PERFORMANCES OF TURBO SAGE BASED EQUALIZER FOR MULTI USER MULTI CARRIER SPACE TIME BLOCK CODED SYSTEM

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ABSTRACT

In this paper, we propose a multi-user detector for space time uplink transmission in multi-carrier CDMA systems. We propose a new combination using STBC, OFDM and CDMA spreading. Furthermore we propose a turbo equalization scheme based on the use of LDPC for channel coding and SAGE algorithm for channel estimation at the LDPC decoder input. Simulation results show that our proposed receiver has comparable performances to EM and MMSE techniques.

1. INTRODUCTION

Estimating multiple inputs multiple outputs (MIMO) channels is challenging since the number of channels to be estimated grows with the number of deployed antennas. Space-time coding (STC) techniques, including space-time trellis coding (STTC) and space-time block coding (STBC) make use of MIMO channel and provide significant capacity gains in wireless channels. A lot of papers ([1]-[3]) have investigated their use particularly for the case of wireless flat-fading channels. However, many wireless channels are frequency-selective in nature, for which the STC design and channels estimation become a complicated issue. The channel estimation complexity can be reduced when using orthogonal frequency-division multiplexing (OFDM) technique which transforms a frequency-selective fading channel into parallel correlated flat-fading channels. But the channel estimation problem is accentuated when we consider uplink multi-user system where each link between each user antennas and the base station antennas need to be estimated.

For OFDM coded mono user systems, Lu et al. have shown in [4] that irregular Low Density Parity Check Code (LDPC), due to their decoding simplicity and their outstanding behavior, are good candidates to interface with soft weighting algorithm for coded symbols such as EM algorithm. The multi user uplink system proposed in [5] is capable to separate all the users in quasi synchronous mode to equivalent multiple single user STBC configurations.

In this paper, we develop a modified SAGE based channel estimator based on [4],[6]-[7] for the multi-user structure proposed in [5]. This paper is organized as follows. We first recall the transmitter and receiver architecture of the multi-user system and then present the turbo equalization adapted to the structure. Finally we show some computer simulation results to illustrate the performance of our system.

2. SYSTEM DESCRIPTION

In this section, we recall the basic multi user system proposed in [5] when each user is assigned a distinct spreading code.

2.1 Transmitter

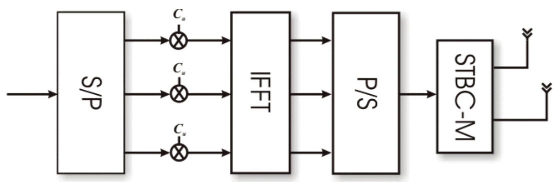

Fig 1 : Transmitter Model

We limit our scope to two transmit antennas system as generalization to N antennas is straightforward. The transmitter model for any user u is represented in Fig 1. The system functions in blocks of $2K$ symbols. K denotes the number of tones in the OFDM modulation. Let $\mathbf{b}_u = \left[\mathbf{b}_{u,q_1}, \mathbf{b}_{u,q_2}\right]$ the $2K$ symbols to be sent by user u with $\mathbf{b}_{u,q_p} = \left[b_{u,q_p}(1) \ldots b_{u,q_p}(K)\right]_{p\in\{1,2\}}$. The index q_p represents invariantly symbols for any timeslot within a STBC codeword. The $2K$ symbols are rearranged to form two vectors of K symbols using the serial to parallel converter (S/P). On each branch of S/P output, each symbol is spread using the code $\mathbf{c}_u = \left[c_u(1) \ldots c_u(\theta)\right]$ of size θ. All chips can be represented as $\mathbf{s}_{u,q_p}$, a $K\times\theta$ matrix where each line represents a distinct subcarrier and each column will form an OFDM codeword.

Let $\mathbf{X}_{u,q_p}$ be the matrix containing θ OFDM codewords of K tones. $\left\{\mathbf{X}_{u,q_1}, \mathbf{X}_{u,q_2}\right\}$ represent a STBC codeword. The matrices forming a STBC codeword are specially formed so that the symbols obtained after multi-user interference (MUI) elimination are orthogonal. In our two transmit antennas configuration, we will send successively $\left\{\mathbf{X}_{u,q_1}, \mathbf{X}_{u,q_2}\right\}$ over the first antenna and $\left\{-\tilde{\mathbf{X}}_{u,q_2}, \tilde{\mathbf{X}}_{u,q_1}\right\}$ over the second antenna, with $\tilde{\mathbf{X}}_{u,q_p} = \mathbf{J}\cdot\mathbf{X}^*_{u,q_p}$ and $\mathbf{J} = \begin{bmatrix} 0 & & 1 \\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & 0 \end{bmatrix}$ is a skew eye matrix representing the OFDM codeword time reversal.

We consider that appropriate OFDM cyclic prefix is used to avoid inter-block interferences. We suppose that the OFDM sub carriers are sufficiently spaced and there is no inter carrier interference. A fully orthogonal real valued spreading code such as Walsh Code is used.

2.2 Receiver

The receiver structure is represented in Fig. 2. In an uplink configuration, each user will have its own frequency selective channel between each transmit and each receive antenna. Considering the channel response of u^{th} user between j^{th} transmit antenna and i^{th} receive antenna, the channel is modeled as

$$h_{j,i}^{u}(\tau)=\sum_{l=0}^{L-1}\alpha_{j,i}^{u}(l)\cdot\delta\left(\tau-\frac{l}{\Delta f}\right) \tag{1}$$

with $L=\left[\tau_m\Delta f+1\right]$, τ_m is the maximum delay in any user frequency selective channel, $\alpha_{j,i}^{u}(l)$ is the l^{th} complex value tap gain, and Δf the whole bandwidth of the OFDM system. All user signals emitted from all transmit antennas, propagate through their respective frequency selective channel and reach the i^{th} receiver antenna. These superposed signals are affected with an additive complex Gaussian noise with variance σ_n^2. It is assumed that all channels are not correlated to each other, and the channel is invariant during our STBC codeword.

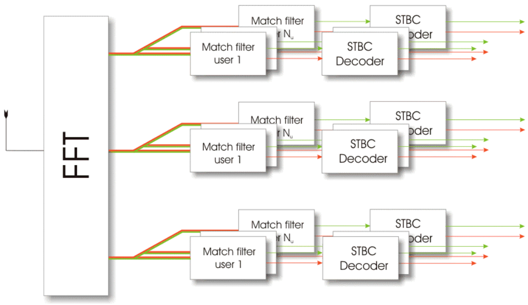

Fig. 2 : Receiver Model

Denote N_u the number of user in the system. Following [5], we consider the quasi synchronous system in a micro cell environment and the received signal after OFDM demodulation can be written as

$$\mathbf{r}_{i,q_1}=\sum_{u=1}^{N_u}\mathbf{\Lambda}_{1,i}^{u}\cdot\mathbf{s}_{u,q_1}-\mathbf{\Lambda}_{2,i}^{u}\cdot\mathbf{s}_{u,q_2}^{*}+\mathbf{n}_{q_1} \tag{2}$$

$$\mathbf{r}_{i,q_2}=\sum_{u=1}^{N_u}\mathbf{\Lambda}_{1,i}^{u}\cdot\mathbf{s}_{u,q_2}+\mathbf{\Lambda}_{2,i}^{u}\cdot\mathbf{s}_{u,q_1}^{*}+\mathbf{n}_{q_2} \tag{3}$$

with $\mathbf{\Lambda}_{j,i}^{u}=diag\left(\mathbf{Q}\cdot\boldsymbol{\alpha}_{j,i}^{u}\right)$, $\boldsymbol{\alpha}_{j,i}^{u}=\left[\alpha_{j,i}^{u}(1)\ \dots\ \alpha_{j,i}^{u}(L)\right]^{T}$ and $\mathbf{Q}$ is the normalized DFT matrix.

After chip match filter, we obtain the following relations.

$$\mathbf{z}_{i,q_1}^{u}=\mathbf{\Lambda}_{1,i}^{u}\cdot\mathbf{b}_{u,q_1}^{T}-\mathbf{\Lambda}_{2,i}^{u}\cdot\mathbf{b}_{u,q_2}^{H}+\mathbf{n}_{q_1}^{u} \tag{4}$$

$$\mathbf{z}_{i,q_2}^{u}=\mathbf{\Lambda}_{1,i}^{u}\cdot\mathbf{b}_{u,q_2}^{T}+\mathbf{\Lambda}_{2,i}^{u}\cdot\mathbf{b}_{u,q_1}^{H}+\mathbf{n}_{q_2}^{u} \tag{5}$$

with $\mathbf{z}_{i,q_p}^{u}$ the $K\times1$ symbols related to user u. Since $\mathbf{\Lambda}_{j,i}^{u}$ is a diagonal matrix, we may write, for individual sub carrier k,

$$z_{i,q_1}^{u}(k)=\Lambda_{1,i}^{u}(k)\cdot b_{u,q_1}(k)-\Lambda_{2,i}^{u}(k)\cdot b_{u,q_2}^{*}(k)+n_{q_1}^{u}(k) \tag{6}$$

$$z_{i,q_2}^{u}(k)=\Lambda_{1,i}^{u}(k)\cdot b_{u,q_2}(k)+\Lambda_{2,i}^{u}(k)\cdot b_{u,q_1}^{*}(k)+n_{q_2}^{u}(k) \tag{7}$$

Equations (6) and (7) are presented in identical form of a single user STBC over a fading channel first proposed by Alamouti [3]. Hence, we only need to know the channel state information (CSI) of the desired user to proceed the decoding. In the next section, we show results of an EM based channel estimation and symbol decision with help of LDPC outer channel encoder.

3. TURBO EQUALIZATION

3.1 Overview

Since we have an equivalent STBC system for each subcarrier for each user, any decoding and channel estimation algorithm known for STBC system can be easily applied. In this section, we present a channel estimation algorithm for each user and a turbo like system to improve the performance of the system using binary LDPC as the outer channel encoder. The turbo equalizer system at the receiver is presented in Fig. 3.

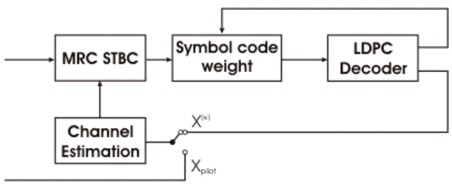

Fig. 3 : Turbo equalizer system model for each user

We use SAGE based channel estimation algorithm using pilot symbols which is a modified version of [4], [6] and [7]. The channel estimation will be used by the Maximum Ratio Combining (MRC) of STBC decoder to estimate STBC symbols. The symbol estimates will be weighed and then be used by the LDPC decoder. Without significant performance loss, the LDPC decoder will not consider the STBC coding constraint during decoding. The LDPC decoder will provide a soft decision that will be used during the symbol weighting operation. The soft decision will also be used to create hard decision used to reestimate the channel. The overall system will be iterated a few times.

3.2 SAGE based channel estimation

We consider here a slowly varying channel between STBC codeword. To conveniently represent the algorithm, (4) and (5) are rewritten into

$$\mathbf{z}_{i,q_1}^{u}=\mathbf{B}_{u,q_1}\boldsymbol{\lambda}_{i,1}^{u}-\mathbf{B}_{u,q_2}^{*}\boldsymbol{\lambda}_{i,2}^{u}+\mathbf{n}_{q_1}^{u} \tag{8}$$

$$\mathbf{z}_{i,q_2}^{u}=\mathbf{B}_{u,q_1}^{*}\boldsymbol{\lambda}_{i,2}^{u}+\mathbf{B}_{u,q_2}\boldsymbol{\lambda}_{i,1}^{u}+\mathbf{n}_{q_2}^{u} \tag{9}$$

where $\boldsymbol{\lambda}_{i,j}^{u}$ is the column vector representation of diagonal matrix $\Lambda_{i,j}^u$, $\mathbf{B}_{u,q_p} = diag\left\{\mathbf{b}_{u,q_p}\right\}$.

The method described below is to be performed at each receive antenna and for each user. Following [8], a natural choice for complete data $\left\{\mathbf{F}_{n,q_p}\right\}_{n,p\in\{1,2\}}$ is selected by decomposing the observed data $\mathbf{z}_{i,q_p}^u$ into two components. We discard the index i and u (8) and (9) to simplify the presentation. We can rewrite those equations as

$$\mathbf{z}_{q_1} = \mathbf{F}_{1,q_1} + \mathbf{N}_{1,q_1} + \mathbf{F}_{2,q_1} + \mathbf{N}_{2,q_1} \tag{10}$$

$$\mathbf{z}_{q_2} = \mathbf{F}_{1,q_2} + \mathbf{N}_{1,q_2} + \mathbf{F}_{2,q_2} + \mathbf{N}_{2,q_2} \tag{11}$$

with $\mathbf{N}_{1,q_p} + \mathbf{N}_{2,q_p} = \mathbf{n}_{q_p}$ and $\mathbf{F}_{n,q_p}$ take values as in Table 1.

TABLE 1 : VALUES OF $\mathbf{F}_{n,q_p}$ FOR EACH VALUE OF *N AND* q_p

Time	1	2
2q	$\mathbf{B}_{q_1}\boldsymbol{\lambda}_{1,q_1}$	$-\mathbf{B}_{q_2}^*\boldsymbol{\lambda}_{2,q_1}$
2q+1	$\mathbf{B}_{q_2}\boldsymbol{\lambda}_{1,q_2}$	$\mathbf{B}_{q_1}^*\boldsymbol{\lambda}_{2,q_2}$

The relation between the complete data $\left\{\mathbf{F}_{n,q_p}\right\}$ and observed data $\mathbf{z}_{q_p}$ is given by $\mathbf{z}_{q_p} = \sum_{n=1}^{2}\mathbf{F}_{n,q_p}$. It is easy to show that the above-described SAGE-based channel estimation algorithm takes the following form

$$\boldsymbol{\lambda}_{n,q_p}^{(e+1)} = \left(\mathbf{B}_{q_p}\right)^{-1}\hat{\mathbf{F}}_{n,q_p}^{(e)} \tag{12}$$

$(\cdot)^{(e)}$ denotes the value at the e^{th} SAGE iteration and

$$\hat{\mathbf{F}}_{n,q_p}^{(e)} = \mathbf{F}_{n,q_p}^{(e)} + \beta_{n,q_p}\left(\mathbf{z}_{q_p} - \sum_{j=1}^{2}\mathbf{F}_{j,q_p}^{(e)}\right) \tag{13}$$

$$\mathbf{F}_{n,q_p}^{(e)} = \mathbf{B}_{q_p}\boldsymbol{\lambda}_{n,q_p}^{(e)} \tag{14}$$

Observe that β_{n,q_p} can be arbitrarily selected due to the decomposition of the independent noise components $\mathbf{N}_{n,q_p}$ with $\sum_{n=1}^{N}\beta_{n,q_p} = 1$ constraint. A typical value is $\beta_{n,q_1} = \beta_{n,q_2} = 0.5$ for the case of two-transmit antennas.

Note that the above result can only be applied for pilot frames, i.e., the transmitted signals are known at the receiver which is exactly the case in [6]. However, in the case of signal transmission, we don't know all the transmitted signals. This is where the symbol hard decision of LDPC comes into action. If we note $\mathbf{B}_{n,q_p}^{(\rho)}$ the symbols matrix deduced from LDPC hard decision at the ρ^{th} turbo iteration, the channel estimation becomes decision directed hence (12) and (14) become

$$\boldsymbol{\lambda}_{n,q_p}^{(e+1)} = \left(\mathbf{B}_{n,q_p}^{(\rho)}\right)^{-1}\hat{\mathbf{F}}_{n,q_p}^{(e)} \tag{15}$$

$$\mathbf{F}_{n,q_p}^{(e)} = \mathbf{B}_{n,q_p}^{(\rho)}\boldsymbol{\lambda}_{n,q_p}^{(e)} \tag{16}$$

To simplify the estimations of $\mathbf{F}_{n,q_p}^{(0)}$, the pilot will be sent alternatively over antenna 1 and antenna 2, i.e. one antenna remains silent while the other is transmitting the pilot. The initial channel estimation will be based over these space alternate pilots.

3.3 MRC STBC

Due to the diagonal form of $\mathbf{B}_{n,q_p}$, each symbol in $\mathbf{z}_{q_p}$ are decoupled to each other. Hence, we can individually decode STBC symbols for each sub carrier k. We have

$$z_{q_1}(k) = b_1(k)\cdot\lambda_{1,q_1}^{(e)}(k) - b_2^*(k)\cdot\lambda_{2,q_1}^{(e)}(k) + n_{q_1}(k) \tag{17}$$

$$z_{q_2}^*(k) = b_1(k)\cdot\lambda_{2,q_2}^{*(e)}(k) + b_2^*(k)\cdot\lambda_{1,q_2}^{*(e)}(k) + n_{q_2}(k) \tag{18}$$

with $b_n(k) = B_n(k,k)$

Using linear combination, we have

$$\begin{aligned}\psi_1(k) &= \lambda_{1,q_2}^{*(e)}(k)\cdot z_{q_1}(k) + \lambda_{2,q_1}^{(e)}(k)\cdot z_{q_2}^*(k)\\ &= b_1(k)\cdot\gamma(k) + n_1\end{aligned} \tag{19}$$

$$\begin{aligned}\psi_2(k) &= \lambda_{1,q_1}^{(e)}(k)\cdot z_{q_2}^*(k) - \lambda_{2,q_2}^{*(e)}(k)\cdot z_{q_1}(k)\\ &= b_2^*(k)\cdot\gamma^*(k) + n_2\end{aligned} \tag{20}$$

where $\gamma(k) = \lambda_{1,q_2}^{*(\kappa)}\cdot\lambda_{1,q_1}^{(\kappa)}(k) + \lambda_{2,q_1}^{(\kappa)}(k)\cdot\lambda_{2,q_2}^{*(\kappa)}(k)$

Hard decision of $\hat{b}_1(k)$ and $\hat{b}_2(k)$ can be obtained by

$$\hat{b}_1(k) = \underset{b_1\in\Omega}{arg\,min}\left(\underbrace{\left|\psi_1(k) - b_1(k)\cdot\gamma(k)\right|^2}_{\varepsilon_1}\right) \tag{21}$$

$$\hat{b}_2(k) = \underset{b_2\in\Omega}{arg\,min}\left(\underbrace{\left|\psi_2(k) - b_2^*(k)\cdot\gamma^*(k)\right|^2}_{\varepsilon_2}\right) \tag{22}$$

Ω denotes the set of all available STBC symbols.

In our case, we need to supply binary soft valued decisions to our binary LDPC decoder. Let $\Gamma_1^{\phi}\left[d_n^j(k)\right]$ the extrinsic a posteriori LLR of the j^{th} LDPC code bit of $b_n(k)$. $\Gamma_1^{\phi}\left[d_n^j(k)\right]$ is computed as presented in (23).

$$\begin{aligned}\Gamma_1^{\phi}\left[d_n^j(k)\right] &= log\left(\frac{P\left(d_n^j(k)=+1\middle|y(k)\right)}{P\left(d_n^j(k)=-1\middle|y(k)\right)}\right) - log\left(\frac{P\left(d_n^j(k)=+1\right)}{P\left(d_n^j(k)=-1\right)}\right)\\ &= log\left(\frac{\sum_{b\in\Omega_j^+}P\left(\hat{b}_n(k)=b\middle|y(k)\right)}{\sum_{b\in\Omega_j^-}P\left(\hat{b}_n(k)=b\middle|y(k)\right)}\right) - \Gamma_2^{\phi,\rho}\left[d_n^j(k)\right]\\ &= log\left(\frac{\sum_{b\in\Omega_j^+}exp\left[-\varepsilon_n + log\,P(b)\right]}{\sum_{b\in\Omega_j^-}exp\left[-\varepsilon_n + log\,P(b)\right]}\right) - \Gamma_2^{\phi,\rho}\left[d_n^j(k)\right]\end{aligned} \tag{23}$$

Ω_j^+ denotes the set of b for which the j^{th} LDPC code bit is '+1' and Ω_j^- is defined the same way. $P(b)$ and $\Gamma_2^{\phi,\rho}\left[d_n^j(k)\right]$ represent the a priori probability of symbol b and the extrinsic a priori LLR respectively which are fed back by LDPC decoder from the previous turbo iteration ρ. At the first turbo iteration,

$$\Gamma_2^{\phi,\rho}\left[d_n^j(k)\right]=0 \tag{24}$$

The calculated $\Gamma_1^{\phi}\left[d_n^j(k)\right]$ values are sent to soft LDPC decoder, which in turn, iteratively computes $\Gamma_2^{\phi,\rho}\left[d_n^j(k)\right]$ and $P(b)$. Hard decisions from LDPC decoder are remapped to symbol of set Ω forming $B_{n,q'}^{(\rho)}$, to be used to reestimate channel in (15)-(16). This completes one turbo-iteration.

4. Simulation Results

For simulation, we use an irregular weight 3 binary LDPC with rate 0.5 and size 1024x2048. 8PSK modulation is used. In the LDPC decoder, we use 30 iterations and a total of five turbo-iterations are applied to each block. The channel estimating algorithms iterate at most five times. The channel is considered invariant for one STBC codeword. A pilot OFDM word is inserted every 10 transmitted STBC codewords. 32 OFDM tones are used and we idealize for this study that adequate prefix cyclic and perfect time and frequency synchronization have been made. Walsh Codes with spreading factor 8 are used to spread the user symbols.

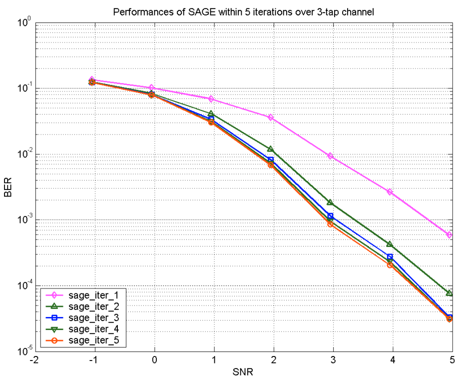

Fig. 4 : Convergence rate of SAGE based channel estimator

We present here the channel estimation adapted to the transceiver structure. The first simulation result shown in Fig. 4 corresponds to a three-path Rayleigh fading channel for an arbitrary user. We observe that SAGE based channel estimator converges fast to stationary values. This is the first interest using SAGE algorithm. The second case shown in Fig. 5 corresponds to a 6-tap uniform Rayleigh fading channel for an arbitrary user. Clearly, BER performances are better in the second case obviously due to the higher order of selectivity of the second channel. We observe furthermore that the performances of SAGE based estimator is comparable to EM and MMSE techniques with appealing less complexity.

5. Conclusion

We have proposed in this paper, a turbo SAGE based equalization for an uplink multi-user and multi-carrier system with space time diversity in a quasi-synchronous system. We showed that the equalization and decoding can be made individual to each user with a relatively simple algorithm. The performances are comparable to other more complex algorithms.

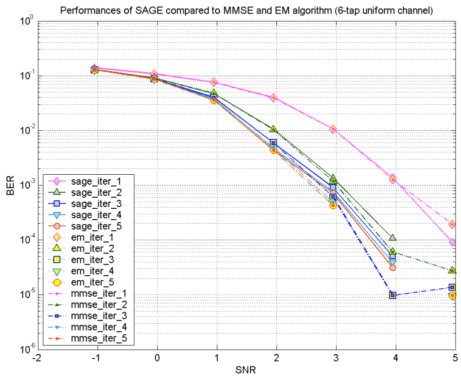

Fig. 5 : Performances of SAGE based channel estimator compared to EM and MMSE techniques

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