JOINT USE OF Σ AND Δ CHANNELS FOR MULTIPLE RADAR TARGET DOA ESTIMATION

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ABSTRACT

This work deals with the problem of estimating the directions of arrival (DOA) of multiple radar targets present in the same range-azimuth resolution cell of a surveillance radar by joint processing the sum (Σ) and delta (Δ) channel data. The AML-RELAX estimator, previously derived by the authors, is extended to a two-channel system, and compared to the classical monopulse system.

1. INTRODUCTION

This work is a follow-up of [1], where we proposed a method making use of only one receiving channel, the sum channel, to estimate the parameters of multiple targets. The proposed algorithm is based on the asymptotic maximum likelihood (AML) technique; it exploits knowledge of the antenna main beam pattern and the fact that the mechanical scanning antenna impresses an amplitude modulation on the signals backscattered by the targets [2]. Here, we generalize the AML estimator to the case of two tightly matched receiving channels (sum and difference) and we compare its performance to that of the monopulse system [2, Ch. 4].

The rest of this paper is organized as follows. The data model and problem statement are briefly introduced in Section 2. In Section 3, we generalize the AML-RELAX estimator to the case of Σ and Δ. In Section 4, the performance of the two-channel AML-RELAX is compared with that of the classical monopulse system for the single and multiple target scenarios. Concluding remarks are reported in Section 5.

2. DATA MODEL AND PROBLEM STATEMENT

In typical phased array radars, a single beam is formed on transmission and two or more beams are formed on reception. In this work, we consider a linear array radar that uses the channel Σ on transmission and two matched channels, Σ and Δ on reception. The two channels, or antenna patterns, are defined as the complex amplitude profiles versus target azimuth angle. For a linear array, the patterns can be determined by the antenna weighting coefficients $\mathbf{w}_\Sigma$ and $\mathbf{w}_\Delta$, the number of elements *K*, the element spacing *d*, and the operating wavelength λ. A target positioned at the angle θ_{TG} will present the directional vector $\mathbf{s}(\theta_{TG}) = \left[1 \quad \exp\left(j2\pi\phi\right) \quad \cdots \quad \exp\left(j2(K-1)\pi\phi\right)\right]^T$ where $\phi = d\sin(\theta_{TG})/\lambda$. The antenna beam patterns Σ and Δ can then be generated as $q_\Sigma(\theta_{TG}) = \mathbf{w}_\Sigma^H \mathbf{s}(\theta_{TG})$ and $q_\Delta(\theta_{TG}) = \mathbf{w}_\Delta^H \mathbf{s}(\theta_{TG})$. The sum beam pattern $q_\Sigma(\theta_{TG})$ is a real and even amplitude function with its maximum at the steering direction and the difference beam $q_\Delta(\theta_{TG})$ is real and odd with zero response at the steering direction. The shapes of the beams depend on the weights $\mathbf{w}_\Sigma$ and $\mathbf{w}_\Delta$. We use here $\mathbf{w}_\Sigma = [1 \cdots 1 1 \cdots 1]^T$ and $\mathbf{w}_\Delta = j[1 \cdots 1 -1 \cdots -1]^T$ [3], so we have

$$q_\Sigma(\theta_{TG}) = \exp\left(j\pi(K-1)\frac{d}{\lambda}\sin(\theta_{TG})\right)\frac{\sin\left(\pi K\frac{d}{\lambda}\sin(\theta_{TG})\right)}{\sin\left(\pi\frac{d}{\lambda}\sin(\theta_{TG})\right)} \quad (1)$$

and

$$q_\Delta(\theta_{TG}) = \exp\left(j\pi(K-1)\frac{d}{\lambda}\sin(\theta_{TG})\right)\frac{1-\cos\left(\pi K\frac{d}{\lambda}\sin(\theta_{TG})\right)}{\sin\left(\pi\frac{d}{\lambda}\sin(\theta_{TG})\right)}. \quad (2)$$

The phase term in (1) and (2) can be neglected without loss of generality [2]. If both the beams are mechanically steered during the *time-on-target* with constant angular velocity ω_R rad/s, the antenna introduces an amplitude modulation on the target signal, in both the channels, that depends on the target azimuth position and on the instantaneous boresight of the array,

$$q_{\Sigma norm}(\theta_{TG}, n) = \frac{\sin\left(\pi K\frac{d}{\lambda}\sin(\theta_{TG} - n\omega_R T)\right)}{K\sin\left(\pi\frac{d}{\lambda}\sin(\theta_{TG} - n\omega_R T)\right)} \quad (3)$$

and

$$q_{\Delta norm}(\theta_{TG}, n) = \frac{1-\cos\left(\pi K\frac{d}{\lambda}\sin(\theta_{TG} - n\omega_R T)\right)}{K\sin\left(\pi\frac{d}{\lambda}\sin(\theta_{TG} - n\omega_R T)\right)}, \quad (4)$$

where $n = 0, 1, \ldots \; N-1$, *T*=1/*PRF* is the radar pulse repetition time (*PRT*) and *PRF* is the pulse repetition frequency. The number *N* of pulses between the one-way -3 dB points is given by $N = \theta_B/(\omega_R T)$, where θ_B is the –3 dB azimuth beam width, i.e. the angle such that $q_{\Sigma norm}^2(\pm\theta_B/2) = 1/2$.

As previously stated, this system, as the monopulse system, uses the Σ channel on transmission and the Σ and Δ channels on reception. Therefore, the overall modulation introduced by the antenna on the target amplitude is represented on the sum channel by the vector $\mathbf{g}_\Sigma$ and on the difference channel by the vector $\mathbf{g}_\Delta$, where

$$g_\Sigma(\theta_{TG}, n) = q_{\Sigma norm}^2(\theta_{TG}, n) \quad (5)$$

and

$$g_{\Delta}(\theta_{TG},n) = q_{\Sigma norm}(\theta_{TG},n) q_{\Delta norm}(\theta_{TG},n) \quad (6)$$

In our analysis we set $d/\lambda = 0.5$ and, to obtain a sum pattern beam width $\theta_B \cong 2^\circ$ we set K=51 receiving elements.

Assume now that M point-like targets, with directions of arrival $\{\theta_{TG\,i}\}_{i=1}^{M}$ and Doppler frequencies $\{f_{Di}\}_{i=1}^{M}$ are present in the range-azimuth resolution cell under test. The $2N \times 1$ data vector $\mathbf{z}$ is composed by the collection of the $2N$ echoes received during the ToT, N on the sum channel and N on the difference: $\mathbf{z} = [\mathbf{z}_{\Sigma}^T\ \mathbf{z}_{\Delta}^T]^T$, where ^{T} is the transpose operator. The nth elements of the $N \times 1$ vectors $\mathbf{z}_{\Sigma}$ and $\mathbf{z}_{\Delta}$ are

$$\begin{cases} z_{\Sigma}(n) = \sum_{i=1}^{M} b_i g_{\Sigma}(\theta_{TGi},n) e^{j2\pi f_{Di} n} + d_{\Sigma}(n), & \text{for} \quad n = 0,1,\ldots N-1, \\ z_{\Delta}(n) = \sum_{i=1}^{M} b_i g_{\Delta}(\theta_{TGi},n) e^{j2\pi f_{Di} n} + d_{\Delta}(n), & \text{for} \quad n = 0,1,\ldots N-1, \end{cases} \quad (7)$$

where b_i is the unknown complex amplitude of the ith target signal, $\theta_{TGi} \in [0,\theta_B)$, and $f_{Di} \in (-0.5,0.5)$ is the Doppler frequency of the ith target normalized to the PRF. The terms $d_{\Sigma}(n)$ and $d_{\Delta}(n)$ model the additive noise in the two channels, composed by the superposition of clutter and thermal noise. The estimation problem is investigated here under the assumptions that they are stationary, mutually independent, complex processes, independent of the signal components. In vector notation, the data model for M targets is given by $\mathbf{z} = \mathbf{A}(\boldsymbol{\theta})\mathbf{b} + \mathbf{d}$, where $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_{TG1},f_{D1})\ \ \mathbf{a}(\theta_{TG2},f_{D2})\ \ \cdots\ \ \mathbf{a}(\theta_{TGM},f_{DM})]$ is the $N \times M$ steering matrix, $\mathbf{b} = [b_1\ \ b_2\ \ \cdots\ \ b_M]^T$ is the $M \times 1$ vector of the unknown deterministic complex amplitudes, $\boldsymbol{\theta} = [\theta_{TG\,1}\ \cdots\ \theta_{TG\,M}\ \ f_{D\,1}\ \cdots\ f_{D\,M}]^T$ is the $2M \times 1$ vector of the unknown DOAs and Doppler frequencies, $\mathbf{a}(\theta_{TGi},f_{Di})$ is a $2N \times 1$ steering vector, factored as $\mathbf{a}(\theta_{TGi},f_{Di}) = \mathbf{g}(\theta_{TGi}) \odot \mathbf{p}(f_{Di})$, where $\odot$ represents the Hadamard product or element-wise multiplication, and

$$[\mathbf{g}(\theta_{TGi})]_n = \begin{cases} g_{\Sigma}(\theta_{TGi},n-1), & \text{for} \quad n = 1,\cdots,N, \\ g_{\Delta}(\theta_{TGi},n-N-1), & \text{for} \quad n = N+1,\cdots,2N, \end{cases} \quad (8)$$

$$[\mathbf{p}(f_{Di})]_n = \begin{cases} e^{j2\pi f_{Di}(n-1)}, & \text{for} \quad n = 1,\cdots,N, \\ e^{j2\pi f_{Di}(n-N-1)}, & \text{for} \quad n = N+1,\cdots,2N. \end{cases} \quad (9)$$

Note that $\mathbf{g}(\theta_{TGi})$ is only function of θ_{TGi}, whereas $\mathbf{p}(f_{Di})$ is only function of f_{Di}. The $2N \times 1$ noise vector $\mathbf{d}$ is given by $\mathbf{d} = [\mathbf{d}_{\Sigma}^T\ \mathbf{d}_{\Delta}^T]^T$, where $\mathbf{d}_{\Sigma}$ and $\mathbf{d}_{\Delta}$ are modelled as independent random vectors, sum of thermal noise $\mathbf{n} = [\mathbf{n}_{\Sigma}^T\ \mathbf{n}_{\Delta}^T]^T$ and clutter $\mathbf{c} = [\mathbf{c}_{\Sigma}^T\ \mathbf{c}_{\Delta}^T]^T$. The thermal noise $\mathbf{n}$ is modelled as a complex zero-mean white Gaussian vector. In shorthand notation, $\mathbf{n} \sim CN(\mathbf{0},\ \sigma_n^2\mathbf{I})$, where σ_n^2 is the variance of each noise component and $\mathbf{I}$ is the $2N \times 2N$ identity matrix. Clutter vectors $\mathbf{c}_{\Sigma}$ and $\mathbf{c}_{\Delta}$ are modelled as a complex Gaussian distributed random vectors having zero-mean and covariance matrix $E\{\mathbf{c}_{\Delta}\mathbf{c}_{\Delta}^H\} = \sigma_{c\Delta}^2 \mathbf{M}_{\Delta}$, and $E\{\mathbf{c}_{\Sigma}\mathbf{c}_{\Sigma}^H\} = \sigma_{c\Sigma}^2 \mathbf{M}_{\Sigma}$, respectively, where ^{H} is the conjugate-transpose operator and $\sigma_{c\Delta}^2$ and $\sigma_{c\Sigma}^2$ are the variances of each clutter component, $\mathbf{M}_{\Delta}$ and $\mathbf{M}_{\Sigma}$ are the normalized covariance matrices: $[\mathbf{M}_{\Delta}]_{i,i} = [\mathbf{M}_{\Sigma}]_{i,i} = 1$ for $i = 1,\ 2,\ \cdots,\ N$. In this work, for ease of treatment, we assume $\sigma_{c\Delta}^2 = \sigma_{c\Sigma}^2 = \sigma_c^2$, and $\mathbf{M}_{\Delta} = \mathbf{M}_{\Sigma}$. Therefore, the disturbance covariance matrix is

$$\begin{aligned} \mathbf{M}_d &= E\{\mathbf{d}\mathbf{d}^H\} = \sigma_c^2 \mathbf{M}_c + \sigma_n^2 \mathbf{I} = \sigma_d^2 \mathbf{M} \\ &= \begin{bmatrix} \sigma_c^2 \mathbf{M}_{\Sigma} + \sigma_n^2 \mathbf{I}_N & \mathbf{0} \\ \mathbf{0} & \sigma_c^2 \mathbf{M}_{\Sigma} + \sigma_n^2 \mathbf{I}_N \end{bmatrix}, \end{aligned} \quad (10)$$

where $\mathbf{I}_N$ is the $N \times N$ identity matrix, $\sigma_d^2 = \sigma_c^2 + \sigma_n^2$ is the total disturbance power and $\mathbf{M}$ is the normalized disturbance covariance matrix, which is given by: $\mathbf{M} = (CNR+1)^{-1}(CNR\,\mathbf{M}_c + \mathbf{I})$, where $CNR = \sigma_c^2 / \sigma_n^2$ is the clutter-to-noise power ratio. In this work we assume that $\mathbf{M}$ is a priori known, whereas σ_d^2 may be unknown. In realistic radar scenarios $\mathbf{M}$ must be estimated from secondary data, as described, as instance, in [4].

The goal here is to estimate jointly vectors $\mathbf{b}$ and $\boldsymbol{\theta}$ based on the observation of $\mathbf{z}$. No *a priori* information for $\boldsymbol{\theta}$ and $\mathbf{b}$ is assumed; they are modelled as unknown deterministic constants. Under this assumption, the data vector $\mathbf{z}$ is complex Gaussian distributed.

3. THE TWO-CHANNEL AML-RELAX ESTIMATOR

Derivation of the ML estimator is similar to that outlined in [1], provided that $\mathbf{z}$ is as in (7). After straightforward manipulations, we find:

$$\hat{\boldsymbol{\theta}}_{ML} = \arg\max_{\boldsymbol{\theta}} \mathbf{z}^H \mathbf{M}^{-1} \mathbf{A} (\mathbf{A}^H \mathbf{M}^{-1} \mathbf{A})^{-1} \mathbf{A}^H \mathbf{M}^{-1} \mathbf{z}, \quad (11)$$

$$\hat{\mathbf{b}}_{ML} = (\mathbf{A}^H \mathbf{M}^{-1} \mathbf{A})^{-1} \mathbf{A}^H \mathbf{M}^{-1} \mathbf{z}, \quad (12)$$

where, for ease of notation, we omitted the dependence of $\mathbf{A}(\boldsymbol{\theta})$ on $\boldsymbol{\theta}$.

Calculation of $\hat{\boldsymbol{\theta}}_{ML}$ requires the $2M$-dimensional ($2M$-D) nonlinear maximization of the functional in (11), where $\boldsymbol{\theta} = [\theta_1\ \cdots\ \theta_M\ \ f_1\ \cdots\ f_M]^T$ denotes the generic parameter vector and the subscript $2N$ points out that the functional depends on the number of samples. Generally, this maximization is computationally cumbersome. To trade off performance with computational complexity, a sub-optimum algorithm was derived in [1] based on the asymptotic (large sample size) maximum likelihood (AML) technique. Under the hypothesis that the Doppler frequencies are sufficiently separated, that is $|f_{Di} - f_{Dj}| \geq 1/N$ when $i \neq j$, the AML estimator calculates an estimate of $\boldsymbol{\theta}$ from the locations of the M highest peaks of the functional in (11), calculated for the single target scenario (M=1).

$$\hat{\boldsymbol{\theta}}_{ML} = \arg\max_{\boldsymbol{\theta}} \frac{|\mathbf{z}^H \mathbf{M}^{-1} \mathbf{a}(\boldsymbol{\theta})|^2}{\mathbf{a}^H(\boldsymbol{\theta}) \mathbf{M}^{-1} \mathbf{a}(\boldsymbol{\theta})}. \quad (13)$$

It is less computationally heavy than the ML of (11), since it replaces the $2M$-D nonlinear search required by the ML with the search of the locations of the M highest peaks of a 2-D functional. The complex amplitudes are then estimated from (12) by making use of $\hat{\boldsymbol{\theta}}_{ML}$ previously derived. To implement efficiently the AML estimator we used an algorithm based on the RELAX method that decouples the multidimensional maximization problem into a series of simpler 2-D problems. The details on the AML-RELAX algorithm are not reported here for lack of space, they can be found in [1]. We investigated the performance of the two channel estimator and compared it with that of the single channel estimator in [1] in the multiple target scenario. Here we focus our attention on the comparison between the two-channel AML-RELAX and the monopulse estimator.

4. THE MONOPULSE ESTIMATOR

The monopulse technique is a classical method to estimate the direction of arrival of targets in tracking and surveillance radar. In principle it can work with just a single pulse and with the two channels Σ and Δ. The estimate $\hat{\theta}_{TG}$ of the target DOA is a function of the ratio of the Δ and Σ channel outputs z_Δ and z_Σ. In detail, the signal processor forms the monopulse ratio defined by $r \triangleq \mathcal{Re}\{z_\Delta / z_\Sigma\}$ where $\mathcal{Re}\{\ \}$ denotes the real part. In absence of disturbance and in presence of a single target, the monopulse ratio reduces to

$$r \triangleq \mathcal{Re}\left\{\frac{g_\Delta(\theta_{TG})}{g_\Sigma(\theta_{TG})}\right\} \tag{14}$$

from which, assuming *r* is monotonic in off-boresight angle, the angular location of the target may be uniquely determined. The statistical characteristics of the monopulse ratio have been studied in presence of disturbance and jammers in many papers (see as instance [2] and references therein).

When the radar receives *N* pulses $\mathbf{z}_\Delta = [z_\Delta(0) \ \cdots \ z_\Delta(N-1)]^T$ and $\mathbf{z}_\Sigma = [z_\Sigma(0) \ \cdots \ z_\Sigma(N-1)]^T$, the monopulse ratio and the estimate of the target DOA can be calculated for each pulse. Finally, we obtain $\hat{\theta}_{TG} = \sum_{n=0}^{N-1} \hat{\theta}_{TG}^{(n)} / N$ where $\hat{\theta}_{TG}^{(n)}$ is the target DOA estimate for each pulse. We treat the situation in which the source can be considered static with respect to the radar during the time-on-target (ToT), that is, during the recording of the *N* pulses. In this case, due to the scanning movement of the antenna, the monopulse estimator is biased and the bias *b* can be calculated. It is easy to verify that $b \triangleq E\{\hat{\theta}_{TG}\} - \theta_{TG} = (N-1)(N-2)\theta_B/2$ then the DOA unbiased estimator is $\hat{\theta}_{TG} = \sum_{n=0}^{N-1} \hat{\theta}_{TG}^{(n)} - b$. The classical monopulse estimator is thought for only one source, then for comparison purposes, we use the AML estimator (13).

The root mean square error (*RMSE*) is derived by running 10^4 Monte Carlo runs. In all the runs, the clutter power spectral density (PSD) is assumed to have a Lorentzian shape symmetrically located around the zero frequency. As a consequence, the autocovariance function (ACF) has exponential shape and the elements of the clutter covariance matrix are given by $[\mathbf{M}_\Sigma]_{ij} = \sigma_c^2 \rho^{|i-j|}$ with $|\rho| < 1$, ρ is the clutter one-lag correlation coefficient. The reference scenario is related to the following set of parameters: azimuth –3 dB beam width: $\theta_B = 2°$; number of integrated pulses: $N = 16$; number of targets: $M = 1$; target DOA: $\theta_{TG1} = 1.5°$; target Doppler frequencies: $f_{D1} = 0.3$; signal-to-disturbance power ratio: $SDR = 20 \ dB$; clutter-to-noise power ratio: $CNR = -\infty \ dB$. The analysed scenarios are obtained by changing only one parameter, while keeping all the others constant. Performance have been investigated as a function of N, SDR, CNR, θ_{TG1}, and f_{D1} and the results are shown in Figs. 1-5. The AML estimator always outperforms the monopulse DOA estimator and the improvement increases with increasing *CNR*. For completeness, in Fig. 6 we report the histogram of the monopulse and AML DOA estimators when two targets are present in the same range-azimuth cell as in Section 3. target DOAs: $[\theta_{TG1} \ \theta_{TG2}] = [1.5° \ 0.9°]$; target Doppler frequencies: $[f_{D1} \ f_{D2}] = [0.3 \ -0.3]$; signal-to-disturbance power ratio: $SDR = SDR_1 = SDR_2 = 20 \ dB$. As known, the monopulse technique provides only one estimate, somewhere in the direction of the "power centroid" of all the targets. Unfortunately, the position of any individual target differs substantially from this average angular position and the measured statistic is not applicable to track any of the targets in the antenna main beam.

5. CONCLUSIONS

In this work, we considered the problem of estimating the DOA of either single or of multiple radar targets present in the same range-azimuth resolution cell. We compared the performance of the AML-RELAX estimator that uses data from both sum and difference channels with that of the monopulse estimator. Our findings can be summarized as follows.

- The use of the AML estimator improves the DOA estimation with respect to the monopulse technique. A suitable example is the following. To obtain an $RMSE(\hat{\theta}_{TG1}) < 3 \cdot 10^{-2}$ with the monopulse estimator we need at least *N*=64, with the AML estimator *N*=16 suffices. The improvement is particularly sensitive when the clutter is dominant with respect to the thermal noise. As a matter of fact, when $CNR \to -\infty$ and *SDR*=20 dB, $RMSE(\hat{\theta}_{TG-AML}) \cong 3 \cdot 10^{-2}$ and $RMSE(\hat{\theta}_{TG-Mono}) \cong 6 \cdot 10^{-2}$; for $CNR \to +\infty$ and *SDR*=20 dB, $RMSE(\hat{\theta}_{TG-AML}) \cong 8 \cdot 10^{-3}$ and $RMSE(\hat{\theta}_{TG-Mono}) \cong 6 \cdot 10^{-2}$.
- Under the hypothesis that $|f_{D1} - f_{D2}| > 1/N$, the AML estimator provides very similar performance in both cases of either one or two targets in the same resolution cell; conversely, as it is well known, the monopulse technique completely fails by providing an erroneous DOA measure, not useful to track any of the targets in the antenna main beam.

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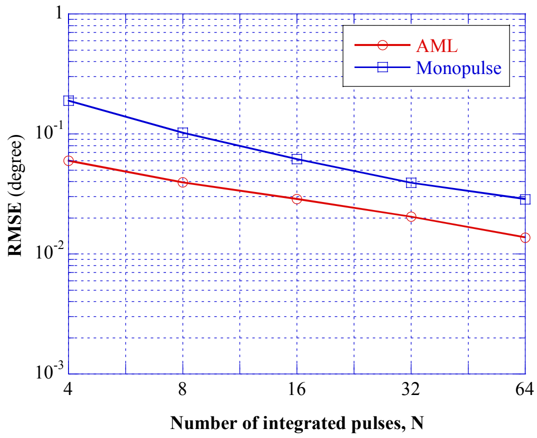

Fig. 1 - RMSE of the DOA estimator versus the number N of integrated pulses.

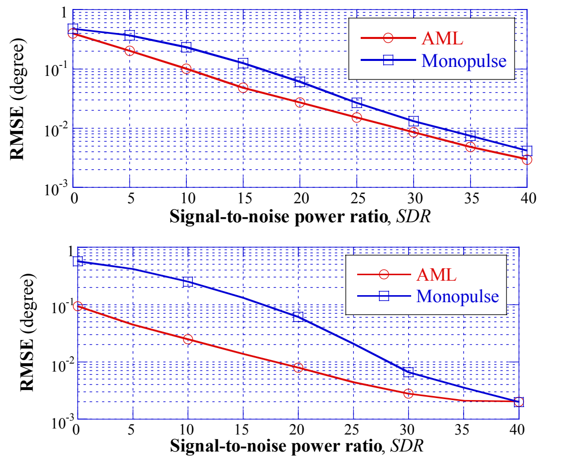

Fig. 2 - RMSE of the DOA estimator versus SDR , $CNR = -\infty\ dB$ and $CNR = +\infty\ dB$, respectively

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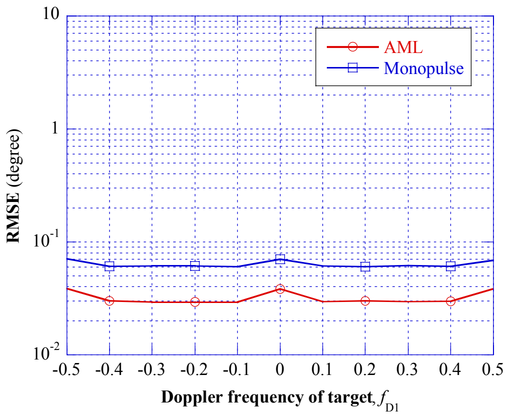

Fig. 3 - RMSE of the DOA estimator versus the Doppler frequency of target, $CNR = -\infty\ dB$.

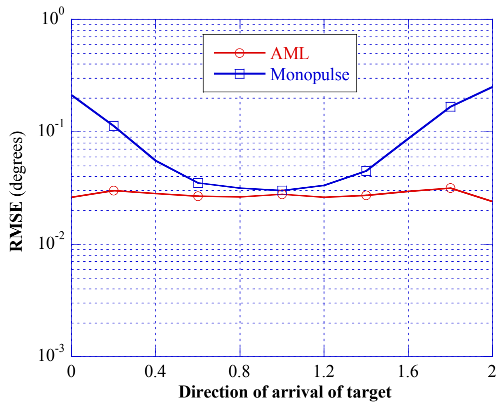

Fig. 4 - RMSE of the DOA estimator versus the DOA of target, $CNR = -\infty\ dB$

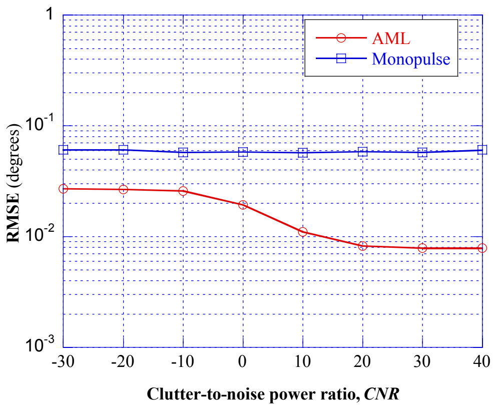

Fig. 5 - RMSE of the DOA estimator versus *CNR*.

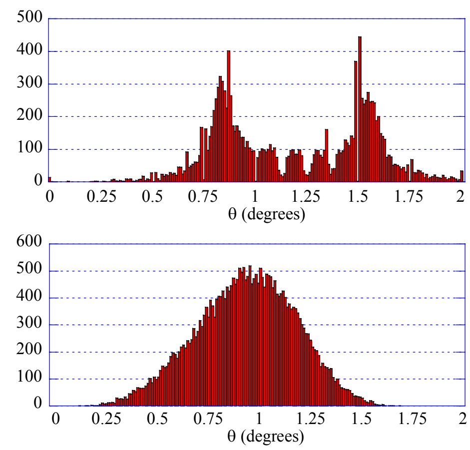

Fig. 6 – Histogram of AML and monopulse estimates.