RECONSTRUCTION OF DAMAGED IMAGES USING RADIAL BASIS FUNCTIONS

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ABSTRACT

The Radial Basis Function method (RBF) can be used not only for reconstruction of a surface from scattered data but for reconstruction of damaged images, filling gaps and for restoring missing data in images, too. The basic idea of reconstruction algorithm with RBF and very interesting results from reconstruction of images damaged by noise are presented. Feasibility of the RBF method for image processing is demonstrated.

1. INTRODUCTION

One of the most interesting problems in image processing is how to reconstruct damaged or incomplete images as well as possible. This problem is referred to in many papers [1, 7]. The main question is: "What value was in a corrupted position and how can I restore it ?". One of the conditions for solving this is to have as much information as possible from the original image. Then methods exist that use this and tray to reconstruct information in gaps [1]. The amount of retained information from the original picture is very important and the quality of the result depends on it.

The Radial Basis Function method (RBF) is based on the variational implicit functions principle and can be used for interpolation of scattered data, see section 3 for details. The possibility of missing data restoration (image inpainting) by the RBF method was mentioned in Kojekine & Savchenko [8]. They used this method for surface retouching and marginally for image inpainting as well. They used compactly supported radial basis functions (CSRBF)[14] for reconstruction and used octree data structure for representation of the parts for reconstruction. The advantage of this method is that the linear system is sparse and can be solved easily [10]. The drawback of this approach is an error which can be obtained with an improper selection of the radius of support of the CSRBF functions see Fig. 11 and Table 2.

In our work we address a global basis function for image reconstruction and reconstruction with constant "window" size. A description of the method is presented in section 4.

2. PROBLEM DEFINITION

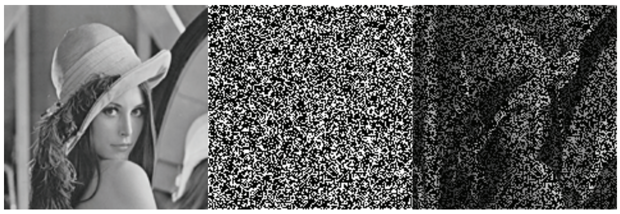
Let us assume that we have an image Ω with resolution $M \, x \, N$ and pixel intensity is presented by the value h.

$$\Omega = \Omega_c + \Omega_i \,, \; \Omega_c \cap \Omega_i = \varnothing \,, \tag{1}$$

$$p = h[x, y] \,, \; p \in \Omega \,, \tag{2}$$

$$x = 0,\dots,M-1 \,, \; y = 0,\dots,N-1$$

Some pixels of the image Ω have incorrect values (missing or overwritten), see Fig. 1c. Thus the image Ω has two parts, one with correct pixel values Ω_c and the second with incorrect pixels values Ω_i, Eq. 1. We would like to restore the original image. Let us assume that we can detect incorrect pixels, too. The "Lena" image [15] (Fig. 1a) corrupted with noise mask (Fig. 1b) was used for explanation of our approach.



a *b* *c*

FIG. 1. *Original image, mask with corrupted pixels and the final image prepared for reconstruction.*

Note that the restoration of the original image is related to the scattered data interpolation problem, as many points are not defined and we want to find a value for them. The mask was randomly generated with a normal noise distribution.

3. RADIAL BASIS FUNCTIONS

Let us describe the RBF method now. The RBF method may be used to interpolate a smooth function given by n points. The resulting RBF interpolating function is defined as [5]:

$$f(\mathbf{x}) = \sum_{i=1}^{n} \lambda_i \phi(\| \mathbf{x} - \mathbf{c}_i \|) + P(\mathbf{x}) \,, \tag{3}$$

$$P(\mathbf{x}) = a c_i^x + b c_i^y + d$$

where $\mathbf{c_i} = [c_i^x, c_i^y]^T$ are coordinates, n is a number of input points (pixels), λ_i are unknown weights, $\mathbf{x}$ is a particular point, $\phi(\|\mathbf{x}-\mathbf{c}_i\|)$ is a radial basis function, $\|\mathbf{x}-\mathbf{c}_i\|=r_i$ is the Euclidean distance of the point $\mathbf{x}$ and the given point $\mathbf{c_i}$., and $P(\mathbf{x})$ is a polynomial of degree m depending on the choice of ϕ ($m = 1$ has been chosen). There are some popular choices for the basis function, e.g. the thin-plate spline $\phi(r) = r^2 * \log(r)$, the Gaussian $\phi(r) = \exp(-\xi r^2)$, the multiquadric $\phi(r) = \sqrt{(r^2+\xi^2)}$, biharmonic $\phi(r) = |r|$ and triharmonic $\phi(r) = |r|^3$ splines, where ξ is a parameter [6]. For those functions the degree m of the polynomial $P(\mathbf{x})$ can be chosen as $m = 1$ as well.

As we have for all points $\mathbf{c_i}$ associated value h_i we have a linear system of equations Eq. 3. with unknowns $\lambda_1, \dots, \lambda_n, a, b, d$. Natural additional constraints for the coefficients λ_i must be included, Eq. 4., to ensure orthogonality of a solution. These equations and constraints determine the linear system:

$$\sum_{i=1}^{n} \lambda_i c^x = \sum_{i=1}^{n} \lambda_i c^y = \sum_{i=1}^{n} \lambda_i = 0 \tag{4}$$

$$\mathbf{B}\begin{bmatrix}\lambda \\ \mathbf{a}\end{bmatrix} = \begin{bmatrix}\mathbf{h} \\ \mathbf{0}\end{bmatrix}, \text{ where } \mathbf{B} = \begin{bmatrix}\mathbf{A} & | \mathbf{P} \\ \mathbf{P}^T & | \mathbf{0}\end{bmatrix} \tag{5}$$

$$\text{and } \mathbf{P} = \begin{bmatrix} c_1^x & c_1^y & 1 \\ \vdots & \vdots & \vdots \\ c_n^x & c_n^y & 1 \end{bmatrix},$$

$$\mathbf{A}_{i,j} = \phi(\| \mathbf{c}_i - \mathbf{c}_j \|), \quad i,j = 1,\dots,n$$

$$\mathbf{a} = [a,b,d]^T, \lambda = [\lambda_1, \lambda_2, ..., \lambda_n]^T, \mathbf{h} = [h_1, h_2, ..., h_n]^T$$

$$\mathbf{0} = [0,0,0]^T$$

The polynomial $P(\mathbf{x})$ in Eq. 3. ensures positive-definiteness of matrix $\mathbf{B}$, see [9]. Afterwards, the linear equation system Eq. 5. is solved and the solution vector with $\boldsymbol{\lambda}$ and $\mathbf{a}$ is known, the function $f(\mathbf{x})$ can be evaluated for an arbitrary point $\mathbf{x}$ (a pixel position in our case), see [4, 9, 11, 12].

4. IMAGE RECONSTRUCTION

We have given the corrupted image Ω, see Eq. 1, and we would like to restore part Ω_i of the original image Ω. The algorithm for reconstruction of incorrect pixels is based on a construction and solution of the linear equation system (Eq. 5.) for a part of the given image. The part of the image "window" is selected in "scan-line" manner. The proposed algorithm processes the k-neighborhood of current pixel p. Note that in our case k is equal to 24, see Fig. 2.

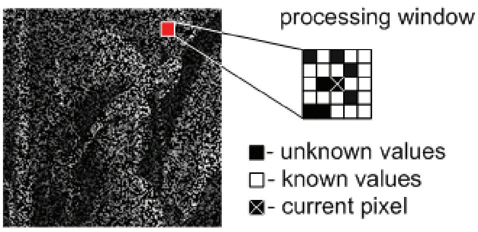


FIG. 2. *Definition of the processing window for current pixel.*

The correct pixels are selected from the window. These pixels are used as an input for the RBF method. The RBF function was computed and used to compute the value of the incorrect pixel. This is repeated for incorrect pixels values. The RBF function differs from case to case as in the window several pixels might be missing. Now we can specify the proposed approach written in pseudo code, see Algorithm 1.

If there are too many incorrect pixel values in the specified window of the size 5 x 5 pixels, then the incorrect pixel is not restored and the algorithm continues, see Fig. 4 and Fig. 6. The incorrect pixels are restored in the next iteration. It is the first modification of the algorithm mentioned above.

The second modification reconstruct the incorrect pixels in the scan-line algorithm and the direction of restoration is changed, see Fig. 5 and Fig. 7.

```
LoadImage(Ω);
DefineNeighborhood(5,5);
Repeat
 For (i,j=1;i<=M,j<=N;i++,j++)
 {
   /* pixel [i,j] is not defined */
   if(Hole(i,j)){
   K = SelectNeighborhoodOfPixel(i,j);
   /* remove all incorrect pixels */
   DeleteHoles(K);
   CreateSystemAndSolveIt(K);
   Pixel[i,j] = ComputeValueFromSystem(i,j);
 }
Until (all pixels reconstructed)
```

ALGORITHM 1.

5. RESULTS

The LU factorization method was used for solving the linear system (Eq. 5.) of equations. The RBF functions used in our experiments are shown in Table 2. The LU factorization is stable but different methods can be used, too.

5.1 Evaluation methods

For evaluation of the results we used Mean Absolute Error (MAE) and Mean Square Error (MSE) methods defined by following criterion:

$$S^k = \left(\sum_{i=1}^{M} \sum_{j=1}^{N} \left| (\Omega_1(i,j) - \Omega_2(i,j)) \right|^k \right) \cdot \frac{1}{T} \tag{6}$$

where: Ω_1 is the original image (without incorrect pixels), Ω_2 is the reconstructed image and T is the number of all image pixels or the number of incorrect pixels, see Table 1. For $k = 1$ is Eq.6 MAE and for $k = 2$ is it MSE. For the value S^k only a part of the image for evaluation was used (Fig. 3a) because in the nearest boundary of the image border an error is accumulated. For the visualization of the difference between the original image and the reconstructed image, see Fig. 3b and Fig. 3c. The problematic parts of the reconstruction can be seen very well on these pictures.

a *b* *c*

FIG. 3. *The example of the selected area for S evaluation (a) and the example of B/W differential image (b,c is negative of b).*

5.2. Scan-line algorithm

We used several corrupted images for our experiments, only the "Lena" image [15] and "Mars" [16] examples are presented here. For results of the one side scan-line algorithm reconstruction of the corrupted image are presented on Fig. 4 and Fig. 6.

a *b* *c*

FIG. 4. *The Lena image with 60% of incorrect pixels (a), intermediate step of the algorithm (b) and the result (c).*

a *b* *c*

FIG. 5. *The Lena image with 60% of incorrect pixels (a), intermediate step of the algorithm (b) and the result of two side scan-line algorithm(c).*

For an evaluation of corrupted image with two sided scan-line method, see Fig. 5., less number of iterations is required. Fig. 5. and Fig. 7. present reconstructed images. Table 1. shows the statistical data. It can be seen that the reconstructed images are usually better as well. Also number of iterations N is smaller.

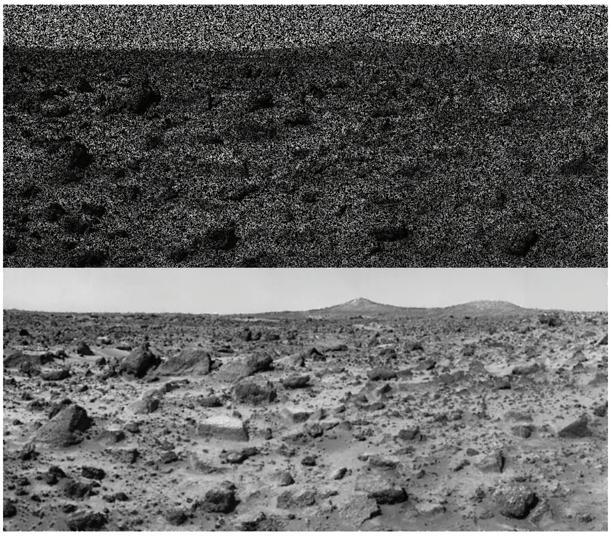


FIG. 6. *The Mars image with 60% of incorrect pixels (top) and the result after reconstruction (bottom).*

The Table 1. presents results obtained for the case when only 60 % of pixels are left from the original image. This table shows the difference between one and the two side scan-line algorithm.

		$T \triangleq$ **Number of all pixels**		$T \triangleq$ **Number of incorrect pixels**		
Method	IM	**S**	**S^2**	**S**	**S^2**	**N**
One-side	Mars	6.72	169.28	11.14	280.30	248
	Lena	4.28	92.63	6.99	151.43	55
Two-side	Mars	6.72	169.14	11.13	280.07	124
	Lena	4.26	91.15	6.96	149.00	28

TABLE 1. Results for 60% of incorrect pixels.

FIG. 7. *The result of two side scan-line algorithm.*

With the increasing number of incorrect pixels the values of S and S^2 increase, too. In the case of the highest percentage of incorrect pixels in the window could happen that there are just few original pixels in the window and the matrix **B** can be singular. Therefore the pixel value of the incorrect pixel is computed in the next cycle. The graph in Fig. 8. shows how S error increases for different percentage of incorrect pixels.

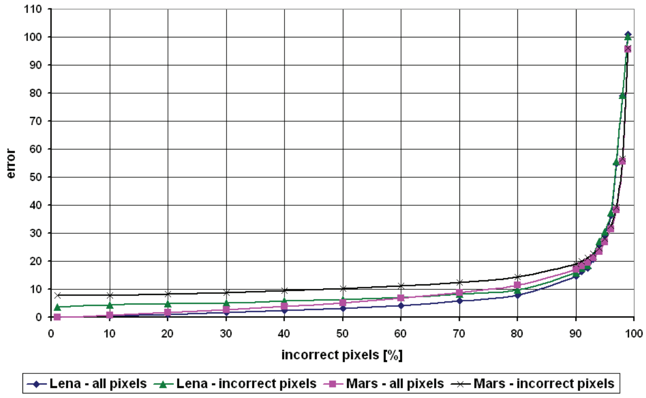


FIG. 8. *The Lena and the Mars image S error comparison.*

You can see that for more than 90% of incorrect pixels the value S has an acceptable value and the result after the reconstruction is still good. The object at the image is still recognizable, see Fig. 9 and Fig. 10.

FIG. 9. *The result of 90% corrupted images reconstruction.*

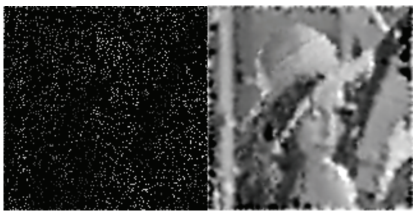


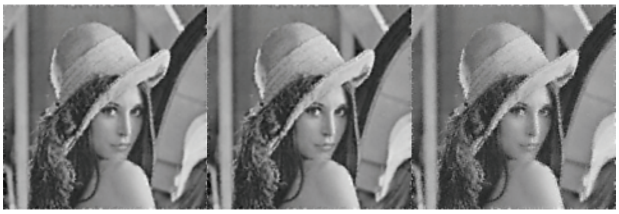
FIG. 10. *The Lena image with 90% of incorrect pixels (left) and the result after reconstruction (right).*

5.3. Different basis functions and the window size

Finally we compared different basis functions in order to check the quality of the reconstruction. Results for different basis functions are summarized in Table 2. and Fig. 11.

	Func.	$r^2 \log r$	r^3	$(1-r)^4(4r+1)$ CSRBF
all pixels	**S**	3.19	3.21	4.11
	S²	62.23	63.69	94.13
corr.pixels	**S**	6.24	6.27	8.05
	S²	121.72	124.56	184.10
Figure		11a	11b	11c

TABLE 2. *Results obtained for different basis functions.*

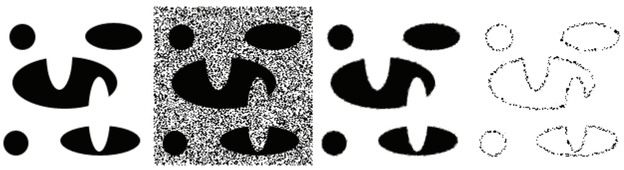


a *b* *c*

FIG. 11. *Results from reconstruction with different basis function.*

6. CONCLUSION AND FUTURE WORK.

It can be seen that the proposed method has a good property, see Fig. 12., where an error of our scan-line reconstruction method is presented. The problem is with edges. Sharp edges raise higher errors in the reconstructed image. Therefore the proposed method is less convenient for images with sharp edges. It is expected that this problem will be solved in future work and the resulting error of the reconstruction will be smaller as well.



a *b* *c* *d*

FIG. 12. *Sharp edges reconstruction: a) the original image, b) the corrupted image, c) the reconstructed image and d) is the differential image of a) and c).*

It can be seen, that the results obtained by RBF application are very good. The preliminary experiments also proved that the RBF interpolation and the presented approach can be used for inpainting problem solution as well [1].

The proposed method for image reconstruction could be helpful for restoration of old scratched images [8] or inpainted images [1].

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