DIFFUSION MODELLING AT THE BOUNDARY OF A DIGITAL WAVEGUIDE MESH

Simon Shelley, Damian T. Murphy

Media Engineering Group, University of York
Heslington, York, YO10 5DD, UK
phone: + (44) 1904432405, fax: + (44) 1904432335, email: sbs107@ohm.york.ac.uk, dtm3@ohm.york.ac.uk
web: http://www.elec.york.ac.uk/ME/

ABSTRACT

The digital waveguide mesh is a method used to simulate the propagation of sound waves in an acoustic system. In order to accurately model such systems, it is important to model the reflection characteristics at the boundaries. A significant property of an acoustic boundary is its diffusivity. This paper presents a new approach to modelling diffusion, built upon a previous method that uses circulant matrices to vary the angle of incident waves prior to reflection. The new model is designed to eliminate the rotation error, inherent in the existing model. The results show that it offers diffusion that is more consistent at different frequencies, as well as a more linear and accurate control over the amount of diffusion, particularly when modelling boundaries with low diffusivity.

1. INTRODUCTION

The digital waveguide mesh is a technique used to model the propagation of sound waves in 2-D and 3-D acoustic systems [1, 2] and can therefore be used in musical instrument and room acoustic modelling. It is a model that, by its nature, incorporates the effects of diffraction and wave interference [2, 3]. This is an advantage when it is compared to other room acoustic modelling techniques such as the image source method [4] and ray-tracing [5].

A specular reflection occurs at a smooth surface when the angle of the reflected sound wave is equal to the angle of incidence. A diffuse reflection occurs when a sound wave reflects from a rough boundary and results in a redistribution of the sound energy across a range of angles. In the most extreme case, the energy is spread evenly in every direction, whatever the angle of incidence [6].

The behaviour of the propagating sound waves in an acoustic system can be affected significantly by the diffusive characteristics of the boundaries. Accurate modelling of this effect in the digital waveguide mesh is therefore required. Previous work details the succesful implementation of a highly diffusive boundary in a 2-D mesh, using a quadratic residue diffuser [7]. However this method limits the amount of control over the diffusivity of the surface and also causes complications if other boundary characteristics are to be modelled, such as frequency dependent absorption.

Another technique has been developed that simulates diffusion by randomly rotating the incident waves as they approach the boundary of the mesh [8]. This model allows the diffusivity of the boundary to be controlled and is lossless, however it introduces an error. This model, and its associated error is reviewed in Section 3. In this paper, a method to avoid this error is introduced and an observation of the effects of this modification is presented.

Two dimensional mesh structures with a triangular topology are used for the purpose of this paper but the diffusion models can theoretically be extended to three dimensions.

2. THE DIGITAL WAVEGUIDE MESH

The digital waveguide mesh is an extension of the 1-D digital waveguide used for physical modelling synthesis [9]. It is made up of discrete time bi-directional delay lines connected by scattering junctions, or nodes, which are arranged to form 2-D or 3-D structures. The scattering junctions act as spatial and temporal sampling points.

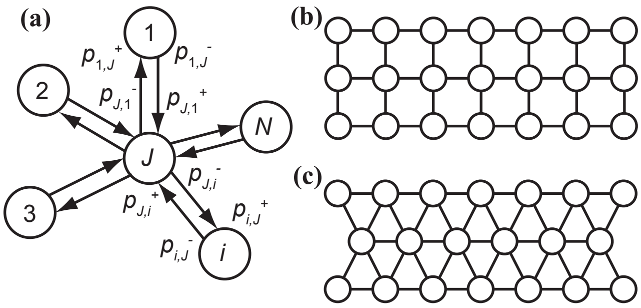

Figure 1: *(a) A general scattering junction J with N connected waveguides for i = 1,2,...N; (b) a 2-D rectilinear mesh structure; (c) a 2-D triangular structure.*

Figure 1(a) shows a scattering junction, J, with N neighbours, $i = 1, 2, \ldots N$. These are connected together by delay lines, or waveguides. The sound pressure in a waveguide is represented by p_i, which is defined by the sum of the two signals p_i^+ and p_i^- :

$$p_i = p_i^+ + p_i^- \qquad (1)$$

These travel in opposing directions along the bidirectional delay line. When two junctions are considered, i and J, the signal $p_{i,J}^+$ represents the incoming signal to junction i along the waveguide connected to junction J. Similarly, the signal $p_{i,J}^-$ represents the outgoing signal from junction i.

By connecting scattering junctions together, it is possible to model wave propagation in 2-D and 3-D spaces. Different mesh topologies can be used to model the same physical structure. For instance a 2-D space can be modelled using either a rectilinear mesh or a triangular mesh, diagrams of which can be seen in Figures 1(b) and 1(c) respectively. The choice of topology dictates the number of neighbours that each scattering junction has.

The sound pressure at a lossless scattering junction, P_J, can be found using (2), where $p^+_{J,i}$ represents the incoming pressure signal at a connecting waveguide and Z_i represents its impedance. Again, N is the number of connecting waveguides at the junction.

$$P_J = \frac{2 \sum_{i=1}^{N} \frac{p^+_{J,i}}{Z_i}}{\sum_{i=1}^{N} \frac{1}{Z_i}} \qquad (2)$$

The scattering junctions are separated by bi-directional unit delay lines. This means that the input to a scattering junction is equal to the output from a neighbouring junction into the connecting waveguide at the previous sampling time step:

$$p^+_{J,i} = z^{-1} p^-_{i,J} \qquad (3)$$

(1), (2) and (3) are collectively termed the scattering equations of the system.

A limitation of the digital waveguide mesh is dispersion error. This results in an inconsistency in the velocity of wave propagation that is dependent on both its frequency and direction of travel. The latter implies that different mesh topologies will have different dispersion characteristics and this has been investigated previously [10] as well as methods to reduce this error [11]. The triangular mesh used in this work has been shown to offer the best dispersion characteristics in the 2-D plane, such that it becomes almost independent of wave direction and is therefore a function of frequency only.

3. A DIFFUSION MODEL USING CIRCULANT MATRICES

At each time step, incoming signals at the scattering junctions are processed according to these scattering equations and new signals are passed out ready to be received by the neighbouring junctions at the next time step. By multiplying the incoming signals with circulant matrices, it is possible to rotate them around a node by any angle, with the result that the direction of the propagating wave at that point is altered. This may be proven mathematically [8], but it is only accurate if the connecting waveguides are uniformly distributed around the junction, an example of which is shown in Figure 2(b).

Similarly the incoming signals at the boundary nodes can be multiplied with circulant matrices in such a way that the directions of the incoming waves are randomly altered just before they are reflected. This effectively simulates diffusion and also ensures that energy is conserved. By varying the range over which the rotation angle is allowed to change, the amount of diffusion can be controlled.

3.1 The Rotation Error

The boundary nodes in a 2-D waveguide mesh with a triangular topology can be connected to 1, 2, 3, 4 or 5 neighbouring nodes, depending on their orientation. However the waveguides connecting these nodes are not distributed uniformly; an example of this is illustrated in the diagram of a 3-port boundary junction shown in Figure 2(a). As a result of this, errors will occur when the incoming signals are rotated. This is referred to as the *rotation error*.

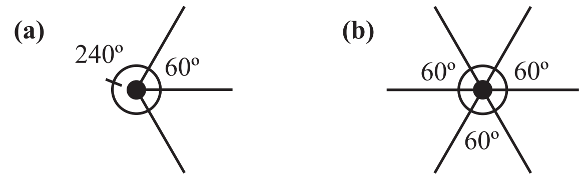

Figure 2: *(a) A 3 port boundary junction and (b) a 6 port junction with connecting waveguides.*

Analysis of the rotation error [8] shows that there is a complex non-linear mapping between the intended (ideal) angle of rotation and the actual (real) angle of rotation that is dependent on two factors. The first is the amount of rotation that is applied in the ideal case and the second is the angle of approach of the incoming waves. The effects of this discrepancy become less extreme as the number of waveguides connected to a boundary junction increases. This means that there are inconsistencies in the diffusion model, as different types of boundary nodes exhibit different diffusive characteristics.

The error becomes particularly apparent when modelling only slightly diffusive boundaries, as small angles of rotation will tend to be distorted into large angles.

4. A DIFFUSION MODEL WITHOUT ROTATION ERROR

The rotation error occurs when the circulant matrix transformation technique is applied to waveguide junctions that are at the boundaries of the digital waveguide mesh. However, if the same method is applied to standard N-port air nodes then the error is eliminated because their connecting waveguides are evenly distributed, being separated by equal angles. In the case of a 2-D waveguide mesh with triangular topology such junctions have six connecting waveguides, separated by angles of sixty degrees, as shown in Figure 2(b). This is the ideal case for the circulant matrix rotation technique. By applying random rotations at junctions adjacent to the boundary, rather than the boundary junctions themselves, it is possible to achieve diffusion without the rotation error.

As a result of this technique, waves that approach the boundary are usually rotated twice. Once as they approach the boundary and a second time as they travel away from it after being reflected. This can be compensated by halving the required rotation angles at the junctions adjacent to the boundary. Undesirable effects may occur, however, when large rotation angles are applied because waves may be rotated more than twice or even just once, depending on the angle of approach and the amount of rotation that is applied.

4.1 Implementation in the 2-D triangular waveguide mesh

The incoming signals at a 6-port junction in a 2-D waveguide mesh can be rotated by an angle φ if they are multiplied with a circulant matrix, $\mathbf{A}$, whose coefficients can be calculated using the following set of eigenvalues, X:

$$X = \left[\begin{array}{cccccc} 1 & e^{j\varphi} & e^{j2\varphi} & -1 & e^{-j2\varphi} & e^{-j\varphi} \end{array} \right] \qquad (4)$$

An inverse Fourier transform, performed on these eigenvalues, yields 6 real numbers that sequentially make up the first row of coefficients, x_0 ... x_{N-1}, in the circulant matrix, **A**. The coefficients in subsequent rows can be calculated as follows:

$$A = \begin{bmatrix} x_0 & x_1 & \dots & x_5 \\ x_5 & x_0 & \dots & x_4 \\ \dots & \dots & \dots & \dots \\ x_1 & \dots & x_5 & x_0 \end{bmatrix} \quad (5)$$

This matrix can then be multiplied with the incoming signal amplitudes at the junction, s_i, to form a new set of rotated signal amplitudes, s'_i, the sum of which is equal to the sum of the original amplitudes:

$$A \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ s_5 \end{bmatrix} = \begin{bmatrix} s'_0 \\ s'_1 \\ \vdots \\ s'_5 \end{bmatrix} \quad (6)$$

These rotations are applied to the junctions adjacent to the perimeter nodes, i.e. all the junctions in the mesh that are directly connected to a boundary junction.

Diffusion is simulated by randomly altering the amount of rotation, φ, of the incoming signals at each of these nodes for each sample step. The amount of diffusion that is modelled can be controlled by limiting the algorithm to a range of angles. As an example, to simulate a relatively smooth wall the maximum random angle can be set to 5 degrees in either direction. More complete diffusion can be achieved by increasing this angle.

5. RESULTS

5.1 Case Study

Two lossless digital waveguide meshes are constructed. In one mesh, Model A, the diffusion method outlined in Section 4 is implemented at every boundary; i.e. circulant matrix rotations are applied at the junctions adjacent to the boundary. In the other mesh, Model B, the original diffusion method described in Section 3 and in [8] is implemented for comparison, where circulant matrix rotations are applied at the boundary nodes. The suggested method for reducing the rotation error [8] is also implemented. During simulation a different angle of rotation, selected at random, is applied at each appropriate junction at the boundaries of the mesh. New angles of rotation are selected at every time step. A random function with a uniform probability distribution is used to select the angle of rotation from within a variable range of angles.

Both meshes are 2-D rectangular structures constructed using a triangular topology, each with a length of 1.91m and a width of 1.10m. The sampling rate is set to 44,100 Hz. A series of 2 second simulations are performed on each mesh, with increasing amounts of diffusion implemented at all boundaries. In each simulation, a low pass filtered impulse is introduced into a corner of the mesh and the outputs are obtained at a junction in the opposite corner. Starting with 0^o, the range of rotation angles is increased for each simulation by $\pm 5^o$ until a maximum possible rotation angle of $\pm 45^o$ is reached.

Figures 3 and 4 are spectrograms of the simulations' outputs concatenated in sequence for Models A and B respectively. The concentrated horizontal lines correspond to the modal frequencies of these systems. These agree with the theoretical modal frequencies of the simulated 2-D rooms. For example the first two theoretical axial modes are 90Hz and 156Hz and these are in agreement with the results presented. The amount of diffusion in each model and for each test can be observed by the concentration of energy found at the modal frequencies. As the amount of diffusivity increases at the boundaries, the energy at these frequencies becomes less concentrated and more energy is spread out into other frequencies.

5.2 Discussion

The results show that, when using random angles of rotation which are limited to the small range of $\pm 5^o$, Model B displays highly diffusive characteristics, but Model A exhibits more subtle diffusive characteristics. As the limit of the angles is increased, the observed diffusivity of Model B increases slightly at higher frequencies, but appears to fluctuate at low frequencies. This behaviour is also observed in Model A but is not so extreme. The diffusivity of Model A increases in a more linear fashion and is more consistent across a range of frequencies.

The difference between the two spectrograms (Figures 3 and 4) is evidence that the rotation error has a significant effect on the diffusion model. The results show that using the method described in Section 4, a more subtle and consistent diffusion model can be achieved. This is particularly true for small ranges of diffusion angles.

The sound of the outputs confirms what is seen in the spectrograms. With no diffusion applied, it is possible to hear the modal frequencies of the rooms. As the range of rotation angles increases in Model A, the modal frequencies diminish and the sound becomes increasing noisy.

The more subtle diffusive effect of the new method can be explained by the elimination of the rotation error, implying that small rotation angles are no longer translated into larger angles. This is an advantage when modelling surfaces with low diffusivity, commonly found in real world materials.

Both models may have practical applications however, because by choosing an appropriate model and diffusion range, it is possible to introduce frequency dependency in highly diffusive surfaces, where high frequencies are scattered more than lower frequencies.

It should be noted that the diffusion model presented in this paper simply alters the incoming signals at the junctions adjacent to the boundary. This *diffusing layer* approach has the advantage that there are no added complications when further processing of these signals, at the boundary itself, is desired. For example if the signals should be filtered in order to model frequency dependent absorption.

6. CONCLUSIONS

The method presented in this paper can be used to effectively model variable diffuse reflections at the boundary of a 2-D triangular waveguide mesh. By eliminating the rotation error, boundaries with relatively low diffusive qualities are modelled more accurately. The modelled diffusion is largely independent of the angle of approach, and of the frequency of the reflected sound waves.

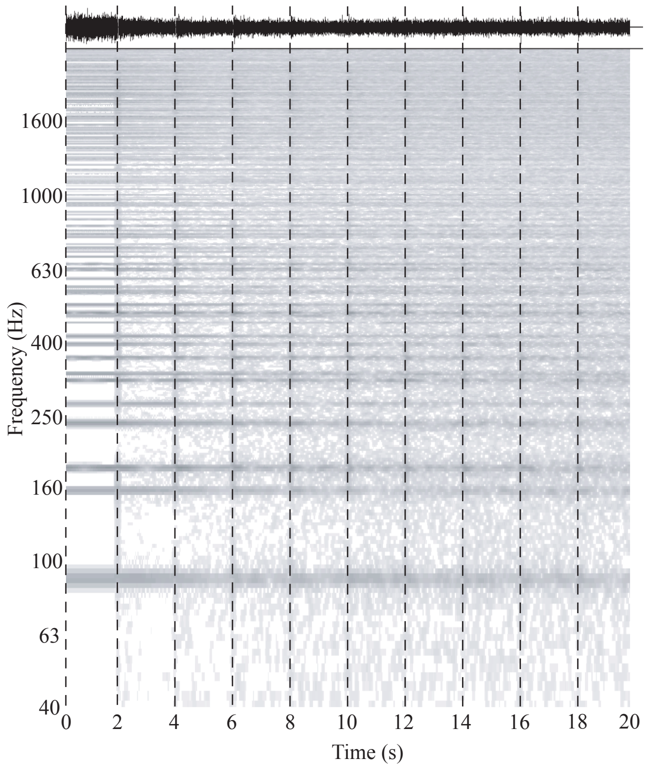

Figure 3: *Spectrogram of the output from Model A. After every 2 seconds the diffusion range is increased by* ± 5 *degrees, the model is reset and an impulse is applied at the input.*

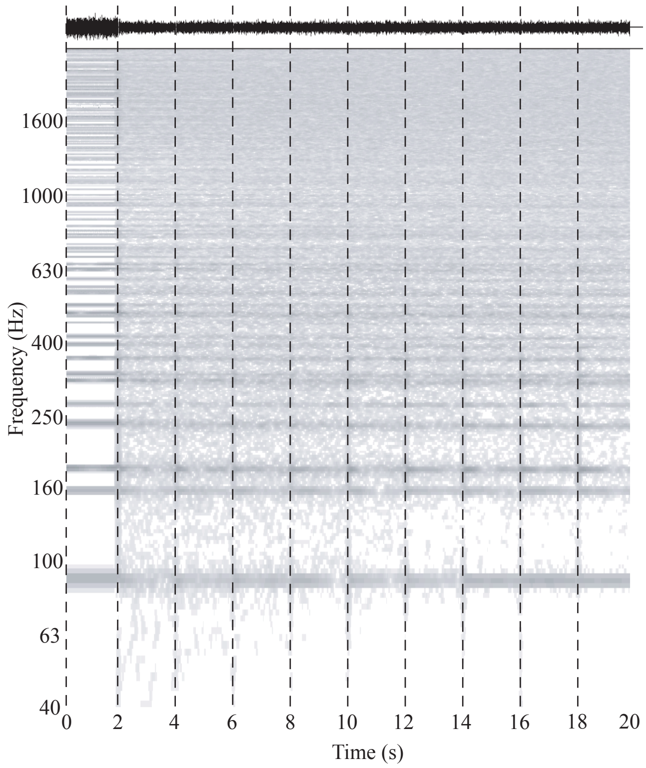

Figure 4: *Spectrogram of the output from Model B. As in Figure 3, the diffusion range is increased by* ± 5 *degrees after every 2 second and the model is reset.*

In future this research will focus on the application of different random probability distributions to the model, simulating frequency dependent diffusive surfaces, extending the model to 3-D digital waveguide meshes, and comparing the effects of the diffusion model with real diffusive acoustic boundaries. This can be achieved, for example, by measuring the *diffusion* coefficient [12].

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