REDUCING NON-ZERO COEFFICIENTS IN FIR FILTER DESIGN USING POCS

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ABSTRACT

The number of nonzero coefficients in an FIR filter determines the number of hardware multipliers that are required to implement the filter. Projection onto convex sets is shown to be an effective method to create linear phase FIR filters with reduced numbers of nonzero coefficients while maintaining filter specifications. The method can be used as an original design method or used to enhance the performance of filters generated by existing design methods.

1. INTRODUCTION

Projection onto convex sets (POCS) has been used in many signal processing applications including: solving systems of linear equations, tomography and image restoration. An advantage of this approach is that it allows a natural way to insert practical constraints into an optimization problem. However, the POCS based methods can guarantee only that the solution lie within the intersection of the specified sets. It cannot explicitly determine the smallest intersection. The usual linear phase FIR filter design problem has been formulated for POCS implementation [1]. The design specifies limits which can be used to define convex sets.

One aspect of FIR filter implementation of interest is the number of multipliers needed to realize the filter. We propose a design method that can produce filters that satisfy specifications while having fewer nonzero coefficients than those produced by common methods. Due to the complexity of multiplications, a realized digital filter with fewer multipliers can lead to a significant reduction in both the physical size and the processing time. The trade-off in such a design is that the total number of delay elements in the new filter is larger than the usual designs.

2. BASIC POCS DESIGN

The specifications for a frequency selective filter are usually given by

passband $\omega \le \omega_p$ with ripple $||H(\omega)| - 1| \le \delta_p$
stopband $\omega \ge \omega_s$ with attenuation $|H(\omega)| \le \delta_s$

The convex sets based on these specifications for the filters, $h(n)$, of length M, are

$$C_M = \{h(n) : h(n) = 0, \text{ for } n < 0, \text{and } n > M - 1\}, \quad (1)$$

$$C_{phase} = \{h(n) : h(n) = h(M-1-n), \ \ 0 \le n \le M-1\}, \quad (2)$$

$$C_{mag} = \{h(n) : |H(\omega) - e^{-j\frac{M-1}{2}\omega}| \le \delta_p, \omega \le \omega_p\ \}, \quad (3)$$

where δ_p is obtained from the passband specification, and

$$C_{mag} = \{h(n) : |H(\omega)| \le \delta_s, \omega \ge \omega_s\ \}, \quad (4)$$

where δ_s is obtained from the stopband specification. The fact that the phase is known makes the definition of the convex constraint sets in the complex domain possible.

The projections onto the convex sets defined above are easily derived. The POCS solution is obtained by projecting sequentially onto K convex sets. The iteration is described by [4, 5]

$$h_{k+1} = P_K(P_{K-1}(...P_1(h_k)...)) \quad (5)$$

where for this problem $K = 4$. The solution can often be obtained faster by a parallel projection method [6]. This method was used to obtain the results shown in this work. Convergence of the POCS iteration and other details are presented well in [7].

The implementation of the algorithm requires that the projections onto the sets defined in the frequency domain be defined for discrete frequencies. There are many ways to define such sets. This work used the simple expedient of a dense set of equally spaced samples. This definition has some interesting consequences. Each of these frequencies defines a convex set, a fact which may aid the convergence of the parallel projection method. A generalization of the convex sets using discrete frequencies is given by

$$C_{mag,k} = \{h(n) : |H(\omega_k) - H_d(\omega_k)| \le \delta_k\} \quad (6)$$

where $H_d(\omega_k)$ is the desired frequency response and δ_k is the bound at frequency ω_k. The number of sets need only be large enough to get reasonable interpolation between sample points of the DFT during the projection and close approximation of the passband and stopband limits.

The POCS design method has the advantage of directly defining the sets which produce a filter that satisfies the specifications, if such a filter exists. The tolerance for the filter error is easily determined from the specifications. As noted from Eq.(6), this can be for general filter shapes. The Parks-McClellan method (Remez in MATLAB software) produces the optimum filter of length M (smallest weighted min/max error)[2],[3]. The major difference in the methods is that the Remez method can work with a continuum of frequencies for frequency selective filters. For a general filter shape, both would require a definition at selected frequencies. For either POCS or Remez, iteration may be necessary to obtain a filter which satisfies the specifications. However, the Remez exchange algorithm is far more efficient and would be preferable in most cases.

3. DESIGN FOR REDUCING THE NUMBER OF NONZERO TERMS

In the case of frequency selective filters that are used extensively after design, the design time is not a significant consideration. Iterative methods are common. Design methods, such as the Parks-McClellan method can produce very efficient filters. However, they are not designed to implement constraints. Constraints of interest include number of nonzero coefficients, maximum value or range of coefficient. The POCS method can implement constraints easily, as long as the constraint can be formulated as a convex set. The number of nonzero coefficients does not define a convex set, i.e., the set of all FIR filters with M nonzero coefficients is not convex. If the positions of the zero coefficients can be specified, then a convex set can be defined. The set of FIR filters, $h(n)$, with $h(n_0) = 0$, is convex.

There are several ways to select a set of candidate indices for the set of zero coefficients. For example, one can start with an FIR design obtained from a common method and set the terms with magnitude less than some threshold value to zero. An alternate method would be to select a pattern of zeros for the filter. This is possible for frequency selective filters with certain passbands.

For frequency selective filters, the zero pattern can be estimated by considering the impulse response of the ideal filter. As an example, let us consider the lowpass filter with passband at $\omega_p = 2\pi f_p$, where f_p is a normalized frequency, $|f_p| < \frac{1}{2}$. The discrete impulse response of the ideal filter is

$$h(n) = \frac{\sin(2\pi f_p n)}{\pi n} \tag{7}$$

For fixed precision arithmetic, the values of $h(n)$ that are nearly zero are those where $2f_p n$ nearly is an integer. For the $f_p = \frac{1}{4}$, the impulse response is exactly zero for n even, $n \neq 0$. For other values of f_p, the indices that are set to zero may depend upon some threshold. Because of the periodic nature of the ideal impulse response, the pattern will be periodic. For impulse responses with exact zeros in the ideal case, it may be advantageous to set other additional positions to zero.

Once the zeros of the filter have been designated, the filter length must be increased. After all, the specifications cannot be satisfied by the optimal filter with modified coefficients. The patterns of zeros must be continued into the extended filter length.

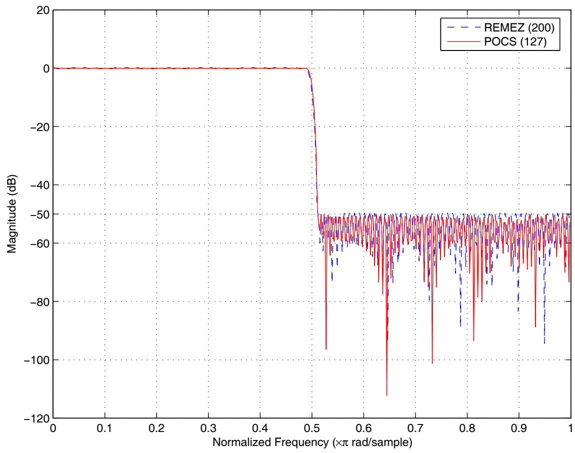

Fig. 1. Comparison of Filter Responses using Parks-McClellan (Remez) and POCS, $f_p = 1/4$

4. EXAMPLES

We begin with a simple halfband FIR, linear phase filter that might be used with any subband decomposition. This is the case mentioned above that $f_p = \frac{1}{4}$, where every other coefficient is zero. Realistic specifications require small ripple in the passband to $\frac{\pi}{2}$ and large attenuation in the stopband. Our method produces a filter with 127 nonzero elements with the

passband $\omega \leq 0.49\pi$ with ripple 0.2dB
stopband $\omega \geq 0.51\pi$ with attenuation 50dB

A filter designed by the Remez algorithm that satisfies the specifications has 200 elements, none of which are zero. The filter responses are shown in Figure 1.

A closer examination of the passband and stopband frequencies in Figure 2 illustrates the performance of the POCS method to satisfy the constraints. Note the significant reduction of the passband ripple. The passband ripple of the FIR filter generated by POCS is approximately 0.03 dB. A filter designed by the Remez algorithm with a modified passband ripple constraint of 0.03 dB required 258 nonzero elements.

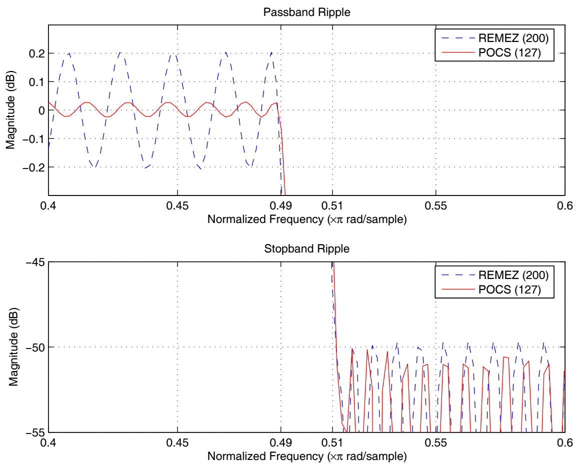

Fig. 2. Comparison of Passband and Stopband Regions using Parks-McClellan (Remez) and POCS, $f_p = 1/4$

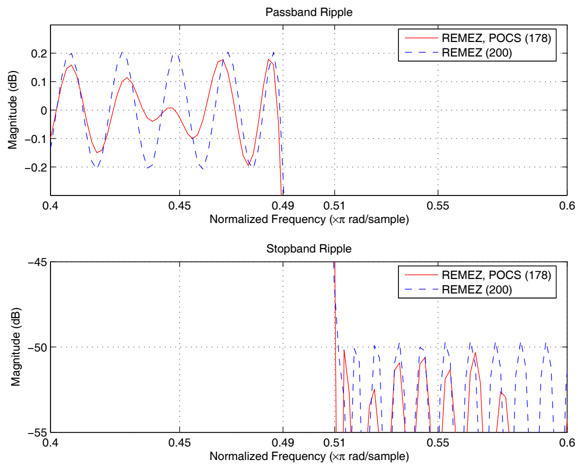

Fig. 3. Comparison of Passband and Stopband Regions using Parks-McClellan (Remez) and the POCS Enhanced Remez, $f_p = 1/4$

We can reduce the number of nonzero elements in the Remez filter at the expense of possibly missing the specifications slightly. To do this, we set elements of the filter that are near zero to zero, then use POCS to optimize the remaining elements. Using a threshold of 0.002 of the maximum coefficient value, the number of nonzero elements is reduced to 178. This filter response is compared to the original Remez in Figure 3. The new filter clearly loses its equiripple properties. Furthermore, the new filter has a slightly lower passband frequency. Though the equiripple properties have been lost, the new filter never exceeds the stopband specifications.

The case of the halfband filter is one of the best cases, since it has natural zeros at nearly half of the delay positions. We consider other cases and find that the POCS method can yield a reduction in the number of nonzero elements, but not with same spectacular results as for the halfband filter. An FIR linear phase filter with a passband frequency of $f_p = \frac{1}{8}$ was designed for comparison to the Remez algorithm according to the following constraints

passband $\omega \leq 0.24\pi$ with ripple 0.2dB
stopband $\omega \geq 0.26\pi$ with attenuation 50dB

The POCS method resulted in a filter with 181 nonzero coefficients, while the Remez method gave 200. The Remez filter could be enhanced to 180 nonzero coefficients using POCS.

An FIR linear phase filter with a passband frequency of $f_p = \frac{3}{16}$ was designed for comparison to the Remez algorithm according to the following constraints

passband $\omega \leq 0.365\pi$ with ripple 0.2dB
stopband $\omega \geq 0.385\pi$ with attenuation 50dB

The filter designed by the Remez algorithm that satisfies the specifications has 200 elements, none of which are zero. The POCS method generated a filter with 201 elements. The 201 elements were not contiguous and represent a longer filter length than the Remez filter. For this case, the POCS method does not perform any better than the Remez algorithm, but it does illustrate that the POCS method can generate similar filters of the same order. Again we note that the passband ripple is better for the POCS filter. The Gibbs phenomena is also evident. The filter responses are shown in Figure 4. The filter responses in the stopband and passband are shown in Figure 5. Using POCS to reduce the number of nonzero coefficients, the enhanced Remez filter has 176 nonzero terms, while maintaining the specifications. This comparison is shown in Figure 6. In fact, we note the same effect of actually improving the passband and stopband properties, while sacrificing the equiripple characteristics.

5. EXTENSIONS

The example of a filter for subband decomposition is a case where the method works very well. It will always be possible to use POCS to reduce the number of nonzero elements by the method of setting those element that are close to zero to zero. However, it is not clear how to determine the advantages from such methods. Another question of interest is how to use POCS on filters other than the frequency selective type. For example, the design of an equalizing filter, $h_e(n)$ for a communication channel, represented by $h_c(n)$, requires the combined system, $h_c(n) * h_e(n)$ to be approximately a delayed delta function, $\delta(n - n_0)$. This problem may be addressed with POCS in a similar way using the

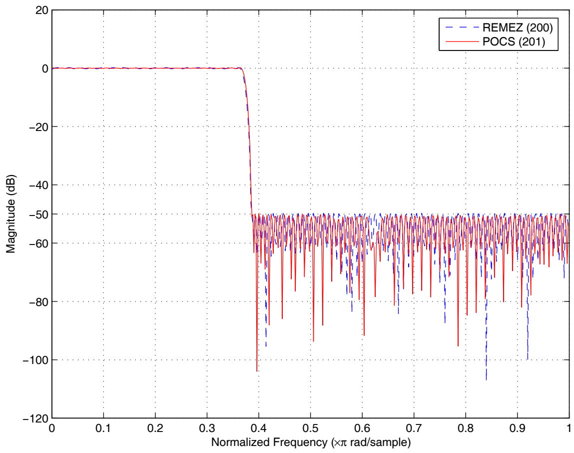

Fig. 4. Comparison of Filter Responses using Parks-McClellan (Remez) and POCS, $f_p = 3/16$

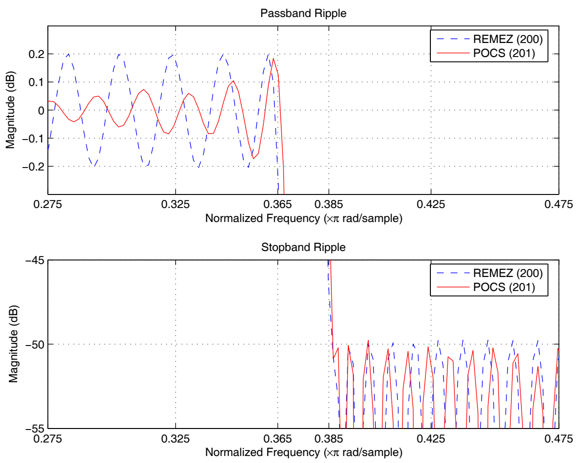

Fig. 5. Comparison of Passband and Stopband Regions using Parks-McClellan (Remez) and POCS, $f_p = 3/16$

frequency domain,

$$C_{mag,k} = \{h(n) : |H_c(\omega_k)H_e(\omega_k) - e^{-jn_0\omega_k}| \leq \delta_k\}, \tag{8}$$

where $H_c(\omega_k)$ and $H_e(\omega_k)$ are the Fourier transforms of the channel and equalization filters, respectively.

6. CONCLUSION

The POCS methods has been shown to be effective in creating linear phase FIR frequency selective filters with a reduced number of nonzero coefficients. The method may be used directly by starting with a truncated impulse response of an ideal filter. In addition, the method may be used to reduce the number of nonzero coefficients in filters designed by other methods.

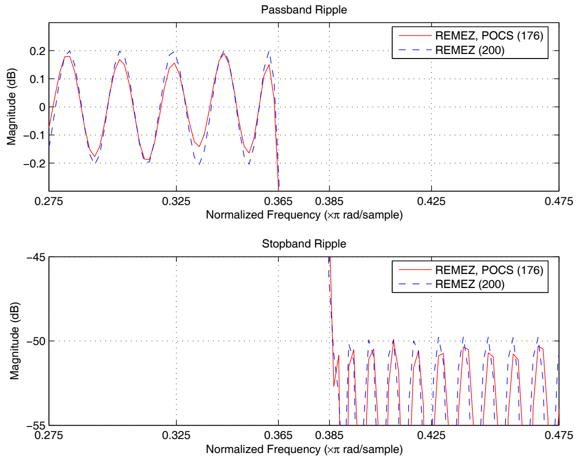

Fig. 6. Comparison of Passband and Stopband Regions using Parks-McClellan (Remez) and POCS, $f_p = 3/16$

7. REFERENCES

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