ARTIFICIAL REVERBERATION USING A HYPER-DIMENSIONAL FDTD MESH

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ABSTRACT

A hyper-dimensional finite-difference time-domain mesh structure and its application to artificial reverberation modeling is introduced. It is shown to produce reverberation with a dense and irregular modal pattern beneficial for modeling the reverberation in rooms or musical instrument bodies at high frequencies. An example design of a four-dimensional FDTD mesh is analysed. The computational cost of the four-dimensional mesh is shown to be lower than that of a bank of second-order resonators producing the same number of modes.

1. INTRODUCTION

Artificial reverberation is widely used in musical performances and recordings. In addition to its use as an effect or simulation of room acoustics, reverberation modeling is needed in synthesis of many musical instruments, where sound is reflected within a resonating body, for example.

Sound pressure waves reflect from boundaries such as walls and furniture in a room. In spaces with many boundaries, the rich set of reflections is perceived as reverberation instead of individual echoes. Where boundaries are located so that sound may be reflected back and forth on one trajectory, standing waves occur. The standing waves determine the modal structure in the frequency response as discussed in Section 2. At low frequencies the modes are sparse, but at frequencies above a critical frequency, often called the Schroeder frequency, the sound field is diffuse and modes are not distiguished individually by the ear [1]. If the modal density created by an artificial reverb is too low at this high frequency region, tonality or a metallic timbre is perceived.

In this paper a new method is introduced for creating diffuse sounding artificial reverberation. The method relies on a *hyper-dimensional finite-difference time-domain mesh* presented in Section 3. The study of hyper-dimensional mesh structures – structures of over three spatial dimensions – has been suggested earlier by Savioja et al. [2] and Rocchesso and Smith [3]. To illustrate the characteristics of a hypermesh, the impulse response of a four-dimensional mesh is analysed in Section 4. It is shown to produce dense and irregularly spaced modes.

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2. NORMAL MODES IN ROOMS WITH RIGID BOUNDARIES

A simple model for a room is a three-dimensional rectangular box with perfectly reflecting, rigid walls. In such a case, the sound pressure inside the box must fulfill the boundary conditions

$$\frac{dp_i}{dx_i} = 0 \qquad (1)$$

at walls perpendicular to coordinate x_i located at $x_i = 0$ and $x_i = L_i$. This holds for all three coordinates x_1, x_2, and x_3. The sound pressure value at a certain modal frequency at any point is given by a solution for (1) that can be written as

$$p_i(x_i) = A_i \cos(k_{n_i} x_i), \qquad (2)$$

where A_i is an arbitrary coefficient, $k_{n_i} = n_i \pi / L_i$, and $n_i = 0, 1, 2, \ldots$ is the integer index of the current mode along dimension x_i [1].

The solution can be extended to N dimensions, where the sound pressure value at point $(x_1, x_2, \ldots, x_N)$ inside the N-dimensional rectangular space at a certain modal frequency is

$$p_{n_1 n_2 \ldots n_N}(x_1, x_2, \ldots, x_N) = C \prod_{i=1}^{N} \cos\left(\frac{n_i \pi x_i}{L_i}\right), \qquad (3)$$

where L_i is the length along the ith dimension and C is an arbitrary coefficient. The modes appear at frequencies

$$f_{n_1 n_2 \ldots n_N} = \frac{c}{2} k_{n_1 n_2 \ldots n_N}, \qquad (4)$$

where c is the sound velocity. The constant $k_{n_1 n_2 \ldots n_N}$ is now a combination of all k_{n_i}:

$$k_{n_1 n_2 \ldots n_N} = \left[\sum_{i=1}^{N} \left(\frac{n_i}{L_i} \right)^2 \right]^{1/2}. \qquad (5)$$

The first standing wave along each dimension with $n_i = 1$ has a frequency whose corresponding wavelength is equal to twice the trajectory length. Other standing waves on the same trajectory are created at multiples of that base frequency. The modal frequencies are inversely proportional to the trajectory length, so for example in large halls the modes start from lower frequencies than in small rooms. As actual rooms and halls are not perfectly rectangular but have furniture and other objects reflecting sound, the effective trajectory lengths are not equal at all frequencies. More propagation trajectories are supported, resulting in a denser modal structure at high frequencies.

3. HYPER-DIMENSIONAL RECTILINEAR FDTD MESH

The finite-difference time-domain mesh structure consists of junctions which have connections to their neighboring junctions and an internal unit delay. Together they provide a model of wave propagation. In one dimension each junction has two neighbors, and input from one neighbor can be passed through the junction, partially transmitted, or reflected back.

The 1-D solution can be extended to higher dimensions by adding more neighbors for each junction. For example, a two-dimensional FDTD mesh structure is shown in Fig. 1. The choice of topology can be made from many possibilities. For its simplicity, a regular rectilinear topology was chosen. In a rectilinear N-dimensional FDTD mesh each junction has $2N$ neigbors. If all junctions are considered to have equal impedances, the junction value at time step t is defined by

$$p_k(t) = \frac{{}_l\, p_l(t-1)}{N} - p_k(t-2), \tag{6}$$

where k represents the index of the junction to be calculated and l is an index for enumerating the neighbors of junction k [2]. The updating method of the mesh values is a function of z^{-2} [10]. This causes the frequency band to be limited at a quarter of the sampling frequency f_s, which is related to the dimensionality N of the mesh by

$$f_s = \frac{c\sqrt{N}}{x}, \tag{7}$$

where c is the wave propagation speed in the mesh and x is the spatial sampling interval corresponding to the distance between two neighboring junctions [4].

In an FDTD mesh, the number of degrees of freedom of the model is equal to the number of junctions, so the maximum number of modes equals the number of junctions in the mesh. Thus, in a mesh of rectangular shape, the highest mode index number n at each dimension is equal to the number of junctions along that dimension. If the reflection coefficient is positive, $n \geq 0$. In the case of phase reversing reflection, $n \geq 1$. In N dimensions, $2N$ additions and one multiplication is needed per junction, so for n modes the computational cost would be $2Nn$ additions and n multiplications. Especially if N is an integer power of two, the multiplication can be implemented as a bit shift operation. In comparison, if the modes would be modeled using a second-order filter for each mode, four additions and five multiplications would be needed per resonator and then all the resonator outputs would have to be added together, resulting in a computational cost of $5n$ additions and $5n$ multiplications. The memory consumption is equal for the FDTD mesh and the resonator bank.

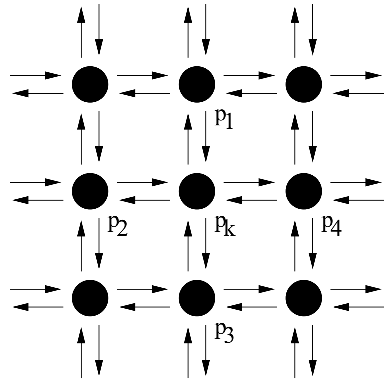

Figure 1: A two-dimensional rectilinear FDTD mesh structure. p_k is the junction currently calculated and p_l, where $l = 1, 2, 3$, or 4, are its axial neighbors as in (6).

4. APPLICATION TO ARTIFICIAL REVERBERATION

4.1 Reverb modeling with the hyper-dimensional mesh

To give a point of reference for modal density in real halls, the measurement data from the Finnish concert halls of Kuopio and Tampere was examined [5]. In these halls there are about 30000 mode frequencies within the frequency band 0 - 24 kHz. The number of modes at $1/3$-octave frequency bands grows almost linearly with frequency. At 10 kHz there are about 2500 modes per band and at 20 kHz about 4000 modes per band.

In artificial reverberation, however, it is not necessary to have so many modes. It has been shown by listening tests that about 1500 modes distributed evenly or along a logarithmic scale over a frequency band of 80 Hz - 10 kHz are enough to produce diffuse and natural sounding artificial reverberation. The same may also be accomplished with an even lower number of modes, but having a randomized distribution [6].

In any real space, the modes are sparse at the low frequencies. These can be modeled using a few adjustable resonators. In the high-frequency range modes are not perceived separately, so only a simple way to produce enough modes over a certain frequency band is needed. In the time domain, the first reflections are sparse but soon get dense and diffuse, which should also be the case with artificial reverberation [7].

The combination of resonators for accurate low-frequency modeling with a structure that produces high-frequency modes less accurately but with a smaller computational cost per mode has been used in previous work. Huang et al. [9] suggested two- and three-dimensional digital waveguide mesh structures for high-frequency reverberation modeling, while Penttinen et al. [8] used a modification of the feedback delay network [7] for the same purpose.

Extending the digital waveguide mesh structure to higher dimensions while using the same number of junctions makes the lengths along each dimension shorter. This results in modal frequencies which are higher and closer together. For maximizing the variation of modal frequencies, the path lengths along each dimension can be chosen close together from a prime number series. This makes the mode distribution inharmonic. The inharmonicity is augmented by perturbation in mode frequencies caused by the numerical dispersion inherent in the rectilinear mesh structure discussed

Table 1: Dimensions of the tested FDTD meshes, excluding the zero valued junctions at the boundaries.

	L_1	L_2	L_3	L_4
1D	1540	-	-	-
2D	20	77	-	-
3D	20	7	11	-
4D	4	5	7	11

in earlier work [10, 11]. Shortening the signal routes also allows the reflections to get denser more quickly. However, increasing the dimensionality of a digital waveguide mesh consumes much more memory. Therefore, a similar but less memory-consuming FDTD mesh structure is now used instead of the digital waveguide mesh.

In an FDTD mesh, simple approximations of lossy boundaries can be implemented using the method introduced by Kelloniemi et al. [12]. For more realistic modeling of air absorption and boundary losses, lowpass filtering should be implemented at the boundaries or within the mesh.

In digital waveguide meshes, traveling wave variables are used, which makes it possible to introduce filters into the mesh. A change to these wave variables can be implemented in the FDTD mesh by using the KW-converters introduced by Karjalainen and Erkut [13]. A more detailed discussion of this solution is out of the scope of this paper, and has been studied earlier [14].

4.2 Hyper-dimensional mesh performance

For creating the dense high-frequency modes, the results given by a 4D mesh were tested as an example of a hyper-dimensional mesh against the results of meshes of equal size but lower dimensionality. The number of free junctions in each mesh was chosen to be 1540. Because of results discussed in Section 4.1, we wanted to have over 1500 junctions. The lengths along each dimension are listed in Table 1. They were chosen so that there are no common factors. The number of modes in each 1/3-octave band at frequencies relative to the sampling frequency f_s are plotted in Fig. 2, calculated using (4). As artificial methods for creating low-frequency sparse modes are well known, these were not implemented in our test.

The reflection coefficient $r = -1$ was used because it is the simplest boundary condition for the FDTD mesh. Thus, all junctions at the mesh boundaries were bound to zero to produce phase reversing reflections. This reduces the computational cost, as inputs from the zero valued boundary junctions may be left uncalculated. Computational costs of each tested mesh type are listed in Table 2. For comparison, the costs for the second-order filter bank solution are also listed. The number of additions is larger for the 4D mesh than for the resonator bank implementation, but considerable savings are obtained in terms of number of multiplications. The overall computational cost of the 4D mesh is also smaller than that of the filter bank.

The simulations were initialized with a unit impulse at a free mesh junction located closest to one corner of each mesh and the output was read at the same location. The results from simulations run for 2^{16} time steps can be seen in Figs. 3 and 4. The initial portion of the impulse responses for each

Table 2: Computational costs of artificial reverberation methods for creating 1540 modes, implemented using FDTD meshes of one to four dimensions and with a second-order resonator bank.

	Additions	Multiplications
1D	3078	1540
2D	5966	1540
3D	8366	1540
4D	11802	1540
Resonators	7700	7700

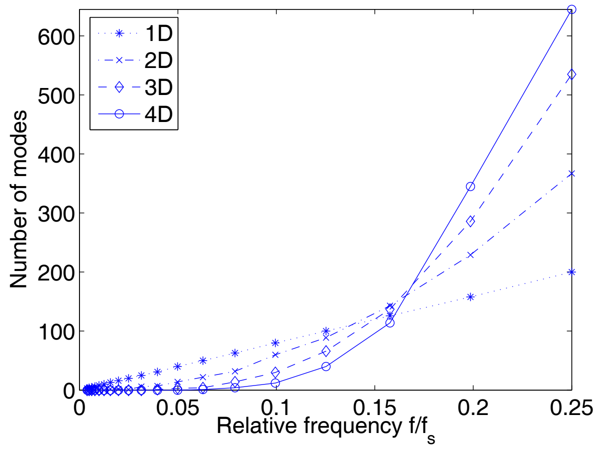

Figure 2: Predicted 1/3-octave band mode distribution of meshes having 1540 junctions.

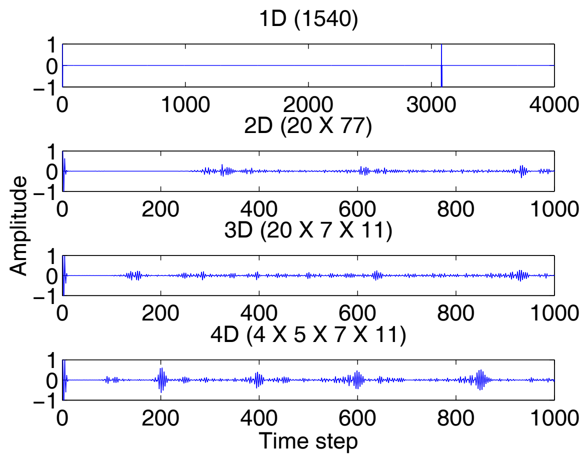

Figure 3: Time domain impulse responses for 1D, 2D, 3D, and 4D solutions. Note the different time scale for the 1D case.

test case are shown in Fig. 3. First the unit impulse is seen and after a while the first reflections appear. The first 1000 time steps are shown except for the 1D case, where it takes 3080 time steps for the first reflection to return from the other end of the line. The figures show that dense reflections begin soon after initialization for 2D, 3D and 4D meshes, soonest

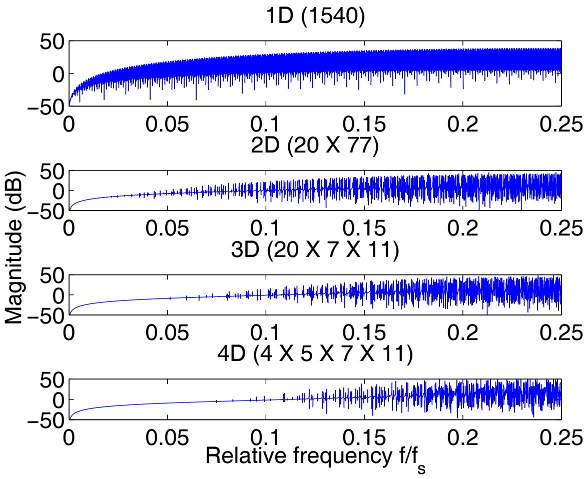

Figure 4: Impulse responses in the frequency domain.

in the 4D case.

The benefit in modal density predicted by (4) can be seen in Fig. 4. In 4D most of the modes are packed to the high frequencies. Note that there are very few sparse modes in the 4D implementation and those too are located at higher frequencies compared to the results from lower order implementations. Thus the output could be used as a model of dense high frequency modes even without highpass filtering or, in case of modeling small resonant structures such as musical instrument bodies, with only low-order highpass filtering.

5. CONCLUSIONS

The hyper-dimensional FDTD mesh structure has been introduced. It is easy to extend the rectangular mesh structure to any N dimensions as explained in Section 3, the only limiting factor being the computational cost growing with the dimensionality. The mesh can be used for artificial reverberation, where the complex modal and temporal structure generated with a four-dimensional FDTD mesh was shown to be much denser than that obtained with meshes of lower dimensionality. Although the computational cost of the 4D mesh is large, it is more efficient than a resonator bank that produces the same amount of modes. To make the algorithm more useful as a reverb, frequency dependent losses should be modeled as discussed in Section 4.1.

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