

# USING A SINES + WAVELETS MIXED DICTIONARY FOR IMPROVING MATCHING PURSUIT-BASED PARAMETRIC AUDIO CODING

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## ABSTRACT

The proposed method improves the matching pursuit (MP) algorithm by using a mixed dictionary of sines and wavelets. The MP algorithm is applied to the signal  $x[n]$  to find the best approximation  $r^m[n]$  in the dictionary  $D$ . The proposed method uses a mixed dictionary  $D$  of sines and wavelets. The MP algorithm is applied to the signal  $x[n]$  to find the best approximation  $r^m[n]$  in the dictionary  $D$ . The proposed method uses a mixed dictionary  $D$  of sines and wavelets. The MP algorithm is applied to the signal  $x[n]$  to find the best approximation  $r^m[n]$  in the dictionary  $D$ .

## 1. INTRODUCTION

The proposed method improves the matching pursuit (MP) algorithm by using a mixed dictionary of sines and wavelets. The MP algorithm is applied to the signal  $x[n]$  to find the best approximation  $r^m[n]$  in the dictionary  $D$ . The proposed method uses a mixed dictionary  $D$  of sines and wavelets. The MP algorithm is applied to the signal  $x[n]$  to find the best approximation  $r^m[n]$  in the dictionary  $D$ . The proposed method uses a mixed dictionary  $D$  of sines and wavelets. The MP algorithm is applied to the signal  $x[n]$  to find the best approximation  $r^m[n]$  in the dictionary  $D$ .

At this point we can conclude that the proposed method improves the matching pursuit (MP) algorithm by using a mixed dictionary of sines and wavelets. The MP algorithm is applied to the signal  $x[n]$  to find the best approximation  $r^m[n]$  in the dictionary  $D$ .

## 2. SPARSE APPROXIMATIONS: MATCHING PURSUIT

Let  $x[n]$  be a signal and  $D = \{g_i[n] ; i = 0, 1, \dots, L\}$  be a dictionary of atoms  $g_i[n]$ . The matching pursuit (MP) algorithm finds the best approximation  $r^m[n]$  of  $x[n]$  in the dictionary  $D$ , where  $m$  is the number of atoms used.

$$r^m[n] = r^{m+1}[n] + \alpha_{i(m)} \cdot g_{i(m)}[n] \quad m \geq 1 \quad (1)$$

where  $\alpha_{i(m)}$  is the coefficient of the atom  $g_{i(m)}[n]$  in the approximation  $r^m[n]$ . The MP algorithm finds the best approximation  $r^m[n]$  of  $x[n]$  in the dictionary  $D$ , where  $m$  is the number of atoms used.

$$\alpha_i^m = \frac{\langle r^m[n], g_i[n] \rangle}{\langle g_i[n], g_i[n] \rangle} = \frac{\langle r^m[n], g_i[n] \rangle}{\|g_i[n]\|^2} = \langle r^m[n], g_i[n] \rangle \quad (2)$$

The MP algorithm finds the best approximation  $r^m[n]$  of  $x[n]$  in the dictionary  $D$ , where  $m$  is the number of atoms used. The MP algorithm finds the best approximation  $r^m[n]$  of  $x[n]$  in the dictionary  $D$ , where  $m$  is the number of atoms used.

$$g_{i(m)}[n] = \arg \min_{g_i \in D} \|r^{m+1}[n]\|^2 \quad (3)$$

From (1) and (3) we can write:

$$g_{i(m)}[n] = \arg \min_{g_i \in D} |\alpha_i^m| \quad (4)$$

As a result, the MP algorithm finds the best approximation  $r^m[n]$  of  $x[n]$  in the dictionary  $D$ , where  $m$  is the number of atoms used. The MP algorithm finds the best approximation  $r^m[n]$  of  $x[n]$  in the dictionary  $D$ , where  $m$  is the number of atoms used.

$$\langle r^{m+1}[n], g_i[n] \rangle = \langle r^m[n], g_i[n] \rangle - \alpha_{i(m)} \cdot \langle g_{i(m)}[n], g_i[n] \rangle \quad (5)$$

On the other hand, the MP algorithm finds the best approximation  $r^m[n]$  of  $x[n]$  in the dictionary  $D$ , where  $m$  is the number of atoms used. The MP algorithm finds the best approximation  $r^m[n]$  of  $x[n]$  in the dictionary  $D$ , where  $m$  is the number of atoms used.

### 3. MATCHING PURSUIT WITH A MIXED DICTIONARY OF SINES + WAVELETS

Let  $e_i[n]$  and  $w_i[n]$  be the elements of the dictionaries  $D_e$  and  $D_w$ .

$$\begin{aligned} \langle e_{i(m)}^*[n], e_i[n] \rangle &= \frac{1}{N} \cdot \sum_{n=0}^{2L-1} e^{-j \frac{2\pi(i+m)}{2L} n} \\ &= \frac{1}{N} \cdot U[\ell(-i(m)) \bmod L] \end{aligned} \quad (12)$$

Let  $x[n]$  be the signal to be processed. The goal is to find the coefficients  $\alpha_i^m$  and  $\alpha_{\{s,p,k\}}^m$  such that the reconstructed signal  $r^m[n]$  is as close as possible to  $x[n]$ .

The signal  $x[n]$  is processed by the L-FFT to obtain  $X[i]$ . The signal  $x[n]$  is then processed by the L-IFT to obtain  $r^m[n]$ . The signal  $x[n]$  is then processed by the L-IFT to obtain  $r^m[n]$ .

#### 3.1 Implementation with sets of complex exponentials

Let  $e_i[n]$  be the elements of the dictionary  $D_e$ . The goal is to find the coefficients  $\alpha_i^m$  such that the reconstructed signal  $r^m[n]$  is as close as possible to  $x[n]$ .

$$e_i[n] = \frac{1}{\sqrt{N}} \cdot e^{j \frac{2\pi}{2L} n}; \quad i=0, \dots, L-1; n=0, \dots, N-1 \quad (6)$$

Let  $r^m[n]$  be the reconstructed signal. The goal is to find the coefficients  $\alpha_i^m$  such that the reconstructed signal  $r^m[n]$  is as close as possible to  $x[n]$ .

$$\begin{aligned} r^{m+1}[n] &= r^m[n] - \alpha_{i(m)} e_{i(m)}[n] - \alpha_{i(m)}^* e_{i(m)}^*[n] \\ &= r^m[n] - 2\text{Re}\{\alpha_{i(m)} e_{i(m)}[n]\} \end{aligned} \quad (7)$$

Let  $e_i[n] \in D_e$ ,  $\alpha_i^m$ ,  $i=0, 1, \dots, L-1$ .

$$\alpha_i^m = \frac{\langle r^m[n], e_i[n] \rangle - \langle r^m[n], e_i[n] \rangle^* \cdot \langle e_i^*[n], e_i[n] \rangle}{1 - |\langle e_i^*[n], e_i[n] \rangle|^2} \quad (8)$$

Let  $\alpha_{i(m)}^m$ ,  $e_{i(m)}[n] \in D_e$ .

$$\begin{aligned} \langle r^{m+1}[n], g_i[n] \rangle &= \langle r^m[n], g_i[n] \rangle - \\ &= \alpha_{i(m)} \langle g_{i(m)}^*[n], g_i[n] \rangle - \\ &= \alpha_{i(m)}^* \langle g_{i(m)}^*[n], g_i[n] \rangle \end{aligned} \quad (9)$$

Let  $e_i[n] \in D_e$ .

$$\langle x[n], e_i[n] \rangle = \frac{1}{\sqrt{N}} \cdot \sum_{n=0}^{2L-1} x[n] \cdot e^{-j \frac{2\pi}{2L} n} = \frac{1}{\sqrt{N}} \cdot X[i] \quad (10)$$

Let  $X[i]$  be the L-FFT of  $x[n]$ . Let  $e_i[n] \in D_e$ .

$$\begin{aligned} \langle e_{i(m)}[n], e_i[n] \rangle &= \frac{1}{N} \cdot \sum_{n=0}^{2L-1} e^{-j \frac{2\pi(i-m)}{2L} n} \\ &= \frac{1}{N} \cdot U[\ell(-i(m)) \bmod L] \end{aligned} \quad (11)$$

#### 3.2 Implementation with sets of wavelet functions

Let  $w_{\{s,p,k\}}[n]$  be the elements of the dictionary  $D_w$ . The goal is to find the coefficients  $\alpha_{\{s,p,k\}}^m$  such that the reconstructed signal  $r^m[n]$  is as close as possible to  $x[n]$ .

$$\begin{aligned} D &= D_e + D_w \\ P &= \{s, p, k\} \\ D_w &= \{w_{\{s,p,k\}}[n]\} \end{aligned}$$

Let  $\alpha_{\{s,p,k\}}^m$ ,  $w_{\{s,p,k\}}[n]$ .

$$\alpha_{\{s,p,k\}}^m = \langle r^m[n], w_{\{s,p,k\}}[n] \rangle \quad (13)$$

$$w_{\{s,p,k\}}[n] = w_{\{s,p\}}[n - 2^p k] \quad (14)$$

Let  $\alpha_{\{s,p,k\}}^m$ ,  $w_{\{s,p,k\}}[n]$ .

$$w_{\{s,p,k\}}[n] = \sum_{w_{\{s,p,k\}} \in D_w} |\alpha_{\{s,p,k\}}^m| \quad (15)$$

Let  $\langle x[n], w_{\{s,p,k\}}[n] \rangle$  and  $\langle w_{\{s_1,p_1,k_1\}}[n], w_{\{s_2,p_2,k_2\}}[n] \rangle$ .

$$\langle w_{\{s_1,p_1,k_1\}}[n], w_{\{s_2,p_2,k_2\}}[n] \rangle = \begin{cases} \delta[k_2 - k_1] & s_1 = s_2, \\ 0 & p_1 = p_2 \\ & s_2 \neq \lfloor \frac{s_1}{2^{p_1-p_2}} \rfloor \\ w_{\{s,p\}}[k_2 - 2^p k_1] & s_2 = \lfloor \frac{s_1}{2^{p_1-p_2}} \rfloor \end{cases} \quad (16)$$

Let  $p = p_1 - p_2$ ,  $s = ((s_1)_{2^p})$  and  $p_1 \geq p_2$ .

#### 3.3 Implementation with the mixed dictionary

Let  $D = D_e + D_w$ ,  $\alpha_i^m$ ,  $\alpha_{\{s,p,k\}}^m$ ,  $\{e_i[n], w_{\{s,p,k\}}[n]\}$ .

Let  $\alpha_i^m$ ,  $\alpha_{\{s,p,k\}}^m$ ,  $\{e_i[n], w_{\{s,p,k\}}[n]\}$ .

$$D = D_e + D_w \text{ is } L \\ e_i[n] \in D_e; \quad 2)$$

### 3.3.1 Correlation between two complex exponentials.

Consider the L-FFT. The signal  $e_i[n] \in D_e$  is  $(1, 1)$  and  $(1, 2)$  is 1. The signal  $e_i[n]$ ,  $i(9)$  is

### 3.3.2 Correlation between a complex exponential and a wavelet function when the complex exponential is chosen.

In this case the signal  $e_i[n]$  is  $(7)$ . The signal  $e_i[n]$  is  $(9)$ . The L-FFT is

$$\langle e_{i(m)}[n], w_{\{s,p,k\}}[n] \rangle = \sum_{n=0}^{N-1} \frac{1}{\sqrt{N}} e^{j \frac{2\pi}{2L} (m)n} w_{\{s,p,k\}}[n] = \frac{1}{\sqrt{N}} W_{\{s,p,k\}}^*[i(m)] \quad (17)$$

by  $W_{\{s,p,k\}}[i(m)]$  is the L-FFT of  $w_{\{s,p,k\}}[n]$ . The L-FFT of  $w_{\{s,p,k\}}[n]$  is  $w_{\{s,p,k\}}[n] = \frac{i(m)}{2L}$ . The L-FFT of  $w_{\{s,p,k\}}[n]$  is  $w_{\{s,p,k\}}[n] = \frac{i(m)}{2L}$ .

### 3.3.3 Correlation between two wavelet functions.

Consider the signal  $w_{\{s,p,k\}}[n] \in D_w$  is  $(16)$ . The signal  $w_{\{s,p,k\}}[n]$  is  $(16)$ ,  $(16)$ ,  $(16)$ .

### 3.3.4 Correlation between a complex exponential and a wavelet function when the wavelet function is chosen.

The signal  $w_{\{s,p,k\}}[n]$  is  $(16)$ . The signal  $w_{\{s,p,k\}}[n]$  is  $(16)$ .

$$\langle w_{\{s,p,k\}}[n], e_i[n] \rangle = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} w_{\{s,p,k\}}[n] e^{-j \frac{2\pi}{2L} n} = \frac{1}{\sqrt{N}} W_{\{s,p,k\}}[i] \quad (18)$$

by  $W_{\{s,p,k\}}[i]$  is the L-FFT of  $w_{\{s,p,k\}}[n]$ . The L-FFT of  $w_{\{s,p,k\}}[n]$  is  $w_{\{s,p,k\}}[n] = w_{\{s,p,k\}}[n - 2^p k]$ .

The signal  $w_{\{s,p,k\}}[n]$  is  $(1)$ . The signal  $w_{\{s,p,k\}}[n]$  is  $(1)$ . The signal  $w_{\{s,p,k\}}[n]$  is  $(1)$ .

## 4. EXPERIMENTAL RESULTS

The signal  $w_{\{s,p,k\}}[n]$  is  $(1)$ . The signal  $w_{\{s,p,k\}}[n]$  is  $(1)$ . The signal  $w_{\{s,p,k\}}[n]$  is  $(1)$ .

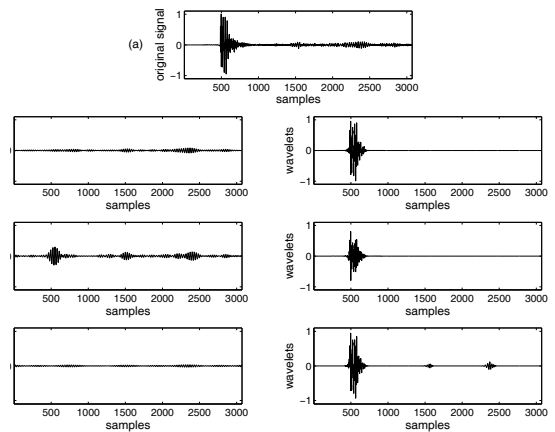


Fig : (a) Audio fragment containing a signal onset. (b) Sinusoids and wavelets extracted from the audio frame by the first approach. (c) The same components extracted by the second approach. (d) Idem by the third approach.

The signal  $w_{\{s,p,k\}}[n]$  is  $(1)$ . The signal  $w_{\{s,p,k\}}[n]$  is  $(1)$ . The signal  $w_{\{s,p,k\}}[n]$  is  $(1)$ .

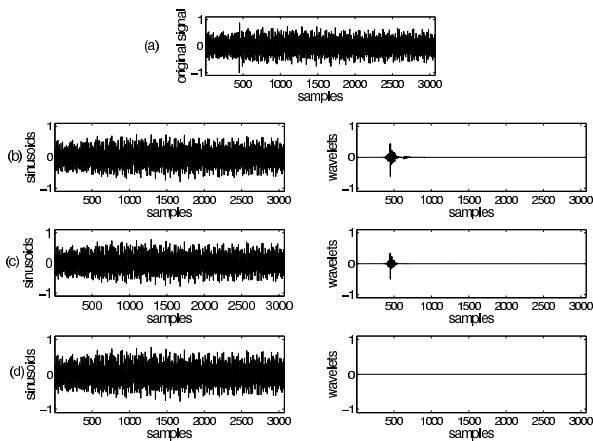


Fig. 2: Audio fragment containing a micro-transient

The original signal is a 3000-sample audio fragment containing a micro-transient. The reconstruction is performed using a mixed dictionary-based approach. The results are compared against a reference signal and a cascaded dictionary-based approach. The results show that the mixed dictionary-based approach achieves a higher MUSHRA score than the cascaded dictionary-based approach.

Table 1: PREFERENCE FOR MIXED DICTIONARY-VS-CASCADED DICTIONARIES (%).

Item	Pre (%)
Reference	100
Ca	100
Pa	52
La	46
Co	100
Ho	70
Lo	56
Co	60
Co	100

The results show that the mixed dictionary-based approach achieves a higher MUSHRA score than the cascaded dictionary-based approach. The results are compared against a reference signal and a cascaded dictionary-based approach. The results show that the mixed dictionary-based approach achieves a higher MUSHRA score than the cascaded dictionary-based approach.

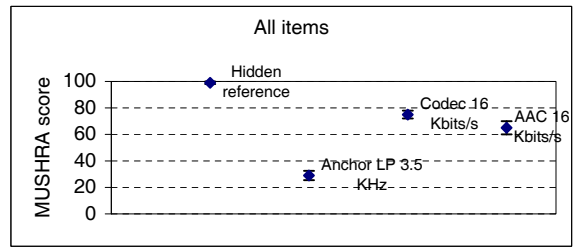


Fig. 3: MUSHRA listening test results showing mean grading and 95% confidence interval.

The results show that the mixed dictionary-based approach achieves a higher MUSHRA score than the cascaded dictionary-based approach.

## 5. CONCLUSIONS

The results show that the mixed dictionary-based approach achieves a higher MUSHRA score than the cascaded dictionary-based approach.

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