FREQUENCY SAMPLING DESIGN OF ARBITRARY-LENGTH FILTERS FOR FILTER BANKS AND DISCRETE SUBBAND MULTITONE TRANSCEIVERS

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ABSTRACT

Recently, a new method for the design of Finite Impulse Response (FIR) prototype filters by frequency sampling for modulated filter banks was presented. In the previous work, we showed the algorithm for obtaining linear-phase prototype filters of restricted length: *N=2mM*, where *m* is an integer and *M* is related to the number of channels. Since fast algorithms of implementation for cosine-modulated filter banks have been proposed when the prototype filter length is *N=(2p+1)M*, we present the general expressions that allow us to obtain the arbitrary-length prototypes. These filters can be used for designing nearly perfect reconstruction modulated filter banks, as well as Transmultiplexers or Discrete Subband Multitone transceivers based on the above kind of filter banks. The analytical and simulation results show again that the designed system performance is extremely good.

Key words.- Channel bank filters, discrete subband multitone transceivers, transmultiplexer, modulated filter banks, filter bank design, filtering theory, frequency sampling technique.

1. INTRODUCTION

Modulated filter banks (MFBs) have been extensively studied and they are used in many applications from data compression (such as audio, image and biosignals coding) to data transmission (by means of multicarrier modulation). In general, modulated filter banks with applications to communications are generally referred to as Discrete Wavelet Multitone or Discrete Subband Multitone (DSBMT) transceivers. These transceivers have been proposed for implementing several multicarrier systems as an alternative solution to modulated transmultiplexer (TMUX) systems based on the Discrete Fourier Transform (DFT) filter bank. MFBs and DSBMT transceivers offer several advantages, such as: (a) High-selectivity and high-discrimination systems can be easily designed; (b) the resulting sub-channel filters can be generated based on the use of a single real-coefficient prototype filter; (c) in particular cases, fast algorithms can be applied for efficiently implementing the analysis and synthesis banks. For the above reasons, the design of prototype filters for MFBs is a topic that has received widespread attention in recent years. Several efficient methods have been proposed which facilitate the prototype filter design for nearly perfect reconstruction (NPR) modulated filter banks [1]-[8].

In this paper, we put forward an arbitrary-length prototype filter design technique. The proposed method is based on the frequency sampling approach for designing FIR filters [10, 11], and the magnitude response values of samples in the transition band for the prototype filter are the only parameters to be optimised. This method was initially proposed in [8] for obtaining linear-phase prototype filters of restricted length: *N=2mM*, with parameter $\alpha = 0$. Since fast algorithms of implementation for modulated filter banks when the prototype filter length is *N=(2p+1)M* have been recently proposed [9], we present the general expressions that allow us to obtain the arbitrary-length prototypes. We also include expressions with parameter $\alpha = 1/2$. The resulting MFBs and DSBMT transceivers closely satisfy the perfect reconstruction property.

2. A UNIFIED SCHEME OF MODULATION

In this section, we present a scheme of modulation by which all the analysis and synthesis (or the receiving and the transmitting) sub-channel filters can be obtained for different families of MFBs (or DSBMT transceivers).

Let $P(z) = \sum_{n=0}^{N-1} p[n] \cdot z^{-n}$ be the system function of the *N*-length real coefficients prototype filter $p[n]$ appropriately designed. We define $S_i(z)$, $0 \le i \le M-1$, as follows:

$$S_i(z) = a_i \cdot P\left(zW_{2M}^{(i+\tau)}\right) + b_i \cdot P\left(zW_{2M}^{-(i+\tau)}\right), \qquad (1)$$

The analysis (receiving) $H_i(z)$ and the synthesis (transmitting) $F_i(z)$ sub-channel filters are obtained as

$$H_i(z)=\begin{cases} S_i(z), & i \text{ even}, \\ z^{-(N-1)}\cdot \tilde{S}_i(z), & i \text{ odd}, \end{cases} \tag{2}$$

$$F_i(z)=c\cdot z^{-(N-1)}\cdot \tilde{H}_i(z). \tag{3}$$

where $\tilde{S}_i(z)=S_i(z^{-1})$. If we apply the inverse z-transform, we obtain the following expressions:

$$f_i[n]=\begin{cases} c\cdot s_i[N-1-n], & i \text{ even}, \\ c\cdot s_i[n], & i \text{ odd}, \end{cases} \tag{4}$$

$$h_i[n]=(1/c)\cdot f_i[N-1-n]. \tag{5}$$

From the above filters, we can obtain several kinds of MFBs, and therefore, different DSBMC transceivers. For example, we can obtain from the above equations the scheme of modulation corresponding to conventional PR and NPR cosine-modulated filter banks (CMFB) as follows:

$$a_i=e^{j\phi_i}, \qquad b_i=a_i^*, \tag{6}$$

$$\tau=1/2, \qquad c=M, \tag{7}$$

$$\phi_i=\frac{\pi}{4}\cdot\left(1-(2i+1)\cdot(N-1)\cdot\frac{1}{M}\right), \tag{8}$$

where $0\le i\le(M-1)$ and M is the number of channels. Operating, we get

$$s_i[n]=2\cdot p[n]\cdot\cos\left(\left(i+\frac{1}{2}\right)\frac{\pi}{M}\left(n-\frac{N-1}{2}\right)+\frac{\pi}{4}\right), \tag{9}$$

and we can rewrite expressions (4) and (5) as

$$f_i[n]=2M\cdot p[n]\cdot\cos\left((2i+1)\frac{\pi}{2M}\left(n-\frac{N-1}{2}\right)-(-1)^i\frac{\pi}{4}\right), \tag{10}$$

$$h_i[n]=2\cdot p[n]\cdot\cos\left((2i+1)\frac{\pi}{2M}\left(n-\frac{N-1}{2}\right)+(-1)^i\cdot\frac{\pi}{4}\right). \tag{11}$$

3. PROTOTYPE FILTER DESIGN

3.1 Conditions for nearly perfect reconstruction

Let $P(e^{j\omega})$ be the frequency response of the prototype filter. The conditions for approximate reconstruction in the M-channel cosine-modulated filter bank and the $2M$-channel Modified-DFT filter bank can be expressed as

$$\left|P(e^{j\omega})\right|\approx 0 \qquad |\omega|>\pi/M, \tag{12}$$

$$\left|T(e^{j\omega})\right|=\left|\sum_{i=0}^{M-1}F_i(e^{j\omega})\cdot H_i(e^{j\omega})\right|\approx 1, \tag{13}$$

where $H_i(e^{j\omega})$ and $F_i(e^{j\omega})$ are, respectively, the analysis and synthesis filters obtained by modulation from $P(e^{j\omega})$.

3.2 FIR filter design by frequency sampling technique

Let $P[k]\equiv P(e^{j\omega_k})$, where $\omega_k=(k+\alpha)\cdot 2\pi/N$, $0\le k\le(N-1)$, and $\alpha=0$ or $\alpha=1/2$, be the samples of the frequency response at N points uniformly spaced in the interval $[0,2\pi)$. The relation between the impulse response coefficients and the frequency response samples is given by

$$p[n]=\frac{1}{N}\sum_{k=0}^{N-1}P(e^{j\omega_k})\cdot e^{j\frac{2\pi}{N}(k+\alpha)\cdot n}, \tag{14}$$

where $0\le n\le(N-1)$. For simplicity, we consider that $p[n]$ is real and symmetric, i.e.: $p[n]=p[N-1-n]$. In this case, the filter coefficients can be obtained using the following equations (see ref. [11]). If $\alpha=0$, we get:

$$p[n]=\frac{1}{N}\left\{H[0]+2\cdot\sum_{k=1}^{U}H[k]\cdot\cos\left((n+1/2)\cdot\frac{2\pi k}{N}\right)\right\}, \tag{15}$$

where $H[k]=P[k]\cdot e^{-jk\pi/N}$ and $U=\lfloor N/2\rfloor-1$ ($\lfloor x\rfloor$ and $\lceil x\rceil$ denote rounding to the next larger or smaller integer, respectively). If $\alpha=1/2$, the filter coefficients can be obtained as

$$p[n]=\frac{2}{N}\cdot\sum_{k=0}^{U}H[k]\cdot sen\left((n+1/2)\cdot\frac{2\pi}{N}\cdot(k+1/2)\right), \tag{16}$$

where $H[k]=P[k]\cdot e^{j(N-2k-1)\pi/(2N)}$.

3.3 Initial Specifications

The initial values of $P[k]$ must be defined before the optimising process starts. Previously, it is important to point out that the number of samples in the transition band must be selected $(L\ge 1)$, and its centre (transition band) must be situated at approximately the frequency $\pi/(2M)$. The closest value to $\omega_c=\pi/(2M)$ is defined as $\omega_r=(r+\alpha)\cdot 2\pi/N$, $r\in Z^+$.

Therefore, given L, r and N, the values of the magnitude response $|P[k]|$ and the phase response $\arg\{P[k]\}$ must be defined as given below in eqs. (17) and (18).

Function $f(k)$ allow us to obtain the values of the transition band samples. It must be carefully chosen in order to ensure the fast convergence of the algorithm to optimum solutions. In [8], we used

$$f(k)=\left(\frac{\omega_s-k\cdot 2\cdot\pi/N}{\omega_s-\omega_p}\right), \tag{19}$$

$$|P[k]|=\begin{cases} 1, & 0\le k\le r-\lceil L/2\rceil, \ \text{(passband)} \\ f(k), & r-\lceil L/2\rceil+1\le k\le r+\lfloor L/2\rfloor, \ \text{(transition band)} \\ 0, & r+\lfloor L/2\rfloor+1\le k\le\lfloor(N-1)/2\rfloor, \ \text{(stopband)} \end{cases} \tag{17a}$$

$$|P[N-1-k]|=|P[k]|, \qquad \lfloor(N-1)/2\rfloor+1\le k\le(N-1), \tag{17b}$$

$$\arg\{P[k]\}=\begin{cases} -\frac{(N-1)}{2}\cdot\frac{2\pi}{N}\cdot(k+\alpha), & 0\le k\le\lfloor(N-1)/2\rfloor, \\ \frac{(N-1)}{2}\cdot\frac{2\pi}{N}\cdot(N-(k+\alpha)), & \lfloor(N-1)/2\rfloor+1\le k\le(N-1). \end{cases} \tag{18}$$

where ω_p is the frequency corresponding to the last sample of the passband and ω_s is the frequency corresponding to the first sample of the stopband. For the design examples explained in Section 4, we have chosen

$$f(k) = 0.95 - \left(\frac{\omega_s - (L+1-k) \cdot 2 \cdot \pi / N}{\omega_s - \omega_p} \right)^2 . \quad (20)$$

3.4 Optimization Procedure

The optimization procedure consists of the following steps:

1. Select the filter length *N*.
2. Select the required number *L* of samples in the transition band.
3. Initialize the *N* samples of the frequency response as described in the precedent subsection (eqs (17) and (18)). The resulting vector $|\mathbf{P}|$, with corresponding magnitude response initial values $|P[k]|$, is

$$|\mathbf{P}| = \Big[\underbrace{1 \;\; 1 \;\; \cdots \;\; 1}_{passband} \;\; \underbrace{l[1] \;\; \cdots \;\; l[L]}_{transition\ band} \;\; \underbrace{0 \;\; \cdots \;\; 0}_{stopband} \Big], \quad (21)$$

   where $0 \le k \le \lfloor (N-1)/2 \rfloor$.

4. Let $\mathbf{l} = [l[1] \;\; l[2] \;\; \cdots \;\; l[L]]$ be the vector whose elements are the samples of the magnitude response at the transition band. Find $\mathbf{l}_{\mathbf{opt}}$, i.e., determine the values of the components of $\mathbf{l}$ that minimize the objective function ψ defined as [4]

$$\psi = \max_{n, n \neq 0} |g[2Mn]|, \quad (22)$$

   where $G(e^{j\omega})$ is the discrete-time Fourier Transform of $g[n]$, defined as $G(e^{j\omega}) = |P(e^{j\omega})|^2$.

5. Calculate the optimum values of the frequency response samples $P_{opt}[k]$. These values are obtained from (17) and (18), by replacing the initial values of the magnitude response in the transition band in (17) by the optimised values $\mathbf{l}_{\mathbf{opt}}$ obtained in the previous step:

$$|P_{opt}[k]| = \Big[\underbrace{1 \;\; 1 \;\; \cdots \;\; 1}_{passband} \;\; \underbrace{l_{opt}[1] \;\; \cdots \;\; l_{opt}[L]}_{transition\ band} \;\; \underbrace{0 \;\; \cdots \;\; 0}_{stopband} \Big],$$

$$0 \le k \le \lfloor (N-1)/2 \rfloor . \quad (23)$$

6. Using $P_{opt}[k]$, obtain the prototype filter coefficients $p[n]$ through equation (14) (or using eq. (15) or (16), as appropriate).

The objective function ψ to be minimized in **step 4** can be defined as in [3], [4] or by defining a different cost function that yields a prototype filter that is a spectral factor of an approximately 2*M*th band filter.

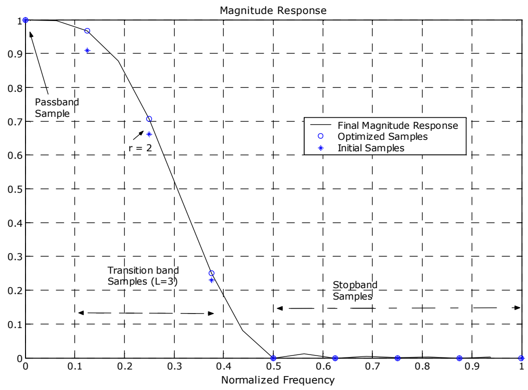

Fig. 1. Prototype filter magnitude response $|P(e^{j\omega})|$ designed using the proposed technique.

We would like to emphasize that since few number of samples *L* in the transition band are taken and the optimization is only performed on these samples, the computational complexity of the proposed algorithm is very low. Fig. 1 depicts an example of distribution of samples (including initial and optimized values), as well as the final magnitude response for a prototype filter designed with the proposed technique.

4. DESIGN EXAMPLES

In this section, we provide two examples to demonstrate our results. The minimization algorithm has been implemented using the MATLAB Optimization Toolbox.

Example 1- In this example a conventional cosine-modulated filter bank is designed with *M=32* and a *480*-length prototype filter. We have chosen the following design parameters: $L = 6$, $r = 3$, and $\alpha = 0$. The results are the following: The peak amplitude distortion is $R_{pp} = 5.23 \cdot 10^{-4}$ and the maximum aliasing error is $E_a = 1.49 \cdot 10^{-4}$ (defined as in [1]). The peak signal-to-noise ratio (PSNR), obtained after applying a random signal as input to the filter bank, is $PSNR = 85.54\,\text{dB}$. Table I shows the magnitude response for the frequencies $\omega_k = k \cdot 2\pi/480$, $0 \le k \le 240$, corresponding to the *480*-length prototype filter designed with the proposed technique.

Example 2- In the second example, we include the results obtained when designing a *1152*-length prototype filter for a *128*-channel cosine-modulated filter bank. The design parameters are: $L = 3$, $r = 2$, and $\alpha = 0$. The resulting filter bank shows a peak amplitude distortion $R_{pp} = 1.499 \cdot 10^{-5}$, a maximum aliasing error $E_a = 8.3124 \cdot 10^{-6}$, and a peak signal-to-noise ratio $PSNR = 67.85\,\text{dB}$, also obtained after applying a random signal as input to the filter bank. Table II shows the optimised samples for the magnitude response (for the frequencies $\omega_k = k \cdot 2\pi/1152$, $0 \le k \le 576$).

Both prototype filters (corresponding to example 1 and example 2) can also be used for designing other kind of efficient filter banks. For example, we can design *64* or *256*-channel Modified-DFT filter banks with the first and second prototype filters, respectively [12-14].

TABLE I

EXAMPLE 1. MAGNITUDE RESPONSE SAMPLES OF THE OPTIMIZED SYMMETRIC PROTOTYPE FILTER.

k	$\left\|P\left(e^{jk2\pi/480}\right)\right\|$
0	1
1	0.99957240722059
2	0.97651856300809
3	0.85986315009771
4	0.64624283821526
5	0.36088356829187
6	0.09807130140825
$7 \le k \le 240$	0

TABLE II

EXAMPLE 2. MAGNITUDE RESPONSE SAMPLES OF THE OPTIMIZED SYMMETRIC PROTOTYPE FILTER.

k	$\left\|P\left(e^{jk2\pi/1152}\right)\right\|$
0	1
1	0.99852489379533
2	0.82584465174373
3	0.26906322304657
$4 \le k \le 576$	0

5. CONCLUSIONS

The paper describes a simple frequency sampling technique for obtaining FIR prototype filters for nearly PR modulated filter banks, transmultiplexers or discrete subband multitone transceivers. We have presented the general expressions: arbitrary-length and $\alpha = 0$, $\alpha = 1/2$. The algorithm is rather simple, and the only parameters to be numerically optimised are the values of the magnitude response in the transition band. The proposed algorithm renders good results as the resulting filter banks have characteristics very close to the PR property. This is because the algorithm converges to optimum solutions when both the cost function to be optimised and the number of samples in the transition band are properly chosen.

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