

BIT RATE COMPARISON BETWEEN MIMO CYCLICALLY PREFIXED SINGLE CARRIER AND MULTICARRIER TRANSMISSIONS

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ABSTRACT

We consider cyclically prefixed block transmission over frequency selective wireless multiple input multiple output (MIMO) channels. We make a bit rate comparison between MIMO cyclically prefixed single carrier (CP-SC) and MIMO orthogonal frequency division multiplexing (OFDM) systems. We analytically prove that MIMO CP-SC with decision feedback equalization (DFE) outperforms MIMO OFDM with linear receiver. Moreover, we show that spatial linear precoding on the OFDM tones, leading to a discrete matrix multitone (DMMT) scheme, enables to close the gap and to perform as well as MIMO CP-SC with DFE.

1. INTRODUCTION

It is now common knowledge that multiple input multiple output (MIMO) channels using multiple antennas, both at the transmitter and receiver, allow an improvement in the quality and data rates of wireless communication. However, the decreasing duration of symbols for higher rates gives rise to frequency selective propagation effects. One efficient approach for dealing with frequency selective fading channels is to use multicarrier (MC) modulation, known as orthogonal frequency division multiplexing (OFDM). It has been pointed out in [1] that cyclically extended single carrier (CP-SC) modulation also allows low complexity equalization of single input single output (SISO) frequency selective channels. Indeed, the cyclic prefix converts the linear convolution into a cyclic one and (inverse) fast fourier transform ((IFFT) operations then allow the equalization to be held in the frequency domain. CP-SC transmission has received much attention because unlike OFDM systems, it does not suffer from the peak-to-average power problem. System-based comparisons between OFDM and SC systems can be found in [2], [3], [4] for SISO schemes, and in [5] for MIMO schemes. Moreover, performances of SISO OFDM and SISO SC systems have recently been compared in [6], [7], [8]. In particular, it was shown in [8] that SISO OFDM with linear frequency-domain equalization (FDE) and SISO CP-SC with decision feedback equalization (DFE) reach equal achievable bit rate for a target bit error rate (BER) and under a high signal to noise ratio (SNR) assumption. A comparison of performances for MIMO systems can also be found in [9].

In this paper, we hold a bit rate comparison between MIMO OFDM and MIMO CP-SC with DFE and thus extend the results of [8] to the MIMO case. The conclusion

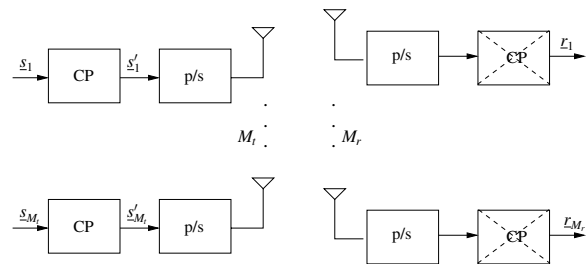


Figure 1: Cyclically prefixed MIMO system.

will be somewhat different from what has been proved [8] for the SISO case. The rest of the paper is organized as follows. We start in section 2 by giving the signal model of a cyclically prefixed MIMO system. Section 3 studies linear pre/decoding in a MIMO CP-SC scheme with minimum mean square error (MMSE) receiver and shows that optimal precoding comes down to a precoded MIMO OFDM scheme. In section 4, we develop a DFE for a MIMO CP-SC system. A bit rate comparison is then held in section 5, while simulation results and conclusion are given in section 6.

Notation: Twice underlined upper case letters denote matrices, underlined lower case letters denote column vectors; $(.)^T$ and $(.)^H$ denote transpose and Hermitian transpose, respectively. The matrix \underline{I}_M denotes the $M \times M$ identity matrix, and $\underline{[A]}_{i,j}$ denotes the (i,j) entry of \underline{A} .

2. SIGNAL MODEL OF A CYCLICALLY PREFIXED MIMO SYSTEM

Let us consider a cyclically prefixed transmission over a MIMO channel with M_t transmit antennas and M_r receive antennas. The vector notations are defined in Fig. 1. We denote by N the block size and by L the maximum length of the $M_t M_r$ impulse responses of the MIMO channel¹. The symbols associated with transmit antenna j ($j = 1, \dots, M_t$) form a $N \times 1$ vector $\underline{s}_j = [s_j(1) \dots s_j(N)]^T$. A prefix of length L is added at each transmit antenna. The transmission of a block involves thus $N + L$ periods. The prefix implies a loss in efficiency but this loss is negligible if $N \gg L$. The cyclic prefix consists in repeating the last L symbols of the block at its beginning to form the following $(N + L) \times 1$ vector which is the one actually sent

$$\underline{s}'_j = [s_j(N - L + 1) \dots s_j(N) s_j(1) \dots s_j(N)]^T$$

¹Each discrete time impulse responses is then represented by $L + 1$ taps.

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At the receiver side, the prefix is removed at each receive antenna: from the $N + L$ samples received during the $N + L$ periods of transmission, we drop the first L samples (which contain block interference) and keep the N following ones to form $N \times 1$ vectors r_i ($i = 1, \dots, M_r$). Introducing the following notations $\underline{r} = [r_1^T \dots r_{M_r}^T]^T$ and $\underline{s} = [s_1^T \dots s_{M_r}^T]^T$, we have the following global signal model

$$\underline{r} = \sqrt{\frac{E_s}{M_t}} \begin{bmatrix} \underline{G}_{c_{1,1}} & \dots & \underline{G}_{c_{1,M_t}} \\ \vdots & \ddots & \vdots \\ \underline{G}_{c_{M_r,1}} & \dots & \underline{G}_{c_{M_r,M_t}} \end{bmatrix} \underline{s} + \underline{n} \quad (1)$$

where $\underline{G}_{c_{i,j}}$ is a $N \times N$ circulant matrix with its first column given by the impulse response between antennas j and i appended by $N - L - 1$ zeros. The taps of the impulse responses will be considered as circularly symmetric complex independent gaussian random variables with zero mean and unitary variance. E_s represents the mean energy transmitted during a symbol period over the set of M_t antennas. The noise vector \underline{n} is composed of the noise vectors associated with each receive antenna $\underline{n} = [n_1^T \dots n_{M_r}^T]^T$, the noise samples are gaussian, uncorrelated and of variance N_0 . By the property of circulant matrices, we have

$$\underline{G}_{c_{i,j}} = \underline{W}^H \underline{\Omega}_{i,j} \underline{W} \quad (i = 1, \dots, M_r)(j = 1, \dots, M_t) \quad (2)$$

where \underline{W} is the $N \times N$ FFT matrix², \underline{W}^H being then the IFFT matrix, and $\underline{\Omega}_{i,j}$ is a diagonal matrix with, on its diagonal, the N -point FFT of the impulse response between antennas j and i . Using (2) we rewrite (1) as

$$\underline{r} = \sqrt{\frac{E_s}{M_t}} \underline{W}_{M_r N}^H \underline{\Omega}_b \underline{W}_{M_t N} \underline{s} + \underline{n} \quad (3)$$

where $\underline{\Omega}_b$ is composed of diagonal blocks:

$$\underline{\Omega}_b = \begin{bmatrix} \underline{\Omega}_{1,1} & \dots & \underline{\Omega}_{1,M_t} \\ \vdots & \ddots & \vdots \\ \underline{\Omega}_{M_r,1} & \dots & \underline{\Omega}_{M_r,M_t} \end{bmatrix}$$

while $\underline{W}_{M_t N}$ (resp. $\underline{W}_{M_r N}^H$) is a bloc diagonal matrix with M_t (resp. M_r) times the matrix FFT \underline{W} (resp. IFFT \underline{W}^H) on its diagonal.

3. FROM MIMO CP-SC TO DMMT

3.1 MMSE receiver

We consider the cyclically prefixed MIMO scheme described in the previous section. We add linear precoding to this scheme, such that $\underline{s} = \underline{X} \underline{d}$. \underline{X} is a $M_t N \times M_t N$ matrix. We develop the MMSE receiver associated with this linear precoded cyclically prefixed MIMO scheme. All computation done (using (3)), the MMSE receiver scheme is the one illustrated in Fig. 2, where

$$\underline{F} = \underline{\Omega}_b^H \left(\frac{E_s}{M_t} \underline{\Omega}_b \underline{W}_{M_t N} \underline{X} \underline{X}^H \underline{W}_{M_t N}^H \underline{\Omega}_b^H + N_0 \underline{I}_{M_t N} \right)^{-1}$$

We see that the MMSE receiver naturally leads to a MIMO

² $[\underline{W}]_{m,n} = (1/\sqrt{N}) \exp(-j2\pi(m-1)(n-1)/N)$; $n, m \in [1, N]$

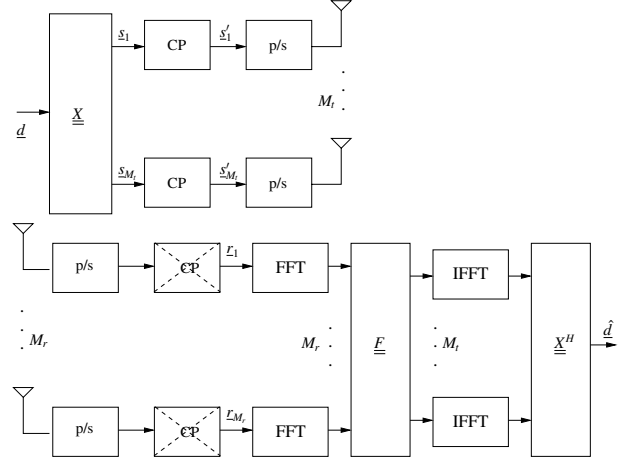


Figure 2: MIMO CP-SC system with linear precoding and MMSE equalization.

CP-SC system with FDE. However, we should keep in mind that the particular precoding $\underline{X} = \underline{W}_{M_t N}^H$ (implementing an IFFT operation at each transmit antenna) converts this linear precoded MIMO CP-SC system into MIMO OFDM. We have the following expression for the error covariance matrix associated with the scheme of Fig. 2:

$$\underline{R}_{\varepsilon\varepsilon} = \left(\underline{I}_{M_t N} + \frac{E_s}{M_t N_0} \underline{X}^H \underline{W}_{M_t N}^H \underline{\Omega}_b^H \underline{\Omega}_b \underline{W}_{M_t N} \underline{X} \right)^{-1} \quad (4)$$

3.2 Linear precoding design

A question naturally follows from the previous section: is there any design of the precoder \underline{X} that makes it possible to minimize the error covariance matrix (in terms of some criterion) under a transmitted power constraint? The response is affirmative. We can be interested in minimizing the determinant³ of the error covariance matrix (4). Such a constrained minimization problem has been solved in [10]. The solution is of the form

$$\underline{X} = \underline{U} \underline{\Phi}$$

where $\underline{\Phi}$ is a diagonal matrix describing the optimal power allocation and \underline{U} is given by the following decomposition

$$\underline{W}_{M_t N}^H \underline{\Omega}_b^H \underline{\Omega}_b \underline{W}_{M_t N} = \underline{U} \underline{\Lambda} \underline{U}^H \quad (5)$$

Let us suppose $\underline{\Phi} = \underline{I}_{M_t N}$ and study what the precoder $\underline{X} = \underline{U}$ involves⁴. We know that $\underline{\Omega}_b^H \underline{\Omega}_b$ is a matrix composed of $M_t M_t$ diagonal $N \times N$ blocks. There exists a permutation matrix \underline{P} such that $\underline{P}^T \underline{\Omega}_b^H \underline{\Omega}_b \underline{P}$ is a block diagonal matrix. More precisely, $\underline{P}^T \underline{\Omega}_b^H \underline{\Omega}_b \underline{P}$ has N blocks of size $M_t \times M_t$ on its diagonal. Since $\underline{P} \underline{P}^T = \underline{I}_{M_t N}$, we can rewrite (5) as

$$\underline{W}_{M_t N}^H \underline{P} \underline{P}^T \underline{\Omega}_b^H \underline{\Omega}_b \underline{P} \underline{P}^T \underline{W}_{M_t N} = \underline{U} \underline{\Lambda} \underline{U}^H \quad (6)$$

³ We will see in section 5 that the determinant criterion is related to the achievable rate.

⁴ In this paper, we assume uniform power allocation. The optimal power allocation is given in [10] but is beyond the scope of this paper.

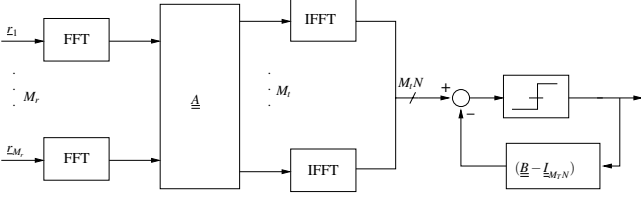


Figure 3: DFE scheme for a MIMO CP-SC system

Let's then denote the following decomposition⁵

$$\underline{P}^T \underline{\Omega}_b^H \underline{\Omega}_b \underline{P} = \underline{V} \underline{\Lambda} \underline{V}^H \quad (7)$$

with \underline{V} also a block diagonal matrix composed of N blocks of size $M_t \times M_t$. Substituting (7) into (6) gives

$$\underline{\mathcal{W}}_{M_t N}^H \underline{P} \underline{V} \underline{\Lambda} \underline{V}^H \underline{\mathcal{W}}_{M_t N}^T = \underline{U} \underline{\Lambda} \underline{U}^H$$

from which we conclude that

$$\underline{U} = \underline{\mathcal{W}}_{M_t N}^H \underline{P} \underline{V}$$

The precoder $\underline{X} = \underline{U}$ can thus be decomposed into three successive operations. The last operations is an IFFT at each transmit antenna ($\underline{\mathcal{W}}_{M_t N}^H$). This is thus a MIMO OFDM scheme. The first two operations are:

- \underline{V} : Some coding is done separately on blocks of M_t symbols.
- \underline{P} : The M_t coded symbols of a block are transmitted on the same OFDM tone (or subcarrier) and spread over the M_t transmit antennas.

In other words, the precoder $\underline{X} = \underline{U}$ converts the precoded MIMO CP-SC scheme into a MIMO OFDM scheme with spatial precoding for each tone, using a singular value decomposition (SVD). This technique was introduced in [11] under the name: discrete matrix multitone (DMMT). The scheme given by Fig. 2 with precoding $\underline{X} = \underline{U}$ thus leads to a DMMT scheme with MMSE linear receiver. We will further use the acronym MMSE DMMT.

4. MIMO CP-SC WITH DFE

In the previous section, we saw that linear processing is not best adapted to a MIMO CP-SC scheme. Indeed, we showed that optimal precoding leads to precoded MIMO OFDM. As a consequence, in this section we focus on the receiver side and study the use of decision feedback equalization (DFE) in a MIMO CP-SC system. Fig. 3 gives the DFE scheme. The forward part is held in the frequency domain while the feedback takes place entirely in the time domain. The transmitter side is omitted in Fig 3 but is the same as in Fig. 1 (it only adds the CP on blocks). The filters are designed to minimize the mean square error. The forward filter is given by

$$\underline{A} = \sqrt{\frac{M_t}{E_s}} \underline{\mathcal{W}}_{M_t N}^H \underline{B} \underline{P}^T \underline{\mathcal{W}}_{M_t N}^H \left(\frac{M_t N_0}{E_s} \underline{I}_{M_t N} + \underline{\Omega}_b^H \underline{\Omega}_b \right)^{-1} \underline{\Omega}_b^H$$

Let's then denote the following Cholesky decomposition

$$\left(\underline{I}_{M_t N} + \frac{E_s}{M_t N_0} \underline{P}^T \underline{\mathcal{W}}_{M_t N}^H \underline{\Omega}_b^H \underline{\Omega}_b \underline{\mathcal{W}}_{M_t N} \underline{P} \right)^{-1} = \underline{L} \underline{D} \underline{L}^H \quad (8)$$

⁵ $\underline{\mathcal{W}}_{M_t N}^H \underline{\Omega}_b^H \underline{\Omega}_b \underline{\mathcal{W}}_{M_t N}$ and $\underline{P}^T \underline{\Omega}_b^H \underline{\Omega}_b \underline{P}$ have identical eigenvalues.

where \underline{L} is a lower triangular matrix with ones on the main diagonal and \underline{D} is a positive diagonal matrix. The feedback filter⁶ is then given by $\underline{B} = \underline{L}^{-1}$, which is such that the error covariance matrix is diagonal

$$\underline{R}_{\varepsilon\varepsilon} = \underline{D}. \quad (9)$$

5. BIT RATE COMPARISON

In this section we hold a bit rate comparison between MIMO CP-SC with DFE, MIMO OFDM and DMMT (both with linear MMSE receiver). In that sense, let us recall that, for the transmission of K symbols, the total achievable bit rate per transmitted symbol is given by⁷

$$b = \frac{1}{K} \sum_{i=1}^K \frac{1}{2} \log_2 \left(1 + \frac{SNR_i}{\Gamma} \right)$$

where SNR_i is the SNR on the i^{th} transmitted symbol while Γ is the SNR gap and depends on the target BER. Under high SNR assumption, we have

$$\begin{aligned} b &= \frac{1}{2K} \log_2 \left(\prod_{i=1}^K \frac{SNR_i}{\Gamma} \right) \\ &= \frac{1}{2K} \log_2 \left(\frac{1}{(\prod_{i=1}^K [\underline{R}_{\varepsilon\varepsilon}]_{i,i}) \Gamma} \right) \end{aligned} \quad (10)$$

Under a high SNR assumption, the product of the diagonal elements of the error covariance matrix is thus a measure of the achievable rate. Moreover, if the error covariance matrix is diagonal, we see that the achievable bit rate is only a function of its determinant, which justifies the use of the determinant criterion when designing pre/decoders. We now compute the product of the diagonal elements of the error covariance matrix of the three schemes we wish to compare.

5.1 MIMO CP-SC with DFE

The error covariance matrix of a MIMO CP-SC system with DFE is given by (9). It is diagonal, hence $\prod_i [\underline{R}_{\varepsilon\varepsilon}]_{i,i} = \det \underline{R}_{\varepsilon\varepsilon}$. We then compute, using (8) and (5)

$$\begin{aligned} \prod_{i=1}^{M_t N} [\underline{R}_{\varepsilon\varepsilon}]_{i,i} &= \det \underline{D} \\ &= \det(\underline{L} \underline{D} \underline{L}^H) \\ &= \det \left(\underline{I}_{M_t N} + \frac{E_s}{M_t N_0} \underline{P}^T \underline{\mathcal{W}}_{M_t N}^H \underline{\Omega}_b^H \underline{\Omega}_b \underline{\mathcal{W}}_{M_t N} \underline{P} \right)^{-1} \\ &= \det \left(\underline{I}_{M_t N} + \frac{E_s}{M_t N_0} \underline{P}^T \underline{U} \underline{\Lambda} \underline{U}^H \underline{P} \right)^{-1} \\ &= \det \left(\underline{I}_{M_t N} + \frac{E_s}{M_t N_0} \underline{\Lambda} \right)^{-1} \end{aligned} \quad (11)$$

The second equality is justified by the fact that \underline{L} is a lower triangular matrix with ones on the main diagonal, while the last equality comes from the orthogonality of \underline{P} and \underline{U} .

⁶ Since the correction on a given symbol estimate can only be constructed from previous decisions, the feedback filter matrix \underline{B} has to be lower triangular with ones on the main diagonal.

⁷ If the residual noise plus interference is assumed to be Gaussian and if PAM or QAM modulation is used.

5.2 MIMO OFDM with MMSE linear receiver

The error covariance matrix of a MIMO OFDM system with MMSE linear equalization is given by (4) with the precoding matrix given by $\underline{X} = \underline{\mathcal{W}}_{M_t N}^H$. It is not diagonal. By Hadamard's inequality $\prod_i [R_{\underline{\varepsilon\varepsilon}}]_{i,i} \geq \det R_{\underline{\varepsilon\varepsilon}}$. We then compute, using (5),

$$\begin{aligned} \prod_{i=1}^{M_t N} [R_{\underline{\varepsilon\varepsilon}}]_{i,i} &\geq \det \left(\underline{I}_{M_t N} + \frac{E_s}{M_t N_0} \underline{\mathcal{W}}_{M_t N} \underline{U} \underline{\Lambda} \underline{U}^H \underline{\mathcal{W}}_{M_t N}^H \right)^{-1} \\ &\geq \det \left(\underline{I}_{M_t N} + \frac{E_s}{M_t N_0} \underline{\Lambda} \right)^{-1} \end{aligned} \quad (12)$$

since $\underline{\mathcal{W}}_{M_t N}$ and \underline{U} are orthogonal matrices.

5.3 MMSE DMMT

We here consider the adding of spatial precoding to the OFDM tones as described in section 3.2, leading to MMSE DMMT. The error covariance matrix associated with such a scheme is given by (4) with precoding $\underline{X} = \underline{U}$. It is diagonal, thus $\prod_i [R_{\underline{\varepsilon\varepsilon}}]_{i,i} = \det R_{\underline{\varepsilon\varepsilon}}$. We then compute, using (5),

$$\begin{aligned} \prod_{i=1}^{M_t N} [R_{\underline{\varepsilon\varepsilon}}]_{i,i} &= \det \left(\underline{I}_{M_t N} + \frac{E_s}{M_t N_0} \underline{U}^H \underline{U} \underline{\Lambda} \underline{U}^H \underline{U} \right)^{-1} \\ &= \det \left(\underline{I}_{M_t N} + \frac{E_s}{M_t N_0} \underline{\Lambda} \right)^{-1} \end{aligned} \quad (13)$$

5.4 Bit rate comparison

From (10), (11), (12) and (13), we conclude the following: In terms of achievable bit rate and under a high SNR assumption, MIMO CP-SC with DFE outperforms MIMO OFDM with linear MMSE equalization. However, MMSE DMMT and MIMO CP-SC with DFE yield equal rates. This confirms our intuition: while the DFE deals with both the spatial and temporal interference, MIMO OFDM only eliminates temporal interference; however, spatial interference is also taken into account in DMMT by adding spatial precoding on tones.

Note that in the SISO case ($M_t = M_r = 1$) we have equality of performances for CP-SC and OFDM schemes, which is what was proven in [8]. We here proved that conclusions are different in the MIMO case.

6. SIMULATION RESULTS AND CONCLUSION

We consider a MIMO frequency selective channel with the following parameters: $N = 32$, $L = 5$, $M_t = M_r = 3$. One thousand realizations of this MIMO channel were used for the simulation. The simulation results are given in Fig. 4 and confirm the analytical result: MMSE DMMT and MIMO CP-SC with DFE achieve equal bit rate at high SNR, while MIMO OFDM with MMSE linear receiver is always outperformed by these two schemes.

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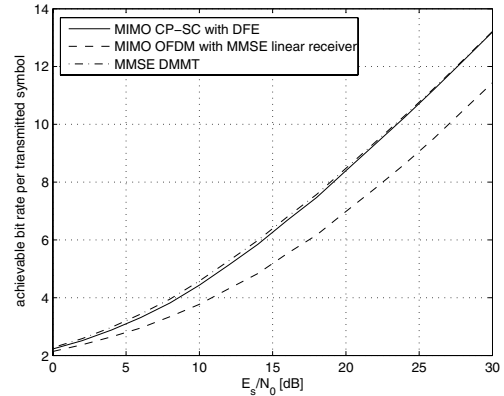


Figure 4: Bit rate per transmitted symbol for a frequency selective MIMO system with $N = 32$, $L = 5$, and $M_t = M_r = 3$. Comparison between the three schemes ($\Gamma = 1$).

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