G}_\mathbf{P}$ denote the top P rows of $\mathbf{G}$, with singular value decomposition $\mathbf{G}_\mathbf{P} = \mathbf{U}\cdot\mathbf{\Sigma}\cdot\mathbf{V}^T$. Define matrix $\mathbf{G}_\mathbf{P}{}^+$ as

$$\mathbf{G}_\mathbf{P}{}^+ = \mathbf{V}\cdot\mathbf{T}\cdot\mathbf{U}^T,$$

with $\mathbf{T} = \begin{bmatrix} \mathbf{D} \\ \mathbf{W} \end{bmatrix}$ of the same size as $\mathbf{\Sigma}^T$.

$\mathbf{D}$ is a diagonal matrix whose non-zero elements are the inverse of the singular values of $\mathbf{G}_\mathbf{P}$. For any matrix $\mathbf{W}$ of appropiate size, $\mathbf{F} = \mathbf{G} \cdot \mathbf{G_P}^+$ is a matrix whose top $P \times P$ submatrix is the identity (if we make $\mathbf{W} = \mathbf{0}$, $\mathbf{G}_P^+$ is the Moore-Penrose pseudoinverse of $\mathbf{G_P}$). Choosing $\mathbf{W}$ conveniently will give $\mathbf{F}$ equal to $\mathbf{C_R}$.

It can be shown that, for $\left[\mathbf{F}^j\right]$, $j = 1, \ldots, P$, to be a base of $\mathbf{K}_P(\mathbf{C}, \mathbf{s}_0)$, $\left[(\mathbf{C} \cdot \mathbf{F})^j\right]$, $j = 1, \ldots, P$, must be a base of $\left\langle \mathbf{F}^j \right\rangle$, $j = 1, \ldots, P$. That is, $\mathbf{C} \cdot \mathbf{F} = \mathbf{F} \cdot \mathbf{R}$ with $\mathbf{R}$ a full-rank $P \times P$ matrix. The values of the elements of $\mathbf{R}$ can be inferred from the special structure of $\mathbf{F}$:

$$\mathbf{R} = \begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & 0 & 1 \\ -a_P & -a_{P-1} & \cdots & -a_2 & -a_1 \end{bmatrix} \tag{8}$$

In absense of noise, there must exist a unique matrix $\mathbf{W_0}$ that makes $\mathbf{C} \cdot \mathbf{F}$ equal to $\mathbf{F} \cdot \mathbf{R}$. $\mathbf{C}$ and $\mathbf{R}$ are built following (7) and (8), whith the elements of their last rows taken from the $(P+1)th$ row of $\mathbf{F}$. As a result, we obtain a quadratic function of the elements of $\mathbf{W}$ which must be zeroed:

$$\mathbf{C} \cdot \mathbf{F} - \mathbf{F} \cdot \mathbf{R} = \mathbf{0} \tag{9}$$

Having found matrix $\mathbf{W_0}$ with (9), the deconvolved source signal results from the coefficients of the linear prediction polinomial $A(z)$, which form the $(P+1)$th row of $\mathbf{F}$:

$$\mathbf{b}^T = \left[-\mathbf{G}_{(P+1)} \cdot \mathbf{G_P}^+ \quad 1 \right],$$

where $\mathbf{G}_{(P+1)}$ stands for the $(P+1)$th row of $\mathbf{G}$.

In the presence of noise, equation (9) will not be satisfied for any matrix $\mathbf{W}$. In this case, the square of the Frobenius norm of the residual $(\mathbf{C} \cdot \mathbf{F} - \mathbf{F} \cdot \mathbf{R})$ can be minimized to find matrix $\mathbf{W_0}$:

$$\mathbf{W_0} = \arg \min_{\mathbf{W}} \left(|\mathbf{C} \cdot \mathbf{F} - \mathbf{F} \cdot \mathbf{R}|^2_{\text{fro}} \right)$$

4. EXPERIMENTAL RESULTS

4.1 Synthetic signals

The algorithm resulting from the method described has been first applied to synthetic signals, obtained through the convolution of a single source with three arbitrary impulse response functions. Two different source signals were designed to drive the multichannel system. One of them takes the shape of half a period of a sinusoidal function, which is the shape expected for the acceleration caused by an elastic collision (test 1). The other one was conceived to show more abrupt changes in its shape (test 2). Gaussian, white noise was added to the synthetized signals, resulting in a maximum signal energy peak to noise power ratio of 93.85 dB.

The algorithm proposed in this work for multichannel blind deconvolution (MCBD) was applied to both test problems, as well as two other blind deconvolution / identification methods: Least-Squares Blind Channel Identification (LSBCI) [3], and Two-Step Maximum Likelihood Identification (TSML) [6].

Methods LSBCI and TSML require previous knowledge (or estimation) of impulse response lengths. In the case of MCBD, source length can be estimated by running the algorithm for several lengths and detecting an abrupt change in the minimun of the objective function. Figure 1 shows a plot of the minimum obtained for different lenghts in test problem 1 and test problem 2. Sources of 27 and 28 samples are estimated in test problems 1 and 2, respectively.

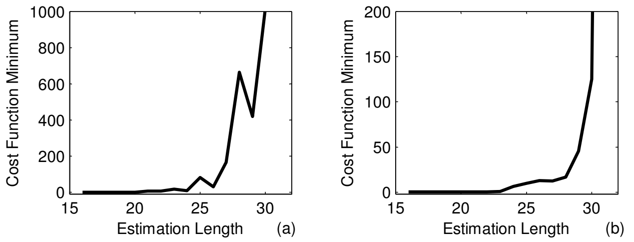

Figure 1: (a, b) Minimum of cost function attained for different lengths of estimated signal for test problem 1 and test problem 2, respectively.

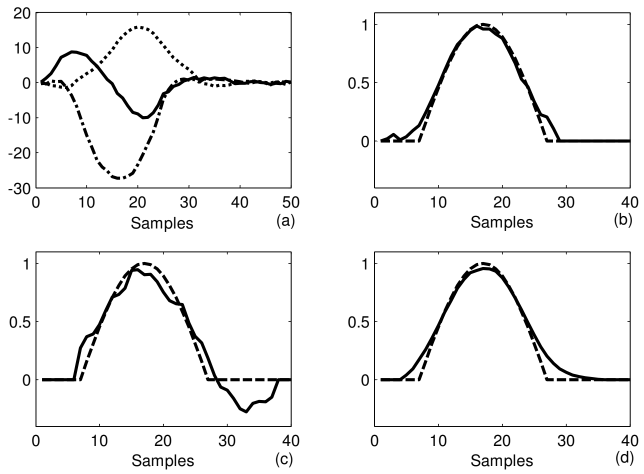

Figure 2: (a) Output signals of test problem 1 simulated multichannel system. (b, c, d) Original (dashed) and estimated (solid) source signals obtained with MCBD, LSBCI and TSML, respectively.

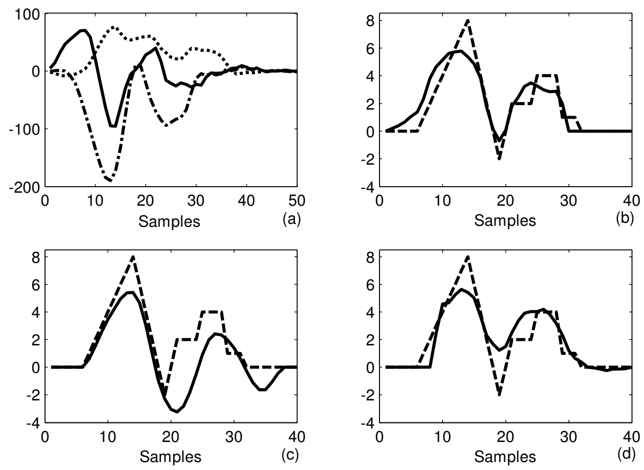

Figure 3: (a) Output signals of test problem 2 simulated multichannel system. (b, c, d) Original (dashed) and estimated (solid) source signals obtained with MCBD, LSBCI and TSML, respectively.

	LSBCI	TSML	MCBD
Test 1	-28.26	-41.10	-45.00
Test 2	-10.14	-20.57	-22.69

Table 1: Estimation error energy to signal energy ratio (dB) for LSBCI, TSML and MCBD.

Figures 2 and 3 show the output signals of the three channels (a) and the real and estimated source signals obtained with MCBD (b), LSBCI (c) and TSML (d) for test problem 1 and test problem 2 respectively. The estimated source signals are conveniently scaled to match the original, as amplitude information cannot be recovered by blind deconvolution techniques.

Table 1 shows the estimation error energy to signal energy ratio obtained with the three algorithms. The proposed algorithm MCBD provides a more accurate estimation for both test problems.

4.2 Real signals

The experimental set-up for the acquisition of real signals consists on a metallic beam hit by a sensorized hammer. The acceleration caused by the impact excites some modes of propagation and is finally detected by three capacitive accelerometers placed upon that beam in different positions. These sensors provide the output of the multichannel system. The accelerometer in the head of the hammer gives us a direct measure of the excitation signal. Figure 4 shows the impact signal and a detail of one of the output signals.

The signals are sampled at 400 samples/second by the acquisition device, which is enough to observe the first two modes of propagation. The infinite length of the signals violates one of the assumptions made in the development of this work. In order to apply the proposed algorithm, the common poles of the signals are first estimated following the method in [7] and their contribution is filtered out. The result are finite-length signals modeling the zeros of the original ones (figure 5: a, b, c).

Figure 5 (d) shows the impact signal provided by the sensor placed inside the hammer and the estimated one, provided by the proposed method (after scaling). The result provided by the algorithm can be considered satisfactory, in the sense it approximates the shape of the excitation signal and allows and estimation of its duration.

5. CONCLUSIONS

This work has introduced a novel blind deconvolution algorithm suitable for recovering transient, impulse-like signals.

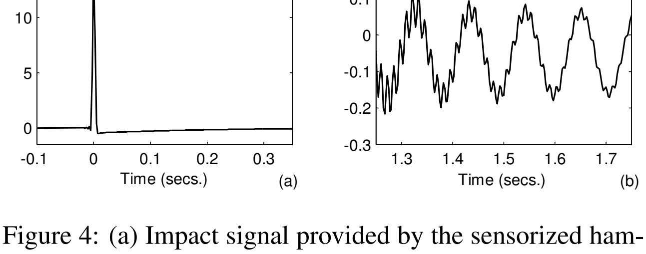

Figure 4: (a) Impact signal provided by the sensorized hammer. (b) Detail of one output signal.

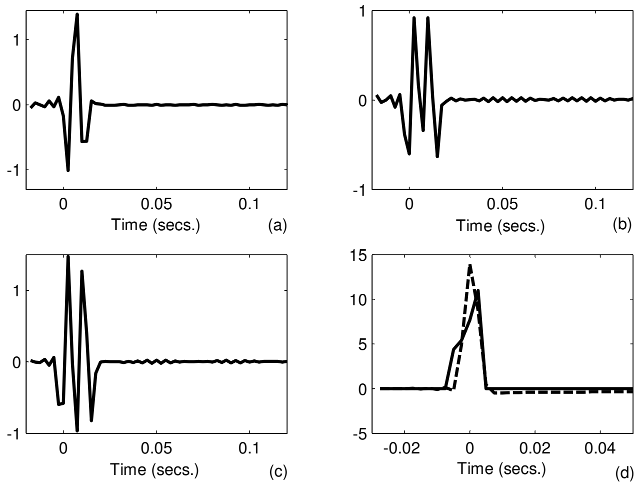

Figure 5: (a, b, c) Output signals after removing poles contribution. (d) Measured (dashed) and estimated (solid) source signal.

This kind of signals appear in many mechanical and acoustical systems and do not fulfill the identifiability conditions of published blind methods. The only conditions imposed by the proposed algorithm are the finite length of signals involved and channel diversity, which is essential to assure uniqueness of solution for any multichannel deconvolution / identification method.

Moreover, the length of the source signal can be found running the algorithm for several length values, whereas other blind methods require previous knowledge or estimation of impulse response lengths.

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