RLS DIRECT EQUALIZER ESTIMATION WITH ASSISTANCE OF PILOTS FOR TRANSMISSIONS OVER TIME-VARYING CHANNELS

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ABSTRACT

We construct in this paper a Decision Feedback Equalizer (DFE) for a time-varying transmission system. The equalizer is imposed with a structure analogous to an Finite Impulse Response Basis Expansion Model (FIR-BEM). For this parametric equalizer, we compute its taps semi-blindly, following a Recursive Least Squares (RLS) approach. Simulation results confirm that the proposed algorithm is robust and can tolerate channel modeling errors.

1. INTRODUCTION

Reliable high data-rate telecommunication between moving transmitters and receivers is gaining more and more attention nowadays. The channel becomes fast fading due to the Doppler spread resulting from the mobility, and makes many existing equalizers that are effective for time-invariant channel less valid. To deal with such rapid-fading and dispersive channels, we need not only a simple and precise channel model, but also a robust and optimal detection algorithm.

The time-varying channel can be tracked by some parametric models, which are usually composed of finite-length channel coefficients expanded by certain time series. With the expansion series defined, we need only estimate those finite-length channel coefficients to reconstruct the channel. Some channel model examples can be found as the Finite Impulse Response Basis Expansion Model (FIR-BEM) in [1] among others, the Discrete Prolate Spheroidal model (DPS) in [2] or the polynomial model in [3] etc. These models are differentiated in their own expansion series and the modeling precision with respect to the Jakes' model [4], which is widely considered as a paradigm model for the time-varying channels.

We follow the practice in [5, Chapter 16] to model the channel as an FIR-BEM. The time-variation of the channel can be efficiently handled with by a Decision Feedback Equalizer (DFE) that bears also an FIR-BEM structure. Although this equalizer can be calculated based on the channel knowledge and renders an outstanding performance, the question arises as to how the receiver is able to access the channel knowledge. The authors in [6,7] presented methods to estimate the FIR-BEM coefficients, but the precision is not completely satisfactory due to the system error accompanying the channel models and the noise disturbance as well. The work in [8] skipped this intermediate step and estimated the equalizer directly. Because the equalizer was obtained solely based on the pilots, this approach had to compromise between the bandwidth efficiency and the equalization performance.

This paper can be deemed as an improvement to [8] by using an adaptive approach for direct equalizer estimation. The update of the equalizer estimate and the data symbol estimate is accomplished iteratively. One of the virtues of this approach is that it is free from any specific channel model assumption and thus can be cast to most real transmission scenarios.

The rest of the paper is organized as follows. In the next section we put forward the system model and briefly describe the FIR-BEM. In Section 3 the equalizer architecture is presented. In Section 4 we show how to estimate the equalizer taps semi-blindly. To support our claim, we exhibit the simulation results in Section 5.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ represent conjugate, transpose and complex conjugate transpose (Hermitian), respectively. $\mathscr{R}(\cdot)$ stands for the real part, and $\mathscr{I}(\cdot)$ for the imaginary part of a complex number. $\otimes$ stands for the Kronecker product. $\mathbf{I}_N$ denotes an $N \times N$ identity matrix and $\mathbf{0}_{M\times N}$ denotes an $M \times N$ all-zero matrix. $\max\{a,b\}$ gives the bigger value between a and b.

2. SYSTEM MODEL

We consider a SIMO system with one input and A outputs. The latter may be obtained by a combination of spatial and temporal oversampling. Suppose we transmit a data symbols sequence $\mathbf{s} = \left[s[0], \cdots, s[N-1]\right]^T$, the received discrete-time sequence at the ath output can be expressed as

$$y^{(a)}[n] = \sum_{\nu=-\infty}^{+\infty} h^{(a)}[n;\nu] s[n-\nu] + w^{(a)}[n], \qquad (1)$$

for $a \in \{1, \cdots, A\}$. Here $w^{(a)}[n]$ denotes the additive noise at the ath output, for $n = 0, \cdots, N-1$. We observe that the channel represented by $h^{(a)}[n;\nu]$ has infinite impulse responses, whose values are time-varying due to the Doppler spreading. Often, the channel can be simplified by assuming it to be an $(L+1)$-tap FIR e.g. $h^{(a)}[n;l] \approx 0$ for $l < 0$ or $l > L$. Further, the time-variation of each tap can be approximated by a truncated Fourier expansion series (FIR-BEM [1]):

$$h^{(a)}[n;\nu] \approx \sum_{l=0}^{L} \delta[\nu - l] \sum_{q=-Q/2}^{Q/2} h_{q,l}^{(a)} e^{j2\pi qn/P}. \qquad (2)$$

Here, P denotes the window size under consideration, within which the channel coefficient $h_{q,l}$ remains constant. In line with the time-frequency duality, the value Q in the above series must satisfy the Nyquist criterion $Q/(2P) \approx f_{max}$, where f_{max} represents the normalized overall Doppler spread among all the A channels. In other words, we approximate the time-varying taps by a superposition of time-invariant coefficients modulated by complex exponentials.

Hence, by substituting (2) in (1), we express the received sequence for the ath output as

$$y^{(a)}[n] = \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} h_{q,l}^{(a)} e^{j2\pi qn/P} s[n-l] + w^{(a)}[n]. \qquad (3)$$

for $n = 0, \cdots, N-1$.

3. PARAMETRIC EQUALIZER

At the receiver, we build a Decision Feedback Equalizer (DFE) whose architecture is depicted in Fig. 1. The serial feedFoward Equalizer (FE), which processes the received samples $y^{(a)}[n]$ for $a = 1, \cdots, A$, respectively, is followed by a closed loop consisting

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Figure 1: The block diagram of a decision feedback equalizer

of a decision device and a serial feedBack Equalizer (BE). The decision device serves in many cases as a memoryless nonlinear quantizer $\mathscr{Q}(\cdot)$, which searches for the closest constellation point to the input signal $\check{s}[n] = \mathscr{Q}(\hat{s}[n])$ and then feed this result back to the BE.

[5, Chapter 16] presented a scheme to construct the FE as well as the BE in a similar FIR-BEM fashion as the channel, i.e., we design the FE to have $L'+1$ taps, with each of the tap being modulated by $Q'+1$ complex exponentials:

$$f^{(a)}[n; \quad] = \sum_{l'=0}^{L'} [\quad - l'] \sum_{q'=-Q'/2}^{Q'/2} e^{j2 \quad q'n/P} f^{(a)}_{q',l'}, \tag{4}$$

and the BE to have $L''+1$ taps, with each of the tap being modulated by $Q''+1$ complex exponentials:

$$b[n; \quad] = \sum_{l''=0}^{L''} [\quad - l''] \sum_{q''=-Q''/2}^{Q''/2} e^{j2 \quad q''n/P} b_{q'',l''}. \tag{5}$$

With the taps of the FE and BE defined as above, we compute the estimate of $s[n]$ as

$$\begin{aligned}\hat{s}[n] = & \sum_{a=1}^{A} \sum_{l'=0}^{L'} \sum_{q'=-Q'/2}^{Q'/2} e^{j2 \quad q'n/P} f^{(a)}_{q',l'} y^{(a)}[n-d'-l'] \\ & - \sum_{l''=0}^{L''} \sum_{q''=-Q''/2}^{Q''/2} e^{j2 \quad q''n/P} b_{q'',l''} \check{s}[n-d''-l''].\end{aligned} \tag{6}$$

Using a little different notation than in [5, Chapter 16], we introduce in the bracket of the first term an FE delay d', which should satisfy $-L \le d' \le L'$ to ensure that the FE input carries the information of $s[n]$. Likewise, we introduce d'' in the second term to denote the BE delay. As we show later on that the equalization takes place off-line, the causality principle observed in [5, Chapter 16] is not necessary in this paper. As a result, when estimating $s[n]$, the preceding as well as the succeeding symbol estimates around it could be used as the BE input. Therefore, we can impose a looser range on the possible values of d'': $-L'' \le d'' \le 0$.

An important streak of (6) is that the proposed equalizer actually modulates the received sequence onto different frequency bins. For each of these frequency bins, there is a conventional transversal FIR filter and their results are superimposed with each other. Since the equalizer works on different frequency band simultaneously, its effective taps as a sum, take on a time-varying character as well. As we shall see in the simulation, only the equalizers equipped with such an architecture can cope with the time variation of the channel competently.

If we stack the in total $A(L'+1)(Q'+1)+(L''+1)(Q''+1)$ equalizer taps in one vector: $\mathbf{v} := [f^{(a)}_{-Q'/2,0}, \cdots, f^{(A)}_{Q'/2,L'},$ $b_{-Q''/2,0}, \cdots, b_{Q''/2,L''}]^T$, (6) can be written in a more concise matrix form:

$$\hat{s}(n) = \mathbf{v}^T \mathbf{u}_n, \tag{7}$$

where $\mathbf{u}_n$ stands for both the FE and BE input:

$$\mathbf{u}_n := [\tilde{\mathbf{y}}_n^T, -\tilde{\mathbf{s}}_n^T]^T. \tag{8}$$

For the FE input $\tilde{\mathbf{y}}_n$, it is easy to verify that

$$\tilde{\mathbf{y}}_n := \left(\mathbf{I}_A \otimes (\boldsymbol{\Delta}_{n,Q'} \otimes \mathbf{I}_{L'+1})\right)[\mathbf{y}_n^{(1)T}, \cdots, \mathbf{y}_n^{(A)T}]^T, \tag{9}$$

with $\boldsymbol{\Delta}_{n,q} := [e^{j\frac{2}{P}n(-\frac{q}{2})}, \cdots, e^{j\frac{2}{P}n(\frac{q}{2})}]^T$ and $\mathbf{y}_n^{(a)} := \left[y^{(a)}[n-d'-L'], \cdots, y^{(a)}[n-d']\right]^T$. Likewise, the BE input $\tilde{\mathbf{s}}_n$ can be expressed as

$$\tilde{\mathbf{s}}_n := (\boldsymbol{\Delta}_{n,Q''} \otimes \mathbf{I}_{L''+1})\check{\mathbf{s}}_n, \tag{10}$$

with $\check{\mathbf{s}}_n := \left[\check{s}[n-d''-L''], \cdots, \check{s}[n-d'']\right]^T$. Note that for above notations, we assume $y^{(a)}[n] = 0$ and $\check{s}[n] = 0$ for $n < 0$ and $n > N-1$.

The entries of the equalizer taps $\mathbf{v}$ can be acquired in term of several criteria. [5, Chapter 16] showed how to compute the MMSE or ZF solution for these equalizer taps based on the knowledge of the channel (FIR-BEM) coefficients given in (2). In this paper we focus on the semi-blind case.

4. DIRECT SEMIBLIND EQUALIZATION

Direct equalization is important in real applications, especially for transmissions over rapidly fading channels. For such cases, channel estimation is less appealing, because it is desired to reduce the modeling error by adopting a larger FIR-BEM model (thus a bigger Q value in (2)), but this will inevitably increase the estimation complexity. Though there is a rich resource of channel estimation algorithms that perform pretty well for time-invariant channels, most of them rely on higher order statistics, which are in general extremely hard to extract for time-varying channels.

In [8], a direct semi-blind equalization approach (without the BE part) for doubly-selective channels is proposed, exploiting the underlying relationship between several distinct equalizers. Recall that these distinct equalizers are related by taking different FE delays d' or modulating the received sequence onto different frequency bands. This idea stems originally from the 'Mutually Referenced Equalizers' (MRE) proposed in [9] for frequency-selective channel. A disadvantage of the MRE is its rather large computational burden. Besides, we are unable to benefit from all the pilots when estimating nonlinear equalizers such as DFE, because for that case, not only the pilot itself but also its neighboring data symbols are required at the same time. Trying to assimilate the merits of the DFE, we propose in this section a training-assisted iterative algorithm, which runs on a symbol level: suppose at one moment we have obtained an equalizer, which yields the data symbol estimates; at the next moment we can use these newly estimated data symbols in turn to update the equalizer. In this way, we are able to construct the BE input by engaging the neighboring symbol estimates obtained from previous iterations. Further, an adaptive approach known as Recursive Least Squares [10] (RLS) will be resorted to prevent the cumbersome matrix inversion involved during each iteration.

A difficulty for a rapidly varying channel is that it is always desired to accomplish some certain operation within a short interval in case the channel situation should drastically change. This means that, unlike the steady channel case, there are in general only a limited amount of data samples available for the adaptation procedure to reach the convergence. We alleviate this problem in this paper by repeating the iteration over the whole received sequence for several times provided that the data symbols are allowed to be processed off-line.

Prior to proceeding, let us first define $\hat{s}^{(k)}(n)$ as the estimate of the nth data symbol before quantization that is obtained at the kth iteration. The relationship between the symbol index n and the iteration index k will become clearer later on. Further, if we assume $\mathbf{v}^{(k-1)}$ to be equalizer taps obtained at the $(k-1)$th iteration, we have:

$$\hat{s}^{(k)}(n) = \mathbf{v}^{(k-1)T}\mathbf{u}_n^{(k)}. \tag{11}$$

Here, $\mathbf{u}_n^{(k)}$ stands for the equalizer input at that time. In comparison with the definition in (8), we introduce a superscript (k) in $\mathbf{u}_n^{(k)}$ to underline the fact that the equalizer input, especially the BE input, is on-line updated with the iteration. Thus the equalizer input must be modified to

$$\mathbf{u}_n^{(k)} := [\tilde{\mathbf{y}}_n^T, -\tilde{\mathbf{s}}_n^{(k)T}]^T, \tag{12}$$

with

$$\tilde{\mathbf{s}}_n^{(k)} := (\mathbf{\Delta}_{n,Q''} \otimes \mathbf{I}_{L''+1})\check{\mathbf{s}}_n^{(k)}. \tag{13}$$

Accordingly, $\check{\mathbf{s}}_n^{(k)}$ contains the quantized estimates from the previous iterations:

$$\check{\mathbf{s}}_n^{(k)} := [\overbrace{\check{s}^{(k-d''-L'')}(n-d''-L''), \cdots, \check{s}^{(k-1)}(n-1)}^{\text{from the present loop}},$$
$$\overbrace{\check{s}^{(\max\{k-N,0\})}(n), \cdots, \check{s}^{(\max\{k-N-d'',0\})}(n-d'')}^{\text{from the previous loop}}]^T,$$

where $\check{s}^{(\tilde{k})}(m)$ denotes the quantized estimate of the mth data symbol obtained at the $\tilde{k}$th iteration. It is worth noting that since the iteration runs over the whole sequence for multiple loops, accordingly, the BE input defined in the above expression will contain the estimates from the present loop (the first $d''+L''$ estimates) as well as from the previous loop (the last $1-d''$ estimate). With the similar motivation, we can characterize the relationship between the iteration index k and the symbol index n as $n = \mathrm{mod}(k,N)$, where $\mathrm{mod}(k,N)$ stands for the remainder of k divided by N.

With the assistance of the above notations and assuming that there are N_t pilots whose values are perfectly known by the receiver, we launch the weighted RLS algorithm [10] as summarized in the following steps:

1. Set $k=0, n=0$ and prepare the equalizer input in the way given by (12). For the inputs to the BE, zeros are filled in $\check{\mathbf{s}}_0^{(0)}$ if the corresponding data symbols are not pilots;
2. Choose a forgetting factor , which is a positive constant close to, but less than 1; Initialize the following parameters

$$\mathbf{v}^{(0)} = \mathbf{0}_{A(L'+1)(Q'+1)+(L''+1)(Q''+1)\times 1},$$
$$\mathbf{P}^{(0)} = \ {}^{-1}\mathbf{I}_{A(L'+1)(Q'+1)+(L''+1)(Q''+1)}.$$

   Here, is a small positive constant;
3. At the kth iteration, compute the following equations:

$$\hat{s}^{(k)}[n] = \mathbf{v}^{(k-1)T}\tilde{\mathbf{u}}_n^{(k)} \tag{14}$$

$$\mathbf{K}^{(k)} = \frac{{}^{-1}\mathbf{P}^{(k-1)}\tilde{\mathbf{u}}_n^{(k)}}{1+\ {}^{-1}\tilde{\mathbf{u}}_n^{(k)H}\mathbf{P}^{(k-1)}\tilde{\mathbf{u}}_n^{(k)}} \tag{15}$$

$$\mathbf{v}^{(k)} = \mathbf{v}^{(k-1)} + \mathbf{K}^{(k)*}\big(g(\hat{s}^{(k)}[n]) - \hat{s}^{(k)}[n]\big) \tag{16}$$

$$\mathbf{P}^{(k)} = \ {}^{-1}\mathbf{P}^{(k-1)} - \ {}^{-1}\mathbf{K}^{(k)}\tilde{\mathbf{u}}_n^{(k)H}\mathbf{P}^{(k-1)} \tag{17}$$

4. Update the BE input in (12) with the newly obtained $\check{s}^{(k)}[n]$ if $s[n]$ is not a pilot; Increment the iteration index and go back to the previous step.

Comparing the procedures listed above with the conventional RLS algorithm, we make some modifications summarized in the following remarks, which prove to be beneficial to the equalizer performance.

Remark 1 When the desired response of the equalizer is unknown (except for the pilots), we switch to a decision-directed fashion. A zero-memory nonlinear function $g(\cdot)$ is therefore deployed to calculate the *a priori* estimation error in (16). Some candidates of $g(\cdot)$ could be:

- Quantization: $\mathscr{Q}(\hat{s}[n])$;
- Sato algorithm: $\ \mathrm{sgn}(\mathscr{R}(\hat{s}[n])) + j\ \ \mathrm{sgn}(\mathscr{I}(\hat{\mathrm{s}}[n]))$, with $\ = \mathrm{E}\{\mathscr{R}(s[n])^2\}/\mathrm{E}\{|\mathbf{s}[n]|\}$.

Note that the Sato algorithm is a coarser quantization, which is preferred for the 'closed eye' situations. The authors in [11] suggested combining the two alternatives to improve the convergence performance. However, there shall be no difference for QPSK signals since in that case equals $1/\sqrt{2}$.

Remark 2 Not only the total number, but also the positioning of the pilots is crucial to this approach. In the simulation, we will parse the pilots into several blocks and interleave them with the data symbol sequence in the way depicted in Fig. 2. As typical to the DFE, once an error arises in estimation, it is fed back and hence propagates. The distribution of the pilots, whose desired response is perfectly known, could help sever such error outbreaks. Empirically, the distance between the pilot blocks, when the number of the pilots is fixed, will also influence the equalization performance. However, so far to our knowledge, these is no literature that qualitatively analyzed this mechanism.

Figure 2: Positioning of the pilots

Remark 3 Though the pilots are not grouped together, we could still launch a pure training phase by having the iteration initially only run over the pilots-related samples. This is only possible if the size of the pilot blocks is greater or equal to the size of the BE. Simulation results show that by this means the algorithm converges much faster than without the pure training phase.

Remark 4 For the implementation of this algorithm, the assumption of the FIR-BEM channel model, on which the MMSE (ZF) equalizer in Section 3 is based, is no longer indispensable here, since no channel knowledge is resorted to for the equalizer computation. More strongly, we can apply this approach without assuming any specific channel model.

5. SIMULATION RESULTS

We present simulations for a SIMO transmission system using one transmit antenna and two receive antennas. Further, the sample stream from each receive antenna is oversampled with a factor of two. Hence, we obtain four channel outputs.

We test the proposed algorithm by transmitting a QPSK sequence of length $N = 400$. It is cast to a time-varying channel with a normalized Doppler frequency $f_{max} = 0.0025$ and a memory length $L = 3$. For the simulation, the channels are generated based on both the Jakes' model and on the FIR-BEM. In order for the FIR-BEM to fit the Jakes' model tightly within a prescribed window $P = N$, we must set $Q = 2$ to satisfy $Q/(2P) \approx f_{max}$. The eventual BER is an average over 1000 Monte Carlo runs, where in each run we generate a different channel, noise and data realization.

Test case 1. We first present the performance of an MMSE equalizer in Fig. 3, which is computed based on the knowledge of the FIR-BEM coefficients as shown in [5, Chapter 16]. The equalizer is equipped with the parameters $[L',Q',L'',Q''] = [7,6,4,4]$. We observe that the MMSE equalizer suffers an error floor in the Jakes' channel. This must be ascribed to the modeling error of the FIR-BEM, whose influence becomes more prominent at the higher SNR than the influence of the disturbance noise. There are several techniques to reduce the modeling error, i.e. by taking a more complex FIR-BEM (thus a bigger Q) or using exponential basis with a finer frequency resolution [12].

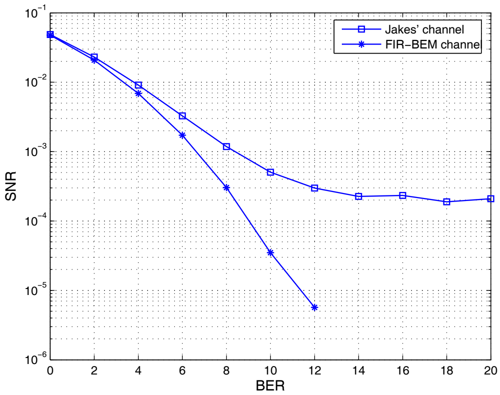

Figure 3: MMSE equalization performance

Test case 2. The performance of the proposed semi-blind equalization is presented in Fig. 4. Here, we use a modest equalizer with parameters $[L', Q', L'', Q''] = [3, 2, 1, 2]$ to decrease the estimation difficulty. Of the transmitted symbols we assume that 25% are pilots, which are grouped in 5 blocks. We insert these blocks equi-distantly to the data sequence in the way as Fig. 2. The iteration procedure runs over the whole sequence for twice (including the pure training phase). It is observed that in comparison with the MMSE equalizer, the semi-blind approach suffers a 4dB loss at a BER 10^{-3}. However, the difference between the Jakes' channel and the FIR-BEM channel is much less pronouncing with respect to the previous case. This difference, though still existing, should be accounted by FIR-BEM structure of the equalizer, which is hence optimal for a similar-structured channel.

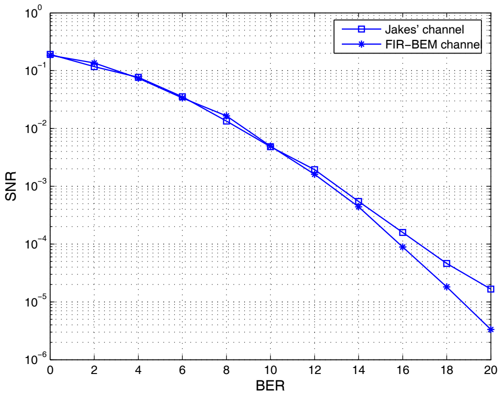

Figure 4: Semi-blind equalization performance

Test case 3. If we set the parameter Q' and Q'' in (6) both to zero, the proposed equalizer degrades to a conventional DFE, which is frequently used for time-invariant channels. Fig. 5 illustrates that the conventional DFE (both the MMSE and semi-blind solution as well) is incapable of capturing the time-variation of the channel.

6. CONCLUSION

In this paper, we construct a DFE for a time-varying channel, with its feedforward and feedback part both taking on an FIR-BEM

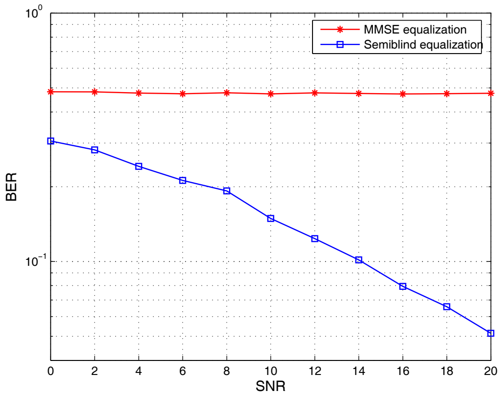

Figure 5: Performance of a conventional DFE

structure. We show how to acquire the value of the equalizer taps by means of semi-blind RLS adaptation. With the assistance of pilots, this approach is cast to test for both practical (Jakes' channel) and ideal (FIR-BEM channel) situations and yields satisfactory performance.

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