PERFORMANCE DEGRADATION DUE TO BLINDNESS IN SEPARATION OF MIMO-FIR SYSTEMS OVER COST207 CHANNELS

Sara Sandberg and James P. LeBlanc

CSEE, Luleå University of Technology
SE-971 87 Luleå, Sweden
sara.sandberg@ltu.se leblanc@ltu.se

ABSTRACT

This paper considers the performance penalty of a blind, compared to a non-blind, separation technique of a MIMO-FIR channel. In the blind method the mixing filters are first identified, while they are assumed to be known in the non-blind case. The blind system identification is performed using a recently proposed method based on cumulant subspace decomposition. Separation is then achieved by the FIR part of the mixing system inverse, which minimizes the cross-channel power. The performance penalty due to blindness is investigated for the case when the channel order is underestimated. Results of average residual cross-channel power of the wireless COST207 channel model are included.

1. INTRODUCTION

Blind source separation techniques in wireless communications have been under active research due to the achievable gains in system performance and capacity. In a commercial cellular communication system there will be different types of interference, from multiuser interference in a single cell to interoperator interference. Blind source separation (BSS) can be used to lower the impact of interference on transmission quality as well as increase the capacity of the system in the uplink, [1, chap. 8.8]. Much of the work in the area of BSS addresses the case of instantaneous mixtures. However, in wireless communications multipath is usually present, which yields convolutive mixtures. Also, only mixing with sufficiently narrowband signals can be approximated as instantaneous, while the more common wideband case leads to convolutive mixtures, [1]. The work herein focuses on the separation of multiple-input multiple-output (MIMO) systems. The remaining ISI can be removed by a number of blind single-channel equalization methods, such as the constant modulus algorithm [2], [3], or by minimization of any of the criteria proposed in [4].

In BSS with convolutive mixtures, the sources are separated and equalized without knowledge of the mixing system or usage of training sequences. Blind methods therefore offer potential improvement in system capacity by eliminating the training sequences which carry no information. However, the performance of the separation is likely to degrade when less *a priori* information is exploited. This work investigates the cost of blindness in terms of residual cross-channel power (that is, after separation).

Blind source separation methods can be divided into direct and indirect approaches. In direct approaches the separated signals are extracted without explicit identification of the mixing system, while indirect methods identify the unknown channels before separation and equalization. A recently proposed method for system identification, [5], exploits second order statistics and decorrelates subchannels of the system. In [6], higher order cumulant matching is used to estimate the channel and a Wiener filter separates the independent sources. Another recent method uses higher order statistics and subspace analysis to identify the MIMO finite impulse response (FIR) system assuming a known channel order, [7]. This is the method considered further here.

This paper investigates the separation performance degradation when the mixing system is blindly estimated and compares it with the performance when the mixing system is known (in terms of remaining cross-channel power). The separation system is the FIR part of the mixing system inverse (known or estimated). The performance loss due to blindness is also investigated when the channel order (that must be known or estimated both for the blind identification and the calculation of the separating system) is underestimated, a common case of practical interest.

2. SYSTEM MODEL

We focus on MIMO-FIR systems as in Figure 1, where N observed signals, $\mathbf{x}[n]$, are the output of a linear channel

$$\mathbf{x}[n] = \sum_{k=0}^{\ell} \mathbf{A}_k \mathbf{s}[n-k] \; + \; \mathbf{n}[n]. \tag{1}$$

The source signals, $\mathbf{s}[n]$, are assumed to be statistically independent and the number of source signals equals the number of observed signals. The additive Gaussian noise, $\mathbf{n}[n]$, is zero-mean and the noise components are mutually independent as well as independent of the source signals. The vectors $\mathbf{x}[n]$, $\mathbf{s}[n]$ and $\mathbf{n}[n]$ are length N vectors. The channel order of the mixing system is denoted by ℓ and the mixing matrix $\mathbf{A}$ can be written as a matrix polynomial

$$\mathbf{A}(z) = \mathbf{A}_0 + z^{-1}\mathbf{A}_1 + \ldots + z^{-\ell}\mathbf{A}_\ell \tag{2}$$

where each matrix $\mathbf{A}_k$ is $(N \times N)$. The separating matrix $\mathbf{B}$ is defined as

$$\mathbf{B}(z) = \mathbf{B}_0 + z^{-1}\mathbf{B}_1 + \ldots + z^{-m}\mathbf{B}_m \tag{3}$$

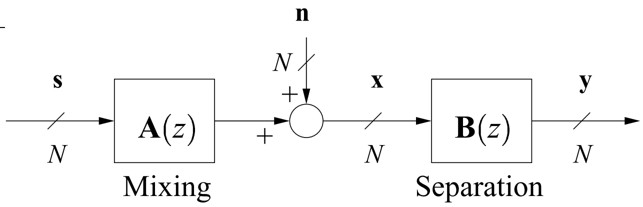

Figure 1: A MIMO-FIR system with N sources and N sensors.

where each $\mathbf{B}_k$ is $(N \times N)$ and the separating filter is of order m. The output of the system can be written as

$$\mathbf{y}(z) = \mathbf{B}(z)[\mathbf{A}(z)\mathbf{s}(z) + \mathbf{n}(z)] \tag{4}$$

and we define the overall system

$$\mathbf{H}(z) = \mathbf{B}(z)\mathbf{A}(z) \tag{5}$$

where

$$\mathbf{H}(z) = \mathbf{H}_0 + z^{-1}\mathbf{H}_1 + \ldots + z^{-(\ell+m)}\mathbf{H}_{(\ell+m)}. \tag{6}$$

3. SEPARATION WITH KNOWN MIXING MATRIX

Assuming that the mixing matrix $\mathbf{A}(z)$ is known or may be estimated, we can find a separating matrix $\mathbf{B}(z)$ by taking the FIR part of the inverse of $\mathbf{A}(z)$. The inverse is given by

$$\mathbf{A}^{-1}(z) = \frac{1}{\det[\mathbf{A}(z)]}\operatorname{adj}[\mathbf{A}(z)] \tag{7}$$

where adj$[\cdot]$ is the adjoint (the transposed cofactor matrix). If the mixing filters in $\mathbf{A}(z)$ are FIR, the filters in adj$[\mathbf{A}(z)]$ as well as the det$[\mathbf{A}(z)]$ will be FIR too. The adj$[\mathbf{A}(z)]$ in this equation can be seen as the separating matrix and the factor $1/\det[\mathbf{A}(z)]$ as the IIR filter that equalizes all channels after separation. Full separation can be achieved if the order of the separating system is at least as large as the order of the filters in adj$[\mathbf{A}(z)]$, that is, if

$$m \geq (N-1)\ell. \tag{8}$$

Remember that full separation still leaves ISI in each separated source.

If the order of the separating system is lower than the order of adj$[\mathbf{A}(z)]$, there will be residual cross-channel power denoted by Q and defined as

$$Q = \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\sum_{k=0}^{\ell+m} |(H_k)_{ij}|^2 \tag{9}$$

where $(H_k)_{ij}$ is element (ij) of $\mathbf{H}_k$. If full separation can not be achieved, we choose the filters of the separating system to be the first $m+1$ taps of the filters in adj$[\mathbf{A}(z)]$. This selection of $\mathbf{B}$ minimizes the cross-channel power.

4. BLIND IDENTIFICATION BASED ON CUMULANT SUBSPACE DECOMPOSITION

If the mixing matrix $\mathbf{A}$ is not known *a priori* it can be blindly estimated from the observed signals by a recently proposed method based on cumulant subspace decomposition, [7]. The proposed algorithm can identify a MIMO-FIR system where

- the source signals are independent, stationary, temporally i.i.d. processes with zero-means and nonzero fourth-order kurtosis
- the channel noises $\mathbf{n}[n]$ are mutually independent zero-mean Gaussian stationary processes and independent of the source signals
- the number of sensors are no less than the number of sources
- the channel order is the same for all sources
- there exists a complex point $z_0 \neq 0$ such that $\mathbf{A}(z_0)$ has full column rank
- the fourth-order kurtosis of all source signals have the same sign.

In [7], some of these assumptions may be relaxed and complemented by other conditions, but this is not considered in the summary given here.

The main idea is to identify $\mathbf{A}(z)$ given only the observed signals $\mathbf{x}[n]$ by employing the fourth-order cumulants. These higher order statistics of $\mathbf{x}[n]$ give enough information to identify the mixing filters up to an arbitrary scaling and permutation and are also not affected by additive white Gaussian noise. The MIMO cumulant subspace - joint diagonalization (MIMOCS-JD) algorithm described in [7] is summarized in three main steps.

1. A set of cumulant matrices containing the fourth-order cumulants with fixed third and fourth argument are defined and the nullspace of this set is estimated.
2. It is shown that a matrix containing the mixing parameters is orthogonal to this nullspace. The estimated mixing filters are obtained via the orthogonality principle.
3. Remaining ambiguities are reduced to scaling and permutation by joint diagonalization of a set of matrices.

A number of simulation examples are given in [7]. For example, the performance is shown for a case where the true channel order is two, but overestimated to three and four. As the authors point out, this algorithm (like other subspace based algorithms) is sensitive to channel order overestimation. However, the interesting case when the channel order is underestimated is not investigated.

5. COST OF BLINDNESS

This paper investigates how sensitive the source separation is to errors in the estimated mixing matrix $\hat{\mathbf{A}}$. The measure of performance used is residual cross-channel power defined in (9). The non-blind case where a separation matrix $\mathbf{B}$ is calculated directly from the known mixing matrix $\mathbf{A}$ is compared with the blind case where the mixing matrix is estimated to $\hat{\mathbf{A}}$ using the blind identification method described in section 4. A separation matrix $\hat{\mathbf{B}}$ is calculated from $\hat{\mathbf{A}}$ and the cross-channel power of the non-blind system $\mathbf{B}(z)\mathbf{A}(z)$ is compared

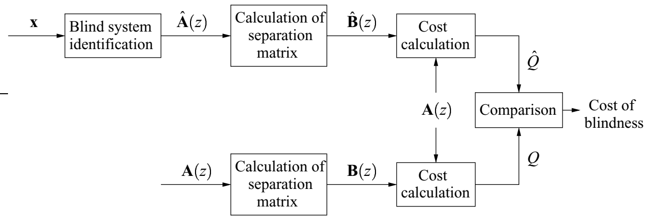

Figure 2: The cost of blindness is estimated by comparing a blind system with a non-blind.

with the cross-channel power of the blind system $\hat{\mathbf{B}}(z)\mathbf{A}(z)$, Figure 2.

It is often the case in communications that the true channel order is higher than the channel order that is tractable to assume for blind identification, [8]. Therefore, we herein investigate the cost of blindness when the channel order is underestimated (when $\hat{\ell} < \ell$), to give an understanding of the robustness of the blind identification method to channel order underestimation.

6. SIMULATION RESULTS

This section presents simulations to demonstrate the performance of the blind versus the non-blind system (the cost of blindness) when the channel order is underestimated. The system model has two sources and two sensors and the channel order ℓ is 6 for the four channels. The channels are randomly generated based on a modified COST207 bad urban wireless channel model [9], where the symbol rate has been increased to give channels with an order of 6. The source signals are mutually independent, temporally i.i.d. QPSK signals and the channel noises are zero-mean, complex, additive white Gaussian processes.

The MIMOCS-JD algorithm, [7], is used for identification of the mixing system. In [7], the sample support used for identifying channels of order 2 is $4,000$. With a higher channel order, there are more parameters in the cumulant matrix that need to be estimated in the blind identification method. A sample support of $70,000$ would keep the ratio of samples to parameters in the cumulant matrix constant, when the order of the channels is 6. The channel is assumed to be constant during the time needed to collect the samples. Other parameters are the same as in [7]. The order of the separating system, m, is chosen equal to $\hat{\ell}$, since this should give perfect separation when there are 2 sources and 2 sensors, see (8). The simulations are performed at an SNR of 20 dB.

Figure 3 shows the average cross-channel power, Q, when $\hat{\ell}$ is varied from 1 to the true channel order 6, for 200 different COST207 channels. The bars show $Q \pm \ /2$, where is the standard deviation. As a comparison it can be noted that the average cross-channel power of the mixing system,

$$Q_{mix} = \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\sum_{k=0}^{\ell} |(A_k)_{ij}|^2 \quad (10)$$

for these 200 channels was 17. As expected, there is a significant residual cross-channel power due to use of the blind technique and it also exhibits greater variance. However, the blind method, which was originally introduced as needing the exact channel order, in fact, is not very sensitive to channel order underestimation for this class of mobile communication channels. The cost, in terms of cross-channel power difference between the blind and non-blind approach, is almost constant when the channel order is underestimated. The relative cost due to channel order underestimation is also small compared to the cost of blindness, at least for reasonably good estimates of the channel order.

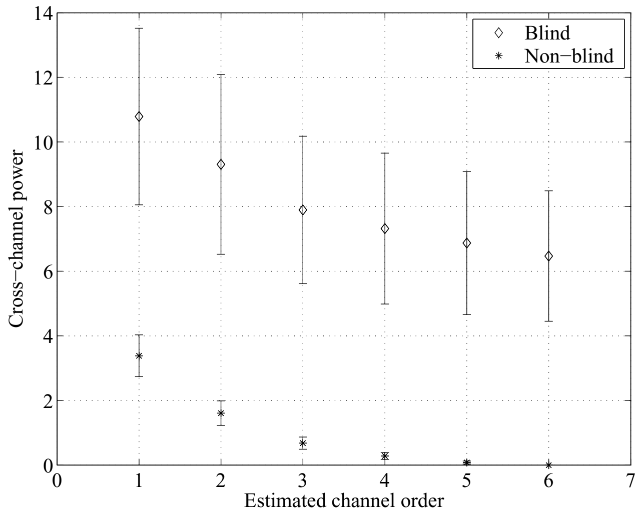

Figure 3: Cross-channel power after non-blind and blind source separation. The true channel order is 6 and $70,000$ samples are used for the blind identification. The bars show $Q \pm \ /2$, where is the standard deviation.

To further interpret the results, we define the cross-channel power reduction, P, as the cross-channel power of the mixing system over the residual cross-channel power after separation, that is,

$$P = \frac{Q_{mix}}{Q}. \quad (11)$$

Figure 4 shows the separation gain in terms of cross-channel power reduction. The last value for the non-blind case is omitted since non-blind separation with the true channel order is perfect, $Q = 0$, and the cross-channel power reduction is infinite. This measure of separation performance demonstrates more clearly the performance penalty due to blindness. The cross-channel power reduction achieved by the blind method does not vary much with estimated channel order. Even if the estimated channel order is just half the true ($\hat{\ell} = 3$), the cross-channel power reduction is almost as high as we can possibly get using the blind method. The difference is less than 1 dB. This is again suggesting that the blind method is not especially sensitive to channel order underestimation.

In practice, the channel may rarely be so slowly varying that it can be assumed to be constant for a support of $70,000$ samples. To shed some light on results applicable to the small sample support case, Figure 5 shows the remaining

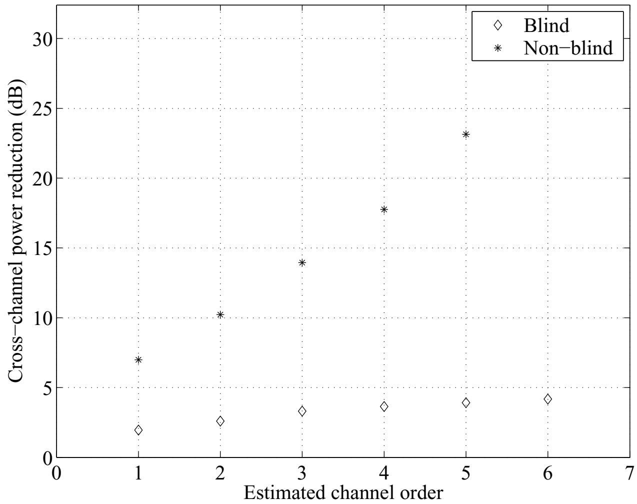

Figure 4: Cross-channel power reduction achieved by blind and non-blind source separation respectively. The last value for the non-blind case is omitted since non-blind separation with the true channel order is perfect and the cross-channel power reduction is infinite.

cross-channel power having support of only $4,000$ samples. With this short sample support the parameters of the cumulant matrices in the blind method are not very accurately estimated for a high channel order, and the cost of blindness increases with an increasing estimated channel order. For this specific simulation a channel order of 5 gives the lowest cross-channel power when the true channel order is 6. This result suggests that channel order underestimation might give lower cross-channel power in cases of limited sample support.

7. CONCLUSIONS

This work investigates the performance penalty of blind separation of a MIMO-FIR system for a class of wireless mobile communication channels. Non-blind separation is used in conjunction with a recently published method for blind system identification to explore the performance degradation due to blindness. Previous work on the blind method investigates its sensitivity to channel order overestimation. Herein the effects due to channel order underestimation are considered, which is important since some form of under-modeling is often the case. Our simulations based on the COST207 channel model show the expected performance degradation due to blindness. Interesting to note is that the performance of the blind method is not very sensitive to channel order underestimation. If the sample support is short, channel order underestimation might even give a lower residual cross-channel power in the blind case. This suggests that even if the true channel order is known, blind identification with a lower channel order than the true may increase the performance of the communication system in terms of reduced cross-channel power.

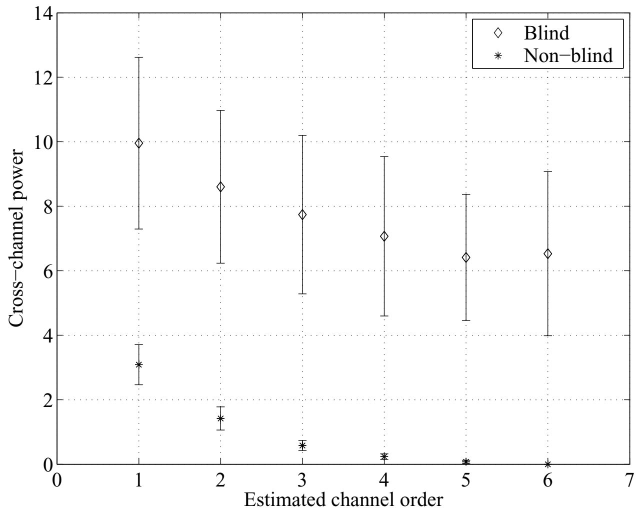

Figure 5: Cross-channel power after non-blind and blind source separation. The true channel order is 6 and $4,000$ samples are used for the blind identification.

REFERENCES

[1] S. Haykin ed. *Unsupervised adaptive filtering, Vol. 1: Blind source separation*. Wiley, 2000.

[2] D. N. Godard. Self-recovering equalization and carrier tracking in two-dimensional data communication systems. *IEEE Trans. on Communications*, pages 1867–75, Nov. 1980.

[3] T. R. Treichler and M. G. Agee. A new approach to multipath correction of constant modulus signals. *IEEE Trans. on Acoustics, Speech, and Signal Processing*, pages 459–472, April 1983.

[4] O. Shalvi and E. Weinstein. New criteria for blind deconvolution of nonminimum phase systems. *IEEE Trans. on Information Theory*, pages 312–321, March 1990.

[5] Y. Hua, S. An, and Y. Xiang. Blind identification of FIR MIMO channels by decorrelating subchannels. *IEEE Trans. on Signal Processing*, pages 1143–1155, May 2003.

[6] J. K. Tugnait. On blind MIMO channel estimation and blind signal separation in unknown additive noise. *IEEE Signal Processing Workshop on Signal Processing Advances in Wireless Communications*, pages 53–56, April 1997.

[7] J. Liang and Z. Ding. Blind MIMO system identification based on cumulant subspace decomposition. *IEEE Trans. on Signal Processing*, pages 1457–1468, June 2003.

[8] J.R. Treichler, I. Fijalkow, and C.R. Johnson Jr. Fractionally spaced equalizers. *IEEE Signal Processing Magazine*, pages 65–81, May 1996.

[9] M. Benthin. *Vergleich kohärenter und inkohärenter Codemultiplex-Übertragungskonzepte für zellulare Mobilfunksysteme (Ph.D. thesis)*. VDI Verlag, 1996.