QUASI-ORTHOGONAL SPACE-TIME BLOCK CODES: APPROACHING OPTIMALITY

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ABSTRACT

In this tutorial we report on recent findings in Quasi-Orthogonal Space-Time Block Codes and show how these codes can be modified to offer high flexibility for many transmission schemes, support antenna selection, water filling and beamforming, approach maximal diversity, provide high capacity and still work with low complexity well suited for implementation. Examples from real-time measurements support our statements.

1. INTRODUCTION

Space-time coding has become very popular after the findings by Foschini [1] and Telatar [2] who showed that multiple antenna systems are capable of providing large capacity increase in wireless transmissions. Seminal papers by Tarokh et al. [3, 4] show criteria based on the Code Distance Matrix (CDM) explaining in which way Space-Time Codes (STC) need to be designed in order to utilize the maximum diversity provided by rich scattering channels. So called Orthogonal Space-Time Block Codes (OSTBC) became quite popular [5]-[7] in the context of Space-Time Transmit Diversity (STTD) since they can provide full diversity with low complexity, however mostly with the drawback of reduced data rates.

In order to gain the advantages of OSTBCs schemes with properties close to such optimal codes but with higher data rates, so called Quasi Orthogonal Space-Time Block Codes (QSTBC) were proposed [8]-[12]. However, soon the interest in such QSTBCs dropped since their loss in diversity appeared too harsh to be acceptable in particular when applying the analysis methods from [3, 4]. Mecklenbräuker et al. [13] provided an analytical result, showing that the diversity loss is not necessarily very large and that it can be overcome by various methods. In more realistic scenarios with correlated channels [14, 15] it turned out that QSTBCs are not necessarily performing poorer than OSTBCs [16]-[20]. This fact together with their flexibility appeared on the other hand so interesting that it was worth to investigate QSTBCs further.

This tutorial provides an overview of the properties of such QSTBCs including recent analytic findings and experimental validation. In Section 2 we introduce the concept of orthogonal space-time block codes and their extension to QSTBCs. We show how families of codes with essentially identical code properties but different transmission properties in spatially correlated channels can be generated. In Section 3 we demonstrate the virtual channel structuring property of such codes and introduce the so called Equivalent Virtual Channel Matrix (EVCM) due to which we can reformulate the transmission problem in an equivalent form, much more suitable for further analysis. We also introduce the modal eigenspace of such EVCMs and show that very simple modal matrices appear, suitable not only for beamforming applications but also applicable to the BER performance analysis of such codes. In Section 4 we explain the various receiver performances under QSTBC transmission and in Section 5 we discuss their capacity properties. Finally, Section 6 will discuss improvements of these codes, preserving high diversity and at the same time offering low complexity decoding.

2. FUNDAMENTALS OF QUASI-ORTHOGONAL SPACE-TIME BLOCK CODES

2.1 Alamouti code

The seminal work by Alamouti [7] started this interesting field. The basic idea is to define a 2×2 block code to receive the vector

$$\mathbf{r} = \mathbf{S}_{12}\mathbf{h} + \mathbf{v}. \tag{1}$$

By applying the following block code

$$\mathbf{S}_{12} = \begin{pmatrix} s_1 & s_2 \\ s_2^* & -s_1^* \end{pmatrix}, \tag{2}$$

the transmission defined by (1) reads explicitly

$$\begin{aligned} r_1 &= s_1 h_1 + s_2 h_2 + v_1 \\ r_2 &= s_2^* h_1 - s_1^* h_2 + v_2. \end{aligned}$$

This in turn is equivalent to:

$$\begin{aligned} r_1 &= h_1 s_1 + h_2 s_2 + v_1 \\ r_2^* &= -h_2^* s_1 + h_1^* s_2 + v_2^*, \end{aligned}$$

or written in vector form:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2^* \end{pmatrix} = \begin{pmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

where we changed the notation for the noise samples as well. In short vector notation we obtain:

$$\mathbf{y} = \mathbf{H}_v \mathbf{s} + \mathbf{n}, \tag{3}$$

where we introduced the so called Equivalent Virtual Channel Matrix (EVCM) $\mathbf{H}_v$ with the advantageous property

$$\begin{aligned} \mathbf{H}_v^H \mathbf{H}_v &= \mathbf{H}_v \mathbf{H}_v^H = h^2 \mathbf{I}_2, \quad \text{with} \\ h^2 &= |h_1|^2 + |h_2|^2. \end{aligned}$$

The Alamouti scheme thus allows to *structure* [21] any given 2×1 transmission system into a virtual orthogonal 2×2 transmission system with double diversity. Further advantages of such a scheme are low complexity receivers since the diagonal transmission scheme (3) can be decoded by a low-complex zero-forcing (ZF) receiver with maximum likelihood receiver (ML) properties. The only drawback is the lower data rate compared to spatial multiplexing, a drawback often taken readily into account.

2.2 Quasi-Orthogonal Space-Time Block Codes for Four Transmit Antennas

It is shown in [5, 22] that higher dimensional full rate schemes that obtain full transmit diversity exist only for binary modulation. Once complex-valued modulation is applied, OSTBCs with full rate and full diversity do not exist. An alternative is to reduce the data rate [23]. It is this data rate limitation that started the interest in QSTBCs to gain increased data rate while preserving most of the diversity advantage of OSTBCs.

Surprisingly, no researcher ever dared to define exactly what a quasi orthogonal code exactly is. The word *quasi* is not well defined in such context. Inspired by Theorem 7.1 in [6], we would thus like to propose the following definition:

Definition 2.1: A QSTBC of dimension $N \times N$ has an EVCM that satisfies $\mathbf{H}_v\mathbf{H}_v{}^H = \sum_1^N |h_i|^2 \mathbf{J}$ with $\mathbf{J}$ being a sparse matrix with ones on its main diagonal, having at least $N^2/2$ zero entries at off-diagonal positions and its remaining entries being bounded in magnitude by $+1$.

We will continue our overview mostly on the basis of four transmit antennas and one receive antenna although we like to point out that the statements we give are equivalently true for more antennas. However, explicit terms are often not as comprehensible as for the four antenna case and four antennas are more likely to be used in the near future than for example 8 or 16 transmit antennas.

2.2.1 Known QSTBCs

We start our overview with a list of well known QSTBCs, the ABBA code proposed by Tirkkonen et al. [8]:

$$\mathbf{S}_{ABBA} = \begin{bmatrix} \mathbf{S}_{12} & \mathbf{S}_{34} \\ \mathbf{S}_{34} & \mathbf{S}_{12} \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3 & s_4 & s_1 & s_2 \\ s_4^* & -s_3^* & s_2^* & -s_1^* \end{bmatrix} \quad (4)$$

the Jafarkhani code [9, 11] which we will call *Extended Alamouti* code since it straightforwardly extends the concept of Alamouti:

$$\mathbf{S}_{EA} = \begin{bmatrix} \mathbf{S}_{12} & \mathbf{S}_{34} \\ \mathbf{S}_{34}^* & -\mathbf{S}_{12}^* \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3^* & s_4^* & -s_1^* & -s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix} \quad (5)$$

and finally the code proposed by Papadias and Foschini [10] that cannot be described by sub-blocks like the first two examples:

$$\mathbf{S}_{PF} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & -s_1 & s_2 \\ s_4^* & s_3^* & -s_2^* & -s_1^* \end{bmatrix}. \quad (6)$$

2.2.2 Other QSTBCs Obtained by Linear Transformations

Although similar in appearance, the codes (4), (5), and (6) show very distinct behavior at particular channels. Thus the question arose how many of such codes exist, and whether there are optimal codes for particular channels. Although the first question of how many codes exist is rather difficult to answer, we will show in the next section by working out the concept of the equivalent virtual channel matrix that there are just a few codes with *distinctly different behavior*. This is due to the fact that all of these codes can be translated into each other by linear transformations of the form:

$$[\mathbf{T}_2\mathbf{C}[\mathbf{T}_1\mathbf{s}]]\,\mathbf{T}_3 = \mathbf{S}. \quad (7)$$

Here $\mathbf{s}$ is the set of symbols (for example $\{s_1, s_2, s_3, s_4\}$) generating the space-time code matrix $\mathbf{S}$. A permutation matrix $\mathbf{T}_1$ changes the order of the symbols. The linear transformation $\mathbf{C}$ maps the vector $\mathbf{T}_1\mathbf{s}$ into a space-time code word by repeating the symbols and placing them in every row differently but in such a way that all symbols appear once in each row and once in each column. The linear transformations $\mathbf{T}_2$, and $\mathbf{T}_3$ finally allow for exchanging rows and columns (permutations) as well as to alter signs in columns or rows and/or change to the conjugate complex values of an entire row. To generate a new quasi-orthogonal space-time code matrix $\mathbf{S}$, the linear transformations described above must be applied in such a way, that the new code also fulfills the constraints given in Definition 2.1. Note that all of these linear transformations (7) are *unitary*, i.e., energy preserving. The consequence of this is that some essential properties like the trace invariance of all these codes remain unchanged.

3. EQUIVALENT VIRTUAL MIMO CHANNEL MATRIX

An important characteristic of QSTBCs is their distinct equivalent, highly structured, virtual MIMO channel matrix $\mathbf{H}_v$. Let us consider an STBC denoted by $\mathbf{S}$, e.g $\mathbf{S}_{\mathbf{EA}}$ from (5), and an 4×1 frequency flat MISO system. Then we obtain $\mathbf{r} = \mathbf{Sh} + \mathbf{n}$, just like in (3) where $\mathbf{r}$ denotes the vector of four temporally successive receive samples. The channel coefficients are denoted by $\mathbf{h} = [h_1, h_2, h_3, h_4]^T$ and $\mathbf{n}$ is the modified noise vector similar to the Alamouti case in Section 2.1.

Complex conjugating some rows of $\mathbf{S}$ leads to the equivalent, highly structured, virtual MIMO channel matrix (EVCM). For example, changing the second and fourth element of the code (4) or (6) leads to a modified received signal vector $\mathbf{y}$ that can be written in the equivalent form

$$\mathbf{y} = \begin{bmatrix} r_1 \\ r_2^* \\ r_3 \\ r_4^* \end{bmatrix} = \mathbf{H}_v\mathbf{s} + \mathbf{n} = \mathbf{H}_v \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \mathbf{n}, \quad (8)$$

introducing $\mathbf{H}_v$ as the 4×1 highly structured EVCM.

3.1 EVCM for known QSTBCs

Computing the EVCM for the three well known QSTBCs (4), (5), (6), provides interesting insight into individual properties of these codes. For all three codes, h^2 denotes the overall *channel gain* (also called overall fading factor) with

$$h^2 = |h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2 \quad (9)$$

indicating a potential system diversity order of four. We obtain for the ABBA code for instance:

$$\mathbf{H}_{v_{ABBA}} = \begin{bmatrix} \mathbf{H}_{12} & \mathbf{H}_{34} \\ \mathbf{H}_{34} & \mathbf{H}_{12} \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ h_3 & h_4 & h_1 & h_2 \\ -h_4^* & h_3^* & -h_2^* & h_1^* \end{bmatrix}. \quad (10)$$

Analyzing the various decoder performances, the Grammian matrix

$$\mathbf{G}_{ABBA} = \mathbf{H}_{v_{ABBA}}^H \mathbf{H}_{v_{ABBA}} = h^2 \begin{bmatrix} 1 & 0 & X_{ABBA} & 0 \\ 0 & 1 & 0 & X_{ABBA} \\ X_{ABBA} & 0 & 1 & 0 \\ 0 & X_{ABBA} & 0 & 1 \end{bmatrix} \quad (11)$$

is essential for the error performance where X_{ABBA} is a channel dependent interference parameter given by:

$$X_{ABBA} = \frac{2\mathrm{Re}(h_1 h_3^* + h_2 h_4^*)}{h^2}. \quad (12)$$

It is this interference parameter that defines how close such a matrix is to being orthogonal. For $X_{ABBA} = 0$ perfect orthogonality is obtained and $\mathbf{G} = h^2\mathbf{I}$. Per definition $X_{ABBA} \in [-1,1]$.

For the extended Alamouti code we obtain:

$$\mathbf{H}_{v_{EA}} = \begin{bmatrix} \mathbf{H}_{12} & \mathbf{H}_{34} \\ -\mathbf{H}_{34}^* & \mathbf{H}_{12}^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & h_1^* & h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \quad (13)$$

$$\mathbf{G}_{EA} = \mathbf{H}_{v_{EA}}^H \mathbf{H}_{v_{EA}} = \mathrm{h}^2 \begin{bmatrix} 1 & 0 & 0 & X_{EA} \\ 0 & 1 & -X_{EA} & 0 \\ 0 & -X_{EA} & 1 & 0 \\ X_{EA} & 0 & 0 & 1 \end{bmatrix}, \quad (14)$$

and

$$X_{EA} = \frac{2\mathrm{Re}(h_1 h_4^* - h_2 h_3^*)}{h^2}. \quad (15)$$

For the Papadias-Foschini code we obtain:

$$\mathbf{H}_{v_{PF}} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3 & h_4 & h_1 & -h_2 \\ -h_4^* & -h_3^* & h_2^* & h_1^* \end{bmatrix}, \quad (16)$$

$$\mathbf{G}_{PF} = \mathrm{h}^2 \begin{bmatrix} 1 & 0 & X_{PF} & 0 \\ 0 & 1 & 0 & -X_{PF} \\ -X_{PF} & 0 & 1 & 0 \\ 0 & X_{PF} & 0 & 1 \end{bmatrix}, \quad (17)$$

and

$$X_{PF} = \frac{2j\mathrm{Im}(h_1^* h_3 + h_2 h_4^*)}{h^2}. \quad (18)$$

3.2 Common Properties of the Virtual Channel Matrices

1. The virtual channel matrix $\mathbf{H}_v$ has a block structure very similar to the corresponding non-orthogonal code matrix $\mathbf{S}$.
2. If any of the linear transformations explained in Section 2.2.2 is worked out in such a way that the quasi-orthogonal structure of $\mathbf{G}$ is not changed, then only the **value** of X changes (with the linear transformation).
3. The interference parameter X is responsible for the non-orthogonality of the code. The closer X is to zero, the closer is the code to an orthogonal code with $\mathbf{G} = h^2\mathbf{I}$.
4. In all 4×4 cases, the Grammian matrix $\mathbf{G}$ has the two pairs of eigenvalues $h^2(1 \pm X)$. Due to the channel properties derived in (9), QSTBCs provide potential full diversity [13] (see also Section 4.1).
5. X is bounded by 1, and thus the eigenvalues of $\mathbf{G}$ are in the range of $h^2[0,2]$ in the 4×4 case.
6. Since X is the only term that changes its value with the correct linear transformations, *only the value of* X that depends on the particular channel realisation influences directly the performance of the code.
7. The values of X can be designed to be either real valued or purely imaginary valued.

3.3 Code Members with Distinct Performance

As mentioned in Section 2.2.2 the number of QSTBCs is not easy to find, however, the number of codes that show distinct behavior on a given wireless channel can be easily derived. In case of a four transmit diversity scheme, only 12 different code types exist, due to 12 different values of X (the negative values of (19, 20) do not count, since they provide the same performance). The 12 distinct X values are listed below. First, six real values of X with

$$\begin{aligned} X_1 &= \frac{2\mathrm{Re}(h_1 h_3^* + h_2 h_4^*)}{h^2} \\ X_2 &= \frac{2\mathrm{Re}(h_1 h_3^* - h_2 h_4^*)}{h^2} \\ X_3 &= \frac{2\mathrm{Re}(h_1 h_2^* + h_3 h_4^*)}{h^2} \\ X_4 &= \frac{2\mathrm{Re}(h_1 h_2^* - h_3 h_4^*)}{h^2} \\ X_5 &= \frac{2\mathrm{Re}(h_1 h_4^* + h_2 h_3^*)}{h^2} \\ X_6 &= \frac{2\mathrm{Re}(h_1 h_4^* - h_2 h_3^*)}{h^2}, \end{aligned} \quad (19)$$

and furthermore six purely imaginary values of X with

$$\begin{aligned} X_7 &= \frac{2j\mathrm{Im}(h_1 h_3^* + h_2 h_4^*)}{h^2} \\ X_8 &= \frac{2j\mathrm{Im}(h_1 h_3^* - h_2 h_4^*)}{h^2} \\ X_9 &= \frac{2j\mathrm{Im}(h_1 h_2^* + h_3 h_4^*)}{h^2} \\ X_{10} &= \frac{2j\mathrm{Im}(h_1 h_2^* - h_3 h_4^*)}{h^2} \end{aligned} \quad (20)$$

$$X_{11} = \frac{2j\text{Im}(h_1h_4^* + h_2h_3^*)}{h^2}$$
$$X_{12} = \frac{2j\text{Im}(h_1h_4^* - h_2h_3^*)}{h^2}$$

exist.
The corresponding QSTBCs are easily found by starting with an arbitrary QSTBC and applying linear transformations. Given a wireless channel, these 12 possible variants of X indicate whether the obtained transmission system is close to an orthogonal one in performance or not.

3.4 Computer Experiments

We simulated the Bit Error Ratio (BER) of 12 code members with the above defined distinct values for X_i as a function of E_b/N_0. Note that the subscript i of the code $\mathbf{S}_i$ corresponds to the subscript of the $X_i, (i = 1, 2, \cdots, 12)$. In our simulations, we have used a QPSK signal constellation with Gray coding. The Rayleigh fading channel has been kept constant during the transmission of each code block but has changed independently between successive blocks (block fading). At the receiver side, we have implemented a ZF receiver (Section 4.2). Each code was simulated on i.i.d. channels and on spatially correlated channels (high correlation, $\rho = 0,95$) using the following correlation matrix:

$$\mathbf{R}_{hh} = \text{E}[\mathbf{h}\mathbf{h}^H] = \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix}. \quad (21)$$

Consider the 12 different values of X_i. The average value of them can be evaluated as

$$\begin{aligned} \text{E}[|X_1|] &= \rho^2 \\ \text{E}[|X_2|] &= 0 \\ \text{E}[|X_3|] &= \rho \\ \text{E}[|X_4|] &= 0 \\ \text{E}[|X_5|] &= (1+\rho^2)\rho/2 \\ \text{E}[|X_6|] &= (1-\rho^2)\rho/2 \end{aligned} \quad (22)$$

while the absolut values of the imaginary valued X_i are all zero in average. For the case of uncorrelated i.i.d. channels $\rho = 0$ and so are all expectations of $|X_i|$. As Figures 1 and 2 show, the performance of all 12 codes on i.i.d. channels is indeed identical. For the strongly correlated channel, however, the situation is different. For $\rho = 0,95$, we have $\text{E}[|X_2|] = \text{E}[|X_4|] = 0$ and $\text{E}[|X_6|] \leq \text{E}[|X_1|] \leq \text{E}[|X_5|] \leq \text{E}[|X_3|]$. The performance of the codes is accordingly. The average values of $\text{E}[|X_i|]$ in (22) explain why some codes are better suited to highly correlated channels than others. Obviously, the code members two and four with the corresponding values X_2 and X_4 and the code members with the imaginary values of X, that are X_7 to X_{12} show the best performance since they approximate in average OSTBCs no matter what the channel correlation ρ is. Thus, assuming a linear antenna array with correlation properties given in (21), codes $\mathbf{S}_2, \mathbf{S}_4$ and $\mathbf{S}_7$ to $\mathbf{S}_{12}$ perform best.

The behavior of the QSTBCs in measured indoor MIMO channels has been investigated in [16, 17] where measurements from [14, 15] were used to simulate the performance of the channels and to estimate the channel matrices modelled by the well known Kronecker model.

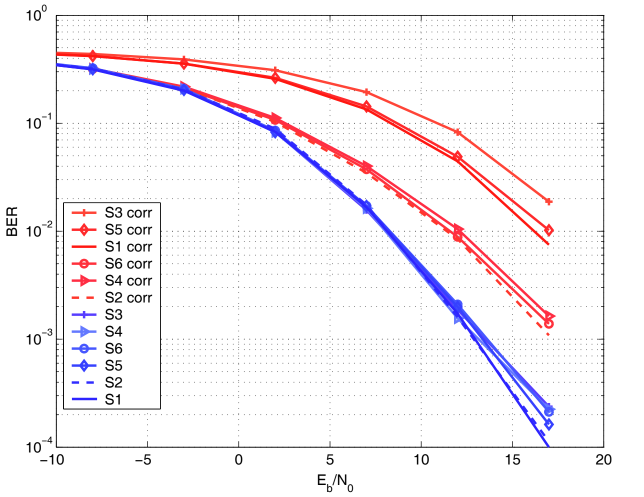

Figure 1: BER-performance of codes with real values of X for i.i.d. and highly correlated channels.

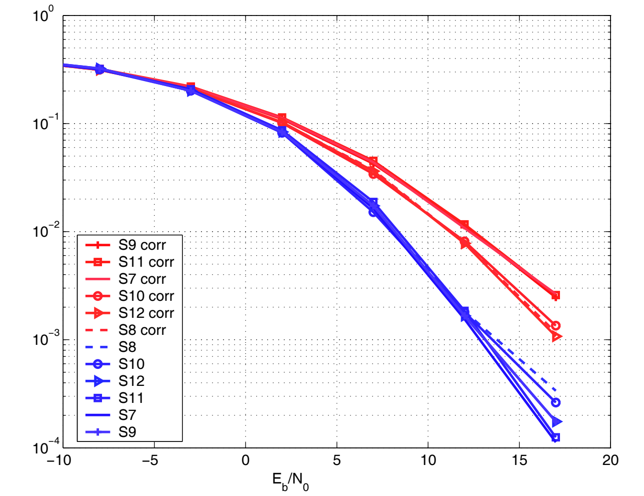

Figure 2: BER-performance of codes with imaginary values of X for i.i.d. and highly correlated channels.

3.5 The Modal Eigenspace of QSTBCs

The distance properties of STBCs are essentially dependent on their distance matrices. The distance matrix $\mathbf{A}_k$ between two valid code matrices $\mathbf{S}_k$ and $\tilde{\mathbf{S}}_k$ is defined as:

$$\mathbf{A}_k = (\mathbf{S}_k - \tilde{\mathbf{S}}_k)(\mathbf{S}_k - \tilde{\mathbf{S}}_k)^H. \quad (23)$$

Good codes should have high rank distance matrices with determinant values as high as possible. These properties can be checked by evaluating the eigenvalues of all possible distance matrices. In [24] it was shown that the EVCM as well as the code distance matrix (CDM) of the EA code share the same modal eigenspace presented in the following theorem:

Theorem 3.1: If $\mathbf{H}_{k-1}^{(1)}$ and $\mathbf{H}_{k-1}^{(2)}$ are two different virtual channel matrices defining a new virtual channel matrix $\mathbf{H}_k$ as done in (13), and $\mathbf{S}_{k-1}^{(1)}$ and $\mathbf{S}_{k-1}^{(2)}$ are two different space-time code word matrices defining the corresponding new code $\mathbf{S}_k$, then the following property for $\mathbf{H}_k$ and $\mathbf{S}_k$ holds correspondingly:

The Grammian of the EVCM $\mathbf{H}_k$ and any CDM $\mathbf{A}_k$ can be diagonalized by the channel independent model matrices $\mathbf{V}_k$ and $\mathbf{W}_k$ starting with $k = 2, 3, ..$:

$$\begin{aligned}\mathbf{\Lambda}_{\mathbf{H}_k} &= 2^{-(k-1)}\mathbf{V}_k^T \mathbf{H}_k^H \mathbf{H}_k \mathbf{V}_k \\ \mathbf{\Lambda}_{\mathbf{A}_k} &= 2^{-(k-1)}\mathbf{V}_k^T \mathbf{A}_k \mathbf{V}_k\end{aligned}$$

where $\mathbf{V}_k$ and $\mathbf{W}_k$ are recursively defined by:

$$\begin{aligned}\mathbf{V}_k &= \begin{pmatrix} \mathbf{V}_{k-1} & \mathbf{W}_{k-1} \\ -\mathbf{W}_{k-1} & \mathbf{V}_{k-1} \end{pmatrix}, \\ \mathbf{W}_k &= \begin{pmatrix} \mathbf{W}_{k-1} & \mathbf{V}_{k-1} \\ \mathbf{V}_{k-1} & -\mathbf{W}_{k-1} \end{pmatrix},\end{aligned}$$

with the initial values

$$\mathbf{V}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \mathbf{W}_1 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}.$$

Since other QSTBCs can be derived by linear transformations of the EA code, it is expected that this theorem holds for all QSTBCs, or at least for those with block structure. From an implementation point of view, we wish to highlight a beneficial property of the recursively defined modal matrices $\mathbf{V}_k$ and $\mathbf{W}_k$: Their elements take only values from $\{-1, 0, 1\}$. Thus, the diagonalization of the virtual channel matrix can be implemented without multipliers. The implications of this theorem are manyfold and will be pointed out in the following sections.

4. RECEIVER ALGORITHMS FOR QSTBCS

Full orthogonal STBCs have the advantageous decoding property that the inverse of the Grammian matrix $\mathbf{G}$ is proportional to the identity matrix. This means that a low-complexity ZF receiver degenerates to a simple maximum ratio combiner (MRC) receiver with $\hat{\mathbf{s}} = 1/h^2 \mathbf{H}^H \mathbf{y}$ and behaves exactly as a possibly high-complex ML receiver. With QSTBCs the hope is that most of the low complexity properties are preserved [26].

4.1 ML Receiver

Assuming Gray-coded QPSK signal modulation normalized to unit power, the BER in case of OSTBCs is given by

$$\text{BER}_{O,ML} = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{h^2}{\sigma_n^2}}\right), \qquad (24)$$

where h^2 now plays the role of a resulting improved fading factor, providing full diversity.

QSTBCs cannot make use of the full channel diversity and their BER behavior depends on the eigenvalues of their CDMs $\mathbf{A}_k$ as shown in [3]. More importantly, the *constant channel independent* modal matrix $\mathbf{V}$ (or $\mathbf{W}$) diagonalizes the CDM $\mathbf{A}_k$, irrespective of the involved code word pairs and symbol alphabet. Due to this property an extraordinary tight BER approximation can be analytically calculated as shown in [27]. The eigenvalues of the CDMs can be calculated with the properties of Theorem 3.1. As an illustrative example, the eigenvalues of the CDMs for four transmit antennas ($k = 2$) are given as

$$\begin{aligned}\lambda_1 = \lambda_2 &= |(s_1 - \tilde{s}_1) + (s_4 - \tilde{s}_4)|^2 + |(s_2 - \tilde{s}_2) - (s_3 - \tilde{s}_3)|^2 \\ \lambda_3 = \lambda_4 &= |(s_1 - \tilde{s}_1) - (s_4 - \tilde{s}_4)|^2 + |(s_2 - \tilde{s}_2) + (s_3 - \tilde{s}_3)|^2\end{aligned}$$

where s_i and $\tilde{s}_i$ are the corresponding signal elements of two competing code matrices.
A direct consequence of such specific eigenvalues is that the maximum rank of most of the CDMs is four but its minimum is two in some special cases where two signals terms differ in a very specific way. It can be seen from the above equations that rank zero is not possible except all eigenvalues are zero and no symbol error occurs. It is interesting to observe that a single symbol error results in a full CDM rank while a two symbol error pattern can lead to a rank deficient CDM with rank two. As already shown in [3], the asymptotic slope of the pairwise error probability (PEP) curves for high signal to noise ratios is determined by the minimum CDM rank. Thus, some PEP curves have a slope of only two decades per 10 dB SNR corresponding to a diversity order 2 and therefore the asymptotic BER performance curve has a flat slope as can be seen in [27]. However, this slope does not show up at low and medium SNR of practical interest. In some cases a slope four BER curve dominates at SNR values of practical interest [27].

Another important aspect in the context of ML receivers for QSTBCs is the *low complexity preserving property*. Due to the large number of zeros in the Grammian matrix $\mathbf{G}$, the ML detection can be separated in two halves. Applying a code alphabet of cardinality P on N_T transmit antennas does not result in P^{N_T} trials in the search space but only in $2P^{N_T/2}$ trials which is approximately the square root in complexity [13].

4.2 ZF Receiver

ZF receivers are highly appreciated for their low complexity. For large signal alphabets this can be of great advantage, while for small signal alphabets the ML receiver can compete in complexity. ZF-decoding of QSTBCs is simply obtained by

$$\hat{\mathbf{s}} = \left(\mathbf{H}\mathbf{H}^H\right)^{-1} \mathbf{H}^H \hat{\mathbf{y}} = \mathbf{s} + \left(\mathbf{H}\mathbf{H}^H\right)^{-1} \mathbf{H}^H \mathbf{n}. \qquad (25)$$

The matrix inverse can require moderate to high implementation complexity if the precision of this inversion is high. In [21] the BER for ZF receiver applying QPSK power normalized signal modulation was shown to be

$$\text{BER}_{Q,ZF} = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{h^2(1-|X|^2)}{\sigma_n^2}}\right), \qquad (26)$$

which suffers from a degradation due to the channel interference parameter X when compared to ML decoding of OSTBCs. For the four transmit antenna case it was shown that this interference term degrades the overall diversity from 4 to 3,2.

4.3 MMSE Receiver

The MMSE receiver behaves similar to the ZF receiver however with an additional term in the matrix inverse due to the noise variance:

$$\hat{\mathbf{s}} = \left(\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I}\right)^{-1}\mathbf{H}^H\hat{\mathbf{y}} \quad (27)$$
$$= \left(\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I}\right)^{-1}\mathbf{H}\mathbf{H}^H\mathbf{s} + \left(\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I}\right)^{-1}\mathbf{H}^H\mathbf{n}.$$

In practice it can be difficult to obtain correct values of σ_n^2. But only for correct values a small improvement compared to ZF can be obtained. Therefore, the MMSE technique is not used in practice.

5. CAPACITY RESULTS

In [10] it has been shown that 4×1 QSTBCs attain most of the open-loop Shannon capacity and most of the outage capacity. The ergodic capacity [28] of a general MIMO channel $\mathbf{H}$ is given by

$$\max_{\mathbf{Q} \leq \mathbf{Q}_o} E\left[\log_2 \det\left(\mathbf{I} + \rho\mathbf{H}\mathbf{Q}\mathbf{H}^H\right)\right] b/s/Hz, \quad (28)$$

with the normalization $E[\text{trace}(\mathbf{H}\mathbf{H}^H)] = E[h^2]N_T = N_T$ and ρ denoting the mean SNR at each receive antenna. Utilizing a QSTBC, we obtain the EVCM $\mathbf{H}_v$ instead and the maximum capacity reads [29]:

$$C_{\max} = \max_{\mathbf{Q} \leq \mathbf{Q}_o} \frac{1}{N_T} E\left[\log_2 \det\left(\mathbf{I} + \rho\mathbf{H}_v\mathbf{Q}\mathbf{H}_v^H\right)\right] b/s/Hz. \quad (29)$$

While precoding methods (see Section 6.2.2) take advantage of finding optimal or suboptimal precoding matrices $\mathbf{Q}$, simple and explicit capacity formulas are not obtained easily. In [29] the mean mutual information of the structured channel with EVCM $\mathbf{H}_v$ was computed instead as (for $N_T = 2, 4, 8$)

$$\bar{I}_{N_T} = \frac{1}{N_T} E\left[\log_2 \det\left(\mathbf{I} + \rho\mathbf{H}_v\mathbf{H}_v^H\right)\right]$$
$$= \frac{1}{\ln 2}\left[1 + \left(1 - \frac{2}{\rho}\right) e^{\frac{2}{\rho}} E_1\left(\frac{2}{\rho}\right)\right] b/s/Hz, \quad (30)$$

with $\mathrm{E}_1(x)$ denoting the exponential integral. Given i.i.d. channels, the pdf of the eigenvalues of $\mathbf{H}_v\mathbf{H}_v^*$ are known [13] and thus the mean mutual information can be computed. Note that (30) indicates that the mean mutual information is independent of the number N_T of transmit antennas. However, also in [13] it is shown, that the outage capacity improves with growing number N_T.

6. IMPROVED DIVERSITY TECHNIQUES FOR QSTBCS

6.1 Symbol Rotation

Applying QPSK in a transmission scheme with a four fold transmit diversity, in total, $4^4(4^4 - 1) = 65280$ error events may occur. Out of all these error events only 2080 events correspond to a rank two distance matrix leading to an effective diversity loss even in case of an ML detector. An efficient way to avoid this rank deficiency in distance matrices has been found in an additional rotation of specific symbols used in the QSTBC [11, 30, 31, 32]. For example, by rotating the symbols $\{s_2, s_4\}$ in the EA-code by $e^{j\pi/4}$, while $\{s_1, s_3\}$ remain unchanged, this diversity loss can be avoided to a large extent [11]. An optimal rotation strategy for a wider class of transmit diversity schemes has been proposed in [29].

6.2 Closed Loop Systems

The use of channel feedback from the receiver to the transmitter is standard in wireline communications. The knowledge of the channel at the transmitter would produce benefits for wireless communications as well. OSTBCs and QSTBCs are designed assuming that the transmitter has no knowledge about the channel. However in some applications, the transmitter can exploit channel state information (CSI) to improve the overall performance of the system, especially in case of spatially correlated channels [14, 15, 25]. If the channel is known at the transmitter, it is optimal to use transmit beamforming [33]-[35]. Research on block codes with partial feedback is beginning to gain more attention [36]-[39].
Partial feedback information can correspond to a quantized channel estimate [36], or to a matrix index in a finite set of precoder matrices [37],[39], to antenna selection [46]-[49] or to code selection [38]-[41]. Each of these partial feedback options returns a limited number of channel information bits from the receiver to the transmitter.

6.2.1 Code Selection

A very simple but clever coding scheme was presented in [38] where new block codes using feedback are described in detail. There, code orthogonality is preserved such that a simple matched filter (MF) receiver can be used for optimal detection. Selecting one of two code matrices at the transmitter leads to full diversity and some coding gain. However, if synchronization between transmitter and receiver based on the feedback information is erroneous, or the feedback information at the receiver is decoded incorrectly this concept looses diversity.

In [40], [41] we presented an even simpler version of code selection in combination with QSTBCs. The receiver returns one or two feedback bits per fading block and, depending on the number of returned bits, the transmitter switches between two or four QSTBCs to minimize the channel dependent interference parameter X. In this way full diversity and nearly full-orthogonality can be achieved with an ML receiver as well as with a simple ZF receiver. This method can be applied to any number of transmit antennas without increasing the required number of feedback bits as shown in [41]. In [17, 42], where our simulations have been based on correlated MIMO channels and MIMO indoor measured channels, we have shown that QSTBCs with our simple feedback scheme are robust against channel variations, and that they perform very well even on highly correlated channels.
In [43] a flexible and scalable testbed for the implementation and evaluation of signal processing algorithms for 4×4 MIMO systems is described. Using the MIMO testbed, the capabilities of QSTBCs can be demonstrated by real-time experiments. Applying the feedback scheme from [18, 40, 41] we compare simulated and real-time measured BER performance of the EAC (5) in Fig. 3. The results are compared with ideal two- and four-path diversity transmissions [13]. Obviously a substantial improvement of the BER can be achieved by sending back only a small amount of information about the channel state. The results in Fig. 3 show that the measured BER curves match very well with the simulated results. These results prove the enormous potential of the QSTBCs.

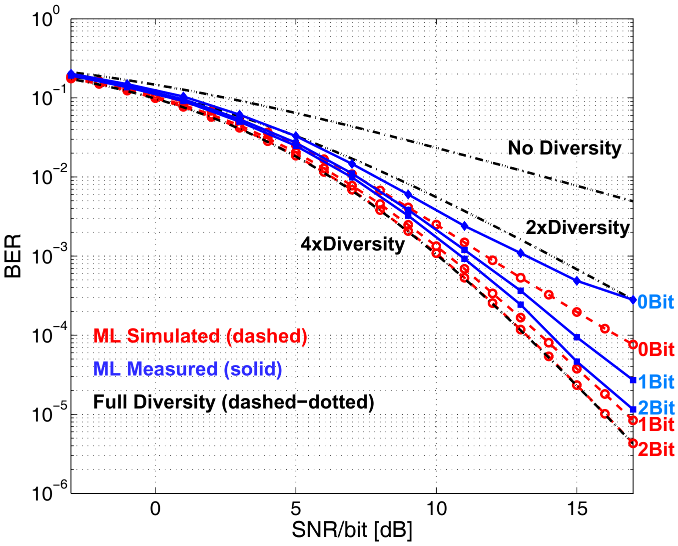

Figure 3: BER-performance for ML decoded EAC, measured and simulated.

6.2.2 *Precoding Methods*

Starting from the capacity formula in (29) it is relatively simple to perform precoding methods by applying a precoding matrix $\mathbf{Q}$. The most relevant method is water filling where the capacity is maximized based on the analytical knowledge of the eigenvalues of the EVCM. The resulting capacity utilizing water filling is given as

$$C_{\max,wf} = \sum_i \log_2 \left(\frac{\rho \lambda_i^{(\mathbf{H}_v^H \mathbf{H}_v)}}{c} \right), \tag{31}$$

with c being a normalization factor such that $\mathrm{tr}(Q) = Q_o$ [28].

A second method approaching maximal capacity is beam-forming. As already mentioned above, the eigenvalues of the distance matrices and the eigenvalues of the equivalent channel matrices appear in pairs. In the case of four transmit antennas they result in $\lambda_1 = \lambda_2 = h^2(1-X)$, $\lambda_3 = \lambda_4 = h^2(1+X)$. (see [11, 13]). Transmitting with QSTBCs, the eigenvectors of the equivalent channel matrices are known in advance, and even more important, they do not depend of the instantaneous channel gain values h_i. Therefore, additional beamforming which has proven especially valuable in the low SNR range can be easily applied. In contrast to the common practice, where the beam is directed only in the direction corresponding to $\lambda_{\max}$, applying QSTBCs one should always transmit at least into two directions, since with QSTBCs the largest eigenvalues always appear in a pair.

Other precoding techniques typically improve the system diversity or more or less directly the BER by channel dependent linear precoders for STBCs [39, 44, 45]. In [39] the precoder minimizes the bound on the conditional error probability with very low rate feedback. In [44] the design of a linear precoder for OSTBC in spatially correlated, quasi-static, flat fading channels with knowledge of the channel covariance at the transmitter is based on minimizing the probability of decoding error. A linear precoder for QSTBCs was presented in [45]. It compensates antenna correlation and minimizes the average pairwise error probability. In this case the exact knowledge of the long-term characteristics of the channel, that is the channel correlation matrix, is required at the transmitter.

6.2.3 *QSTBCs with Antenna Selection*

In order to reduce the implementation complexity of MIMO systems (e.g. the high number of radio-frequency (RF) chains on both link ends) channel adaptive antenna selection (CAAS) at the transmitter and/or at the receiver side has been proposed in [46]-[49], where only a subset of "best" antennas is used for transmitting the data. CAAS was first combined with OSTBCs in [46, 47]. In [46], the transmit selection criterion was based on maximization of the Frobenius norm of the channel transfer matrix. It has been shown that this scheme achieves full diversity, as if all the transmit antennas were used.

In [48, 49] transmit and receive CAAS with QSTBC for four transmit antennas and using a ZF receiver has been proposed. The selection criterion is based on the analytic expression for the BER given in (26) and minimizes the term $h^2(1-|X|^2)$. This criterion maximizes diversity, and at the same time minimizes the channel dependent interference parameter X [49].

6.3 More Receive Antennas

So far we only considered one receive antenna. However, using QSTBCs at the transmitter, decoding techniques can easily be extended to more, say N_R, receive antennas. Denoting the various receive signals as $\mathbf{y}_i$, with $i = 1, 2, ..N_R$, these signals can be combined to

$$\mathbf{r} \;=\; \sum_{i=1}^{N_R} \mathbf{H}_i^H \mathbf{y}_i = \sum_{i=1}^{N_R} \mathbf{H}_i^H \mathbf{H}_i \mathbf{s} + \mathbf{H}_i^H \mathbf{n}_i, \tag{32}$$

leading to a new Grammian matrix $\mathbf{G} = \sum_{i=1}^{N_R} \mathbf{G}_i = \sum_{i=1}^{N_R} \mathbf{H}_i^H \mathbf{H}_i$ with the same sparse structure as before. However, the degrees of freedom in X are increased. This in turn leads to pdfs of X better concentrated around zero. Asymptotically, it can be shown that for $N_R \to \infty$, $X \to 0$ with probability one and the transmission appears perfectly orthogonal. In practice such behavior is observed much earlier. In [26] it was shown that for ZF techniques and four receive antennas already a diversity factor 16 and for $N_R = 16$ even a diversity factor 64 is obtained.

7. CONCLUSIONS

In this tutorial we presented a unified view of the various QSTBCs presented in the literature up to now. We explained the high potential of these codes in some detail and discussed some interesting extensions to improve diversity and coding gain or to reduce the implementation complexity. In summary QSTBCs is a promising technique to be used extensively in future wireless communication systems.

REFERENCES

[1] G.J. Foschini, M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas", Wireless Personal Communications, vol. 6, no. 3, pp. 311-335, March 1998.

[2] I.E. Telatar, Capacity of multi-antenna Gaussian channels, European Trans. Telecommunications, vol. 10, no. 6, pp. 585-595, 1999, Technical Memorandum, Bell Labs, Lucent Technologies, Oct. 1998.

[3] V. Tarokh, N. Seshadri, A.R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," IEEE Trans. Inform. Theory, vol. 44, no. 2, pp. 744–765, March 1998.

[4] V. Tarokh, H. Jafarkhani, A.R. Calderbank, "Space-time block coding for wireless communications: Performance results," IEEE Journal on Sel. Areas in Com., vol.17, no. 3, pp. 451-460, Mar. 1999.

[5] V. Tarokh, H. Jafarkhani and A.R. Calderbank, "Space-time block codes from orthogonal designs," IEEE Trans. Inform. Theory, vol. 45, pp. 1456-1467, July 1999.

[6] E.G. Larsson, P. Stoica, *Space-Time Block Coding for Wireless Communications,* Cambridge, U.K. Cambridge Univ. Press, Aug. 2003.

[7] S.M. Alamouti "A simple diversity technique for wireless communications," IEEE Journal on Sel. Areas in Com., vol. 16, no. 8, pp. 1451-1458, Oct. 1998.

[8] O. Tirkkonen, A. Boariu, A. Hottinen, "Minimal nonorthogonality rate one space-time block codes for 3+ Tx antennas," IEE International Symposium on Spread Spectrum Techniques and Applications (ISSSTA), vol. 2, New Jersey, USA, pp. 429-432, Sept. 2000.

[9] H. Jafarkhani, "A quasi orthogonal space-time block code," IEEE Trans. Comm., vol. 49, pp. 1-4, Jan. 2001.

[10] C. Papadias, G. Foschini, "Capacity-Approaching Space-Time Codes for System Employing Four Transmitter Antennas", IEEE Trans. Inf. Theory, vol. 49, no. 3, pp. 726-733, March 2003.

[11] C.F. Mecklenbräuker, M.Rupp, "On extended Alamouti schemes for space-time coding,", *WPMC'02*, Honolulu, pp. 115-119, Oct. 2002.

[12] A. Boarui, D. M. Ionescu, "A Class of Nonorthogonal Rate-One Space-Time Block Codes with Controlled Interference", IEEE Trans. Wireless Comm. vol. 2, no. 2, pp. 270-276, March 2003.

[13] C.F. Mecklenbräuker, M. Rupp, "Flexible space-time block codes for trading quality of service against data rate in MIMO UMTS," EURASIP J. on Appl. Signal Proc., no. 5, pp. 662-675, May 2004.

[14] H. Özcelik, M. Herdin, H. Hofstetter, E. Bonek, "A comparsion of measured 8×8 "MIMO system with a popular stochastic channel model at 5.2 GHz," 10th International Conference on Telecommunications (ICT'2003), Tahiti, March 2003.

[15] H. Özcelik, M. Herdin, R. Prestros, E. Bonek, "How MIMO capacity is linked with single element fading statistics", International Conference on Elektromagnetics in Advanced Applications, Torino, Italy, pp. 775-778, Sept. 2003.

[16] B. Badic, M. Herdin, G. Gritsch, M. Rupp, H. Weinrichter "Performance of various data transmission methods on measured MIMO channels," 59th IEEE Vehicular Technology Conference VTC Spring 2004, Milan, Italy, May 2004.

[17] B. Badic, M. Herdin, M. Rupp, H. Weinrichter, "Quasi Orthogonal Space-Time Block Codes on Measured MIMO Channels," SympoTIC 04, Bratislava, pp. 17-20, Oct. 2004.

[18] B. Badic, H. Weinrichter, M. Rupp, "Comparison of Non-Orthogonal Space-Time Block Codes in Correlated Channels," SPAWC 04, Lisboa, July 2004.

[19] F.C. Zheng, A.G. Burr, "Robust non-orthogonal space-time block codes over highly correlated channels: a generalization,"IEE El. Letters, vol. 39, no. 16, pp. 1190-1191, Aug. 2003.

[20] B. Friedlander, J. Francos, "Space-Time Block Coding for Correlated Fading," 38. Asilomar Conference on Signals, Systems and Computers, Pacific Grove, Nov. 2004.

[21] M. Rupp, C.F. Mecklenbräuker, "Improving Transmission by MIMO Channel Structuring," IEEE International Conference on Communications, ICC'03, Anchorage, pp. 3066-3070, May 2003.

[22] B. Hochwald, T.L. Marzetta, C.B. Papadias, A transmitter diversity scheme for wideband CDMA systems based on space-time spreading, IEEE Journal on Sel. Areas in Com., vol. 19, no. 1, pp. 48-60, 2001.

[23] C. Xu, Y. Gong, K.B. Letaief, "High rate complex orthogonal space-time block codes for high number of transmit antennas", IEEE International Conference on Communications, ICC'04, Paris, France, vol. 2, pp. 823 - 826, June 2004.

[24] M. Rupp, C.F. Mecklenbräuker, G. Gritsch, "On Modal Subspaces of Extended Alamouti Space-Time Block Codes", 38. Asilomar Conference on Signals, Systems and Computers, Pacific Grove, Nov. 2004.

[25] Eko N. Onggosanusi, Anand G. Dabak, Timothy M. Schmidl, " High Rate Space-time Block Coded Scheme: Performance and Improvement in Correlated Fading Channels," Wireless Communications and Networking Conf. 2002, WCNC, pp.194-198, March 2002.

[26] C.F. Mecklenbräuker, M. Rupp, G. Gritsch, "High Diversity with Simple Space-Time Block-Codes and Linear Receivers", IEEE Global Telecommunications Conference, GLOBECOM'03, San Francisco (CA), USA, pp. 302-306, Dec. 2003.

[27] G. Gritsch, H. Weinrichter, M. Rupp, "A Tight Lower Bound for the Bit Error Performance of Space-Time Block Codes", 59th IEEE Vehicular Technology Conf. VTC Spring 2004, Milan, Italy, May 2004.

[28] A. Hottinen, O. Tirkkonen, R. Wichman. Multiantenna Transceiver Techniques for 3G and Beyond. Wiley, 2003.

[29] M. Rupp, C.F. Mecklenbräuker, G. Gritsch "On Mutual Information and Outage for Extended Alamouti Space-Time Block Codes," SAM'04, Barcelona, July 2004.

[30] M. Marques da Silva, A. Correia, "Space-Time Block Coding for 4 antennas with coding rate 1," IEEE Int. Symp. on Spread Spectrum Tech. and App., vol.2, pp. 318-322, Sept. 2002.

[31] N. Sharma, C. Papadias, "Improved quasi-orthogonal codes through constellation rotation," ICASSP02, Orlando, May 2002.

[32] W. Su, W.G. Xia, "Quasi-orthogonal space-time block codes with full diversity", IEEE Global Telecommunications Conference, GLOBECOM '02, vol. 2, pp. 10981102, Nov. 2002.

[33] K. Mukkavilli, A. Sabharwal, E. Erkip, B. Aazhang, "On beamforming with finite rate feedback in multiple antenna systems," IEEE Trans. Inf. Theory, vol. 49, no. 10, pp. 2562-2579, Oct. 2003.

[34] S. Zhou, Z. Wang, G.B. Giannakis, "Performance analysis for transmit-beamforming with finite-rate feedback," Conf. on Information Sciences and Systems, Princeton University, March 2004.

[35] L. Liu, H. Jafarkhani, "Combining beamforming and quasi-orthogonal space-time block coding using channel mean feedback", IEEE Global Telecom. Conference GLOBECOM '03, vol. 22, no. 1, pp. 1925-1930, Dec. 2003.

[36] G.Jöngren, M.Skoglund, "Utilizing quantized feedback information in orthogonal space-time block coding," IEEE Global Telecom. Conference GLOBECOM '02, vol. 2, pp. 1147-1151, Nov. 2002.

[37] E.G. Larsson, G. Ganesan, P. Stoica, W.H. Wong, "On the performance of orthogonal space-time block coding with quantized feedback," IEEE Comm. Letters, vol. 6, pp. 487-489, Nov. 2002.

[38] J. Akhtar, D. Gesbert, "Partial feedback based orthogonal block coding," The 57th IEEE Vehicular Technology Conf., 2003, VTC 2003-Spring, vol. 1, pp. 287-291, April 2003.

[39] D. Love, R. Heath, "Diversity performance of precoded orthogonal space-time block codes using limited feedback," IEEE Comm. Letters, vol. 8, pp. 308-310, May 2004.

[40] B. Badic, M. Rupp, H. Weinrichter, "Quasi-orthogonal space-time block codes for data transmission over four and eight transmit antennas with very low feedback rate," Int. ITG Conf. on Source and Channel Codig (SCC), Erlangen, Germany, pp. 157-164, Jan. 2004.

[41] B. Badic, M. Rupp, H. Weinrichter, "Adaptive Channel-Matched Extended Alamouti Space-Time Code Exploiting Partial Feedback," ETRI, vol. 26, no. 5, pp. 443-451, Oct. 2004.

[42] B. Badic, M. Rupp, H. Weinrichter, "Extended Alamouti codes in correlated channels using partial feedback", IEEE International Conf. on Com. ICC'04, vol. 2, pp. 896 - 900, Paris, France, June 2004.

[43] E. Aschbacher, S. Caban, C. Mehlführer, G. Maier, M. Rupp, "Design of a Flexible and Scalable 4×4 MIMO Testbed," 11th DSP Workshop, USA, Aug. 2004.

[44] Y. Zhao, R. S. Adve, T. J. Lim, "Optimal STBC precoding with channel covariance feedback for minimum error probability, EURASIP-JASP, no. 9, pp. 1257-1265, Sept. 2004.

[45] A. Alexiou. M. Qaddi, "Re-configurable Linear Precoders to Compensate for Antenna Correlation in Orthogonal and Quasi-Orthogonal Space-Time Block Coded Systems", 59th IEEE Vehicular Technology Conf. VTC Spring 2004, Milan, Italy, May 2004.

[46] D. Gore, A. Paulraj, "Space-time block coding with optimal antenna selection," Proc. of the IEEE International Conf. Acoustics, Speech, Signal Processing, vol 4, pp. 2441-2444, May 2001.

[47] Z. Chen, J. Yuan, B. Vucetic, Z. Zhou, "Performance of Alamouti scheme with transmit antenna selection,' IEE El. Letters, vol. 39, no. 23. pp. 1666-1668, Nov. 2003.

[48] B. Badic, P. Fuxjäger, H. Weinrichter, "Performance of a Quasi-Orthogonal Space-Time Code with Antenna Selection", IEE El. Letters, vol. 40, no. 20, pp. 1282-1284, Sep. 2004.

[49] B. Badic, P. Fuxjäger, H. Weinrichter , "Optimization of Coded MIMO-Transmission with Antenna Selection," VTC Spring, Sweden, May 2005.