A SUBSPACE METHOD FOR THE BLIND EXTRACTION OF A CYCLOSTATIONARY SOURCE

Roger Boustany, and Jérôme Antoni

Mechanical Engineering Department, University of Technology of Compiègne
Centre de Recherche de Royallieu, 60205, Compiègne, France
email:roger.boustany@utc.fr

ABSTRACT

The need for blindly separating mixtures of source signals arises in many signal processing applications. The solution to this problem was found using emerging blind source separation (BSS) techniques which rely on the knowledge of the number of independent sources present in the mixture. This paper deals with the case where the number of sources is unknown and statistical independence may not apply, but where there is only one signal of interest (SOI) to be separated. We propose a method for extracting this SOI by exploiting its cyclostationarity through a subspace decomposition of the observations. This method is first developed for instantaneous mixtures and is then extended to the convolutive case in the frequency domain where it does not suffer from the permutation problem as does classical BSS. Experiments on electrocardiogram and industrial data are finally performed and illustrate the high performance of the proposed method.

1. INTRODUCTION

The problem of blindly separating instantaneous or convolutive mixtures of sources is encountered in many signal processing applications. Among these, one can cite communications, radar and sonar systems, biomedical applications, the "cocktail party effect" and many other fields. The solution to this problem was found using emerging blind source separation techniques which usually rely on the assumption of the mutual statistical independence of the sources present in the mixture. Ways of ensuring the mutual independence of the recovered sources include exploiting their non-gaussianity [1], their non-whiteness [2], or their non-stationarity — and particularly their cyclostationarity [3], [4], [5]. As a matter of fact, many (quasi-)periodic physical phenomena generate (quasi-)cyclostationary signals. Examples of these are the electrocardiogram (ECG) measuring the electrical activity of the heart and the vibrations caused by rotating or reciprocating mechanical systems [6].

However, in many situations, the number of sources is often unknown and may be greater than the number of sensors. Since classical BSS needs the information on the number of sources and usually relies on a greater number of measurement signals, it fails to deal with such cases. Mutual statistical independence of the sources is another restrictive condition which might not be respected in some applications, like those involving mechanical systems.

The purpose of this work is to propose a substitute approach to BSS in order to extract one cyclostationary source labelled "SOI" (for signal of interest) drowned by an unknown number of interfering sources and noise. This subject was addressed in the communications context for instantaneous mixtures in [7] where the authors proposed a class of Spectral Self-Coherence Restoral (SCORE) algorithms which take advantage of the fundamental cyclic frequency of the SOI. We propose a novel approach still exploiting the cyclostionarity of the SOI but using a subspace decomposition of the data. Unlike SCORE, the method decribed in this paper easily allows the use of several cyclic frequencies which yields a more accurate estimation of the SOI, and it requires the determination of only half the number of unknowns of SCORE. A frequency-domain extension to the convolutive case is also presented in order to cover a broad range of applications.

The paper is organized as follows. In section 2, we review the problem of BSS, its formulation and its limitations. After a brief review of cyclostationary properties in section 3, a more suitable approach for mixtures with an unknown number of sources and one cyclostationary SOI is developed in section 4. An extension to convolutive mixtures is then described in section 5. Finally, we present experiments carried out on real world signals to illustrate the efficiency of the approach.

2. BLIND SOURCE SEPARATION AND EXTRACTION

Blind source separation consists of retrieving m unknown sources, $\mathbf{s}(t) = (s_1(t), s_2(t), \ldots, s_m(t))^T$ solely from the knowledge of n observation signals $\mathbf{x}(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$, $t \in \mathbb{R}$. Depending on the application, the mixture model may be either instantaneous

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

or convolutive

$$\mathbf{x}(t) = \mathbf{H}(t) * \mathbf{s}(t) + \mathbf{n}(t), \tag{2}$$

where $*$ is the convolution product, $\mathbf{A}$ and $\mathbf{H}(t)$ denote respectively full rank "scalar" and linear filter matrices modelling the transfers between sources and sensors and $\mathbf{n}(t)$ is an additive noise uncorrelated with the sources. A typical condition for the BSS to work well, is to have $n \geq m$, that is more equations than unknowns. Another simplifying assumption is to consider the noiseless mixture model ($\mathbf{n}(t) \equiv \mathbf{0}$).

However, in many situations, the number of sources m is unknown and the noise is considerable. We will focus on the problem where there is only one SOI. In this case, the BSS problem reduces to a blind signal extraction (BSE) problem. In particular, we are interested in solving the situation where the mixture contains only one cyclostationary source at some cyclic frequency α. Then, instead of using an independence criterion as in classical BSS, one can take advantage of the cyclostationarity of the SOI.

The next section reviews some basics about cyclostationary signals before presenting the principles of our BSE approach.

3. CYCLOSTATIONARY SIGNALS AND THEIR SECOND ORDER DESCRIPTORS

In the following all signals are assumed of finite-power and stochastic. A second-order cyclostationary signal $s(t)$ (CS signal) with cyclic frequencies $\alpha_i \in \mathscr{A}$ is such that its auto-correlation function is periodic and therefore admits a Fourier series expansion

$$\begin{aligned} R_s(t,\tau) &\triangleq \mathbb{E}\{s(t)s^*(t-\tau)\} \\ &= \sum_{\alpha_i \in \mathscr{A}} R_s^{\alpha_i}(\tau) e^{j2\pi\alpha_i t}, \end{aligned} \tag{3}$$

where the so-called cyclic auto-correlation functions $R_s^{\alpha_i}(\tau)$ are non-identically zero over the set $\mathscr{A}$.

Second-order spectral statistics are very useful and will be needed later. For instance, the auto-spectral correlation is defined by the double Fourier transform of the auto-correlation function with respect to t and as

$$S_s(\ ,f) = \int\int R_s(t, \ )e^{-j2 \ \ t}e^{-j2 \ f} \, dtd \ ,$$

and becomes upon inserting Eq.(3)

$$S_s(\ ,f) = \sum_{i \in \mathscr{A}} S_s^{\ i}(f) \ (\ - \ _i), \tag{4}$$

which means that the power of a cyclostationary signal is distributed along spectral lines parallel to the f-axis and positioned on the cyclic frequencies $= \ _i$. The spectral quantity $S_s^{\ i}(f)$ is known as the cyclic power spectrum and will turn out very useful for the BSE method relative to convolutive mixtures derived in section 5. The BSE method for instantaneous mixtures described hereafter will only take advantage of the cyclic auto-correlation function.

4. THE SUBLEX METHOD

4.1 Principles — general case

We first consider the noisy mixture described in Eq.(1). As stated previously, the SOI results from a single CS source with cyclic frequency — all other sources and the noise are either stationary or CS with different cyclic frequencies. The cyclic correlation matrix of the observed mixture is:

$$\begin{aligned} \mathbf{R}_x(\ ) &\triangleq \left\langle \mathbb{E}\{\mathbf{x}(t)\mathbf{x}^H(t- \ )\}e^{-j2 \ \ t}\right\rangle \\ &= \mathbf{A}\left\langle \mathbb{E}\{\mathbf{s}(t)\mathbf{s}^H(t- \ )\}e^{-j2 \ \ t}\right\rangle \mathbf{A}^H \\ &= \mathbf{A}\mathbf{R}_s(\ )\mathbf{A}^H, \end{aligned}$$

with $\langle(\ldots)\rangle = \lim_{T\to} T^{-1}\int_{-T/2}^{T/2}(\ldots)dt$. This is a rank 1 matrix because $\mathbf{R}_s(\ )$ has only one non-zero element on its diagonal and this is true independently of the noise $\mathbf{n}(t)$. Therefore, it can be rewritten as

$$\mathbf{R}_x(\ ) = R_s(\ )\mathbf{a} \ \mathbf{a}^H,$$

where $R_s(\ )$ is the cyclic auto-correlation function at lag of the cyclostationary source $s \ (t)$, and $\mathbf{a}$ is the corresponding column of matrix $\mathbf{A}$. Thus the eigenvalue decomposition of $\mathbf{R}_x(\ )\mathbf{R}_x(\ )^H$ will give column $\mathbf{a}$ of $\mathbf{A}$ up to a scaling factor but this is not sufficient to separate the mixture. In order to extract the SOI, a complementary orthogonal subspace to $\mathbf{a}$ is constructed. Let $\mathbf{b}_2,\ldots,\mathbf{b}_n$ be $n-1$ mutually orthogonal and unitary vectors spanning an $(n-1)$-dimensional vector space $\mathscr{B}$ orthogonal to $\mathbf{a}$. Then, the signals $r_i(t) = \mathbf{b}_i^H\mathbf{x}(t)$ where $i = 2,\cdots,n$, are $n-1$ mutually orthogonal references on the interferences and orthogonal to $s \ (t)$. In a matrix form, we have

$$\mathbf{r}(t) = \mathbf{B}^H\mathbf{x}(t), \tag{5}$$

where $\mathbf{r}(t)$ is the $(n-1)$-dimensional vector of references on the interferences. Note that the $n\times(n-1)$ matrix $\mathbf{B}$ is easily found by using the QR method which is a fast, stable and numerically efficient implementation of the Gram-Schmidt orthogonalisation.
The contribution of the interfering sources on the sensors denoted by $\widehat{\mathbf{x}}_{\backslash s} \ (t)$ (i.e. $\widehat{\mathbf{x}}_{\backslash s} \ (t)$ are the observation signals when the cyclostationary source s is switched off) is then estimated by mean squares:

$$\widehat{\mathbf{x}}_{\backslash s} \ (t) = \mathbf{C}\mathbf{r}(t),$$

where

$$\mathbf{C} = \mathbf{R}_{xr}^0(0)\mathbf{R}_r^0(0)^+,$$

with

$$\begin{aligned} \mathbf{R}_{xr}^0(0) &= \left\langle \mathbb{E}\{\mathbf{x}(t)\mathbf{r}^H(t)\}\right\rangle \\ \mathbf{R}_r^0(0) &= \left\langle \mathbb{E}\{\mathbf{r}(t)\mathbf{r}^H(t)\}\right\rangle, \end{aligned}$$

and the superscript $^+$ denoting the Moore-Penrose pseudoinverse of a matrix. The SOI contribution $\widehat{\mathbf{x}}_{/s} \ (t)$ (i.e. the observation signals when only the SOI s is switched on) is finally extracted from the sensor signals as:

$$\begin{aligned} \widehat{\mathbf{x}}_{/s} \ (t) &= \mathbf{x}(t) - \widehat{\mathbf{x}}_{\backslash s} \ (t) \\ &= \left(\mathbf{I} - \mathbf{C}\mathbf{B}^H\right)\mathbf{x}(t). \end{aligned} \tag{6}$$

4.2 Assessment in the noiseless case

The proposed approach is designed to replace BSS algorithms in the noisy case, when there is only one (cyclostationary) SOI. However, it now remains to prove that it does just as well in the noiseless case — i.e. it achieves *perfect* separation of the SOI as BSS will then do. In order to prove it, let us first assume — without loss of generality — that the first source in vector $\mathbf{s}(t)$ is the SOI $s \ (t)$ and that $\mathbb{E}\{\mathbf{s}(t)\mathbf{s}(t)^H\} = \mathbf{I}$.
Then, let

$$\mathbf{A} = \mathbf{Q}\mathbf{\Omega} = \begin{pmatrix} \frac{\mathbf{a}}{\|\mathbf{a} \ \|} & \mathbf{B} \end{pmatrix}\mathbf{\Omega} \tag{7}$$

be the QR decomposition which provided matrix $\mathbf{B}$ in Eq.(5), where is the $n\times m$ upper triangular matrix. Upon inserting Eq.(7) into Eq.(5), we get $\mathbf{r}(t) = \begin{pmatrix} \mathbf{0} & \mathbf{\Omega}_{22} \end{pmatrix}\mathbf{s}(t)$ with $\mathbf{\Omega}_{22}$ the $(n-1)\times(m-1)$ block of matrix of such that $\mathbf{\Omega} = \begin{pmatrix} _{11} & \mathbf{\Omega}_{12} \\ \mathbf{0} & \mathbf{\Omega}_{22} \end{pmatrix}$.
This yields after some manipulations $\mathbf{C} = \mathbf{A}\begin{pmatrix} \mathbf{0}^T \\ \mathbf{\Omega}_{22}^+ \end{pmatrix}$, where $\mathbf{\Omega}_{22}^+ = \mathbf{\Omega}_{22}^H(\mathbf{\Omega}_{22}\mathbf{\Omega}_{22}^H)^+$ is the pseudoinverse of $\mathbf{\Omega}_{22}$.
Finally, substituting $\mathbf{C}$ in Eq.(6) by the former expression yields:

$$\begin{aligned} \widehat{\mathbf{x}}_{/s} \ (t) &= \mathbf{A}\mathbf{s}(t) - \mathbf{A}\begin{pmatrix} \mathbf{0}^T \\ \mathbf{\Omega}_{22}^+ \end{pmatrix}\mathbf{B}^H\mathbf{Q}\mathbf{\Omega}\mathbf{s}(t) \\ &= \mathbf{A}\mathbf{s}(t) - \mathbf{A}\begin{pmatrix} 0 & \mathbf{0}^T \\ \mathbf{0} & \mathbf{I} \end{pmatrix}\mathbf{s}(t) \\ &= \mathbf{A}\begin{pmatrix} s \ (t) \\ \mathbf{0} \end{pmatrix}, \end{aligned}$$

provided that $\mathbf{\Omega}_{22}^+\mathbf{\Omega}_{22} = \mathbf{I}$, i.e $\mathbf{\Omega}_{22}^+$ is the left inverse of $\mathbf{\Omega}_{22}$ or $n \geq m$. Hence in this case and in the noiseless mixture situation, the error on $\mathbf{x}_{/s} \ (t)$ is null.

4.3 Practical considerations

4.3.1 Conditioning the empirical cyclic correlation matrix

In real situations, we only have finite-length measurement signals and thus we deal with empirical statistics. The empirical matrix $\widehat{\mathbf{R}}_x(\ )$ might be of a rank greater than 1 which yields a bad estimation of $\mathbf{a}$. To overcome this difficulty, the method will be rather performed on the pre-whitened observations

$$\begin{aligned} \widetilde{\mathbf{x}}(t) &= \mathbf{W}\mathbf{x}(t) \\ &= \mathbf{W}\mathbf{A}\mathbf{s}(t) + \mathbf{W}\mathbf{n}(t), \end{aligned}$$

where $\mathbf{W} = (\widehat{\mathbf{R}}_x^0(0)^+)^{1/2}$. The algorithm is applied to these whitened data without changing the principle of the SUBLEX method, but it now makes $\widehat{\mathbf{R}}_{\widetilde{x}}(\ )$ closer to a rank-one matrix.

4.3.2 Joint diagonalisation

In order to make the estimation of $\mathbf{a}$ more accurate (up to a scaling factor), we may perform the joint diagonalisation of a set of hermitian matrices $\widehat{\mathbf{R}}_{\widetilde{x}}(\ )\widehat{\mathbf{R}}_{\widetilde{x}}(\ )^H$ indexed by different values of and .

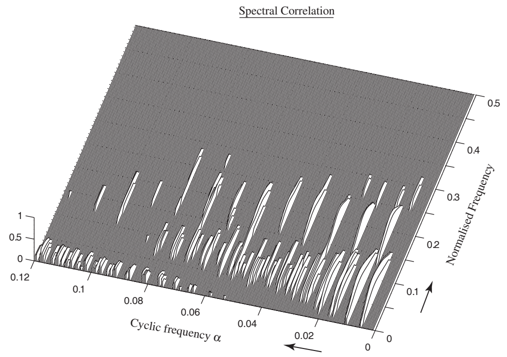

Figure 1: Spectral correlation of an abdominal sensor signal computed with a half-sine window of 32 samples and 75% overlap showing a foetal cyclic frequency at $= 8.8 \times 10^{-3} f_s$. The signal is pre-whitened and only values above a 5% significance level are displayed.

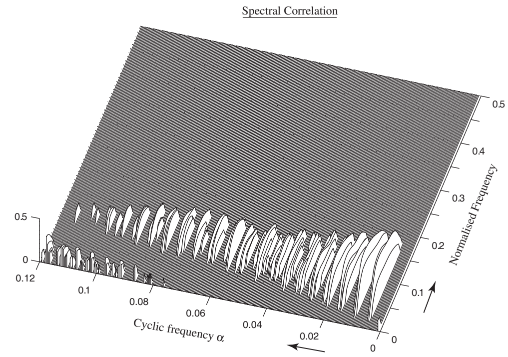

Figure 2: Spectral correlation of a thoracic sensor signal computed with a half-sine window of 32 samples and 75% overlap. The signal is pre-whitened and only values above a 5% significance level are displayed.

5. EXTENSION TO THE CONVOLUTIVE MIXTURES PROBLEM

Taking Eq.(2), the convolutive mixture model may also be written in the frequency domain as[1]

$$\mathbf{dX}(f) = \mathbf{H}(f)\mathbf{dS}(f) + \mathbf{dN}(f).$$

The extended SUBLEX method is then performed using the cyclic power spectrum $S_s\ (f)$ instead of the cyclic correlation function $R_s\ (\ )$. The method is then applied for all frequencies in order to reconstruct the SOI spectrum. Practically, here again, a pre-whitening of the observations is performed and a joint diagonalisation of $\widehat{\mathbf{S}}_{\widetilde{x}}\ (f)\widehat{\mathbf{S}}_{\widetilde{x}}\ (f)^H$ over a countable set $\mathscr{A}$ of cyclic frequencies is suitable to ensure an accurate estimation of the vector $\mathbf{a}\ \ (f)$ of matrix $\mathbf{H}(f)$ corresponding to the SOI. It remains to note that unlike classical BSS, the frequency-domain SUBLEX does not suffer from the permutation problem since only one SOI is reconstructed.

6. APPLICATION TO REAL WORLD DATA

6.1 ECG data

The first application illustrates the use of the SUBLEX method in the instantaneous mixture case. We consider the problem of extracting the foetal electrocardiogram from recordings on the mother's skin, a signal which contains important indications about the health of the foetus. Although classical BSS was successfully applied to this problem where the number of sources present in the mixture is known *a priori* [8], the aim of this experiment is to show the efficiency of our approach before applying it to convolutive mixtures. The data consists of 8 recordings with a sampling frequency $f_s = 500$Hz and 2500 samples. The method assumes the knowledge of at least one cyclic frequency of the SOI. In order to determine , we computed the spectral correlation of one of the sensor signals, as given by Eq.(4). Figure (1) shows a high level of cyclostationarity of the mixture at the fundamental cyclic frequency $= 8.8 \times 10^{-3} f_s$ and its harmonics. By examining the spectral correlation of a recording coming from an electrod placed on the thorax of the mother (Fig.(2)), the $= 8.8 \times 10^{-3} f_s$ cyclic frequency is not present. This means it can be attributed to the foetal electric activity. For a better accuracy in the estimation of the $\mathbf{a}$ vector, we used several harmonics of the cyclic frequency and preformed a joint diagonalisation as advocated before — Fig.(1) shows that a good choice is $N\ \ = 9$ harmonics. We then applied SUBLEX on the first three recordings coming from the abdominal electrodes. Results are plotted in Fig.(3) showing a very good performance of the method.

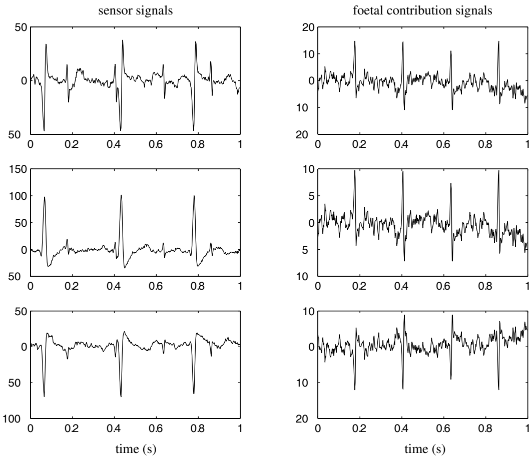

Figure 3: Extraction of the foetal electrocardiogram from 3 sensor recordings.

6.2 An industrial application

In a second application, we consider measurements taken on a mechanical system consisting of a one stage gearbox with a ratio of 32 : 49, an input shaft speed of $f_r = 3$Hz, and a torque of 60N.m. The driving shaft is supported by a double-row self-aligning ball bearing whose characteristics are the following:

- Outer race diameter = 44.85mm
- Inner race diameter = 32.17mm
- Ball diameter B_d = 7.12mm
- Number of balls N_b = 12/row

The objective is to extract the contribution of a fault caused by a slot on the inner race of the ball bearing. We used a data set of 7 sensor recordings of 100000 points each with a sampling frequency $f_s = 16384$Hz. The ball-pass frequency on the inner race of an ideal bearing with a contact angle and a pitch diameter P_d is given by the formula

$$BPFI = \frac{f_r N_b}{2}\left(1 + \frac{B_p}{P_d}\cos\ \right) \approx \frac{f_r N_b}{2}\left(1 + \frac{B_p}{P_d}\right) \qquad (8)$$

[1] dX denotes the spectral increment of Cramér's decomposition of a stochastic signal: $x(t) = \int_{-}^{+} e^{j2\ \ ft} dX(f)$

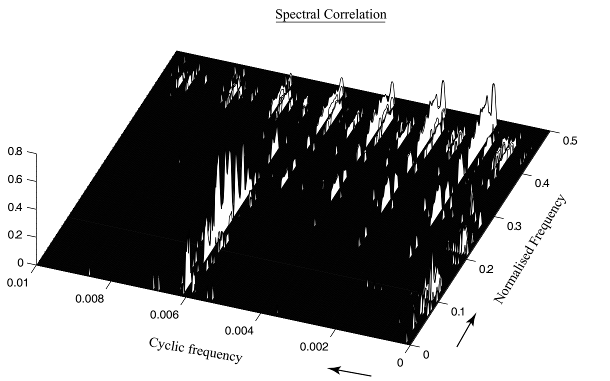

Figure 4: Spectral correlation of a sensor signal computed with a half-sine window of 256 samples and 75% overlap showing a ball bearing fault cyclic frequency at $= 1.36 \times 10^{-3} f_s$. The signal is pre-whitened and only values above a 5% significance level are displayed.

which gives a BPFI ≈ 22.39 Hz. In order to determine more accurately the cyclic frequency of the damaged ball bearing, we computed the spectral correlation of one of the recordings [6]. Figure (4) shows high correlation values at the normalised cyclic frequencies $= 6.12 \times 10^{-3} f_s$ corresponding to the gear mesh frequency and $= 1.36 \times 10^{-3} f_s = 22.28$Hz and its harmonics corresponding to the fault which verifies the frequency obtained using Eq.(8). Next, we used the convolutive frequency-domain version of SUBLEX for extracting the fault contribution with $= 1.36 \times 10^{-3} f_s$ and $N = 4$. The first column of Fig.(5) shows the sensor signals, and the second one the corresponding fault contribution signals obtained by the proposed method. A zoom on some cycles is shown in Fig.(6) which illustrates the impacts due to the slot on the inner race of the ball bearing.

7. CONCLUSION

In this paper, we proposed a subspace method (SUBLEX) for the blind extraction of a cyclostationary source mixed with an unknown number of interfering sources — a typical situation where classical blind source separation fails. We showed that our approach performs perfect separation of the SOI in the noiseless case provided that the number of sensors is greater than the number of the sources; the error in the noisy case is still to be theoretically assessed although it was found very small in simulations. We successfully applied the method to ECG data to extract the foetal electrocardiogram, and its frequency-domain extension to industrial data to extract the signature of a fault in a complex convolutive mixture.

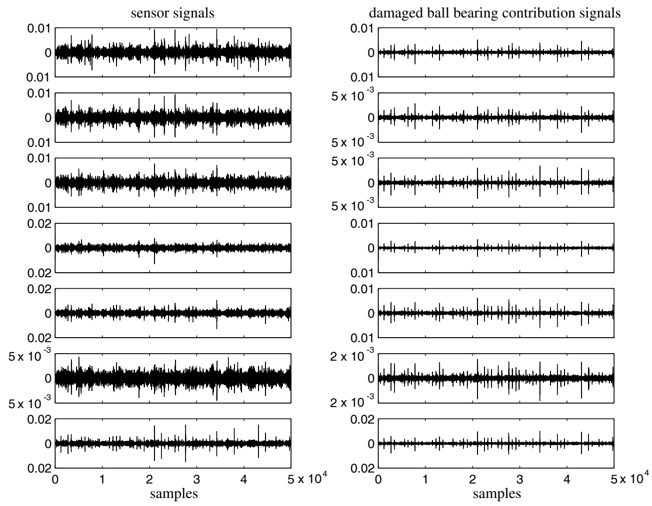

Figure 5: Extraction of the damaged ball bearing signals from 7 sensor recordings.

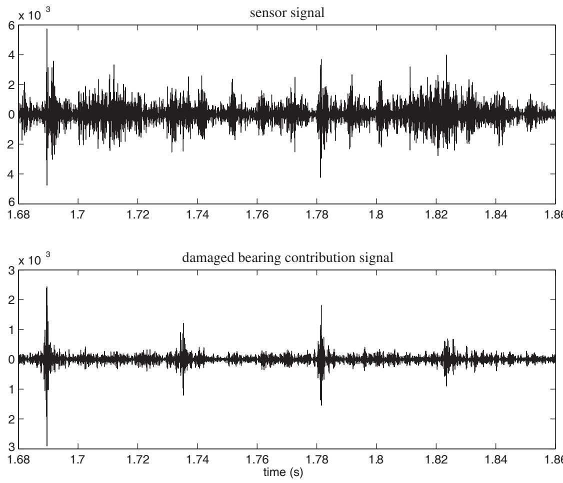

Figure 6: Zoom on some cycles showing the typical impacts characterising the fault in the bearing.

REFERENCES

[1] J.-F. Cardoso and A. Souloumiac, "Blind beamforming for non gaussian signals," in *IEE Proceedings-F*, vol. 140, pp. 362–370, Dec. 1993.

[2] A. Belouchrani, K. Abed-Meraim, J.-F. Cardoso and E.Moulines, "A blind source separation technique using second order statistics," *IEEE Trans. on Signal Processing*, vol. 45, no. 2, pp. 434–444, Feb. 1997.

[3] K. Abed-Meraim, Y. Xiang and Y. Hua, "Blind source separation using second-order cyclostationary statistics," in *IEEE Trans. on Signal Processing*, vol. 49, no. 4, pp. 694–701, Apr. 2001.

[4] A. Ferréol and P. Chevalier, "On the behavior of current second and higher order blind source separation methods for cyclostationary sources," in *IEEE Trans. on Signal Processing*, vol. 48, no. 6, pp. 1712–1725, Jun. 2000.

[5] J. Antoni, F. Guillet, M. ElBadaoui and F. Bonnardot, "Blind separation of convolved cyclostationary processes," in *Signal Processing*, 85, pp.51–66, 2005.

[6] R. B. Randall, J. Antoni, S. Chobsaard, "The Relationship between Spectral Correlation and Envelope Analysis for Cyclostationary Machine Signals. Application to Ball Bearing Diagnostics," in *Mechanical Systems and System Processing*, vol. 15, no. 5, pp. 945–962, 2001.

[7] B.G. Agee, S.V. Schell and W.A. Gardner, "Spectral self-coherence restoral: A new approach to blind adaptive signal extraction using antenna arrays," in *Proceedings of the IEEE*, vol. 78, no. 4, pp. 753–767, Apr. 1990.

[8] J.-F. Cardoso, "Multidimensional independent component analysis," in *Proc. ICASSP'98*, Seattle, USA, May. 12-15. 1998, pp. 1941–1944.