IMPROVING THE ABILITY OF MATCHING PURSUIT ALGORITHM IN DETECTING SPIKES

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ABSTRACT

Detection of signal transients, or spikes, is a suitable application of time-frequency signal processing. One such time-frequency method for spike detection is matching pursuit, incorporating a redundant time-frequency dictionary. However, problems arise when using matching pursuit to detect repetitive rhythmical spiking, which is a common characteristic in an application such as newborn EEG seizure detection. In this paper we investigate the ability of matching pursuit to detect spikes both in synthetic signals and real newborn EEG seizure. It is shown that repetitive spikes may be recognised by matching pursuit as harmonic patterns rather than individual spikes. Consequently, these spikes cannot be located in the matching pursuit time-frequency domain representation. However, we have found that the relationship between the length of a repetitive spike sequence and interval between successive spikes in the sequence plays a pivotal role in the ability of matching pursuit to detect these spikes.

1. INTRODUCTION

Spike detection has found applications in many areas such as mechanical and biomedical engineering. In analysing brain functioning using the electroencephalogram (EEG), some types of abnormality, such as newborn EEG seizure, are characterised by spikes [1].

Matching pursuit (MP) decomposition using a time-frequency (TF) dictionary is a suitable tool for detecting spikes [2]. MP provides a signal representation using waveforms (atoms) selected from a redundant collection of atoms, referred to as a dictionary [3].

Unlike the unique representation from an orthogonal basis, multiple signal representations are possible when incorporating a redundant dictionary. However, using a redundant TF dictionary, MP can provide an adaptive TF representation, which can provide high TF resolution of signal components [3].

The number of spikes in EEG and distance between successive spikes can be used to detect EEG seizure [4]. The accuracy of a spike detection technique can be vital in detecting EEG seizures. In [2], the author proposed a new TF based spike detection technique using MP. It was reported that this technique has a better performance in detecting EEG spikes compared to the other existing methods. We have observed that the MP decomposition technique is very capable of detecting single spikes. However, in the case of successive spikes, which are a characteristic of newborn EEG seizure, the MP algorithm may consider these as harmonic patterns [5]. Consequently, these spikes cannot be located in the MP TF domain representation.

In this paper, we investigate the ability of the MP algorithm in detecting spikes. Using synthetic signals, we initially explain the mechanics of the MP algorithm. The results and observations from the synthetic examples, which indicate the requirements for MP to detect individual spikes from a repetitive spike sequence, are then applied to real newborn EEG seizure.

2. MATCHING PURSUIT

The MP atomic decomposition algorithm, using a redundant dictionary $\Phi = (\phi_\gamma)_{\gamma\in\Gamma}$, was introduced as a method of providing a signal approximation using a linear superposition of atoms from Φ [3]. The subscript γ refers to a parameter or multi-index parameter which uniquely defines each individual atom.

An approximation of a signal x using m atoms can be given as

$$\hat{x} = \sum_{i=0}^{m-1} \alpha_{\gamma i}\phi_{\gamma i} \tag{1}$$

where $\alpha_{\gamma i}$ is the atom coefficient. The approximation error, also referred to as the residual, is $R^m x = x - \hat{x}$ [3], where $R^0 x = x$.

The MP algorithm is an iterative process where the atom selected at each iteration is the one for which the projection of the residual is largest. That is, for iteration $i+1$ the index γi associated with the atom $\phi_{\gamma i}$ is determined by

$$\gamma i = \arg\{\sup_{\gamma\in\Gamma}\langle R^i x, \phi_\gamma\rangle\} \tag{2}$$

Using (2), the MP decomposition of signal x is given as

$$x = \sum_{i=0}^{m-1} \langle R^i x, \phi_{\gamma i}\rangle \phi_{\gamma i} + R^m x \tag{3}$$

where the inner product, $\langle R^i x, \phi_{\gamma i}\rangle$, is the coefficient value, $\alpha_{\gamma i}$, associated with the atom $\phi_{\gamma i}$. The dictionary is normalized so that the ℓ^2 norm of each atom is $||\phi_\gamma||_2 = 1$. This normalization removes any magnitude bias in the projection of the residual vector $R^i x$ onto any atom vector ϕ_γ.

A TF representation can then be obtained by the summation of the estimated TF contribution of each atom selected in the decomposition. A method of estimating the TF contribution is the Wigner-Ville distribution (WVD) [3] such that a TF representation can be obtained as

$$E(t,f) = \sum_{i=0}^{m-1} |\langle R^i x, \phi_{\gamma i}\rangle|^2 WVD_{\phi_{\gamma i}}(t,f)$$

3. SYNTHETIC SPIKE SEQUENCE REPRESENTATION USING MP

3.1 Fourier/Spike dictionary

There are a variety of TF dictionaries that have been previously used with the MP algorithm for signal decomposition. In our analysis of MP for detecting spikes we first consider a basic dictionary $\Phi = \Phi_1 \cup \Phi_2$, [6], where Φ_1 is the orthonormal spike basis

$$\phi_{1,\tau}(t) = 1_{\{t=\tau\}}, \qquad \tau = 0,1,\ldots,N-1$$

and Φ_2 is the orthonormal Fourier basis

$$\phi_{2,\xi}(t) = \frac{1}{\sqrt{N}} e^{j2\pi\xi t/N}, \qquad \xi = 0,1,\ldots,N-1$$

For this dictionary the atom indexer γ is the vector $[a,b]$ where a indicates whether it is a spike or Fourier atom and b is the index for that atom. Using this dictionary, spikes can either be represented with Fourier atoms, shown as horizontal lines in a TF plot (see Figure 1(a)), or spike atoms, shown as vertical lines in a TF plot (see Figure 1(b)).

The synthetic signals to be analysed in this section are spike trains, represented as

$$\mathbf{III}_N^T(n) = \begin{cases} 1 : n = l\cdot T, & l = 0,1,\ldots,N_t-1 \\ 0 : \text{else} \end{cases} \tag{4}$$

where $N = T \cdot N_t$, with period T samples between non-zero elements (spikes) and N_t number of spikes.

The ability of MP to detect periodic spikes using the Fourier/Spike dictionary can be better understood using the uncertainty principle for discrete signals defined in [7]. This principles states that for a discrete signal, $x(n)$, of length N and its discrete Fourier transform (DFT), $X(k)$, that

$$N_t \cdot N_f \geq N \tag{5}$$

where N_t is the number of non-zero points of $x(n)$ and N_f is the number of non-zero points of $X(k)$. It should be noted that the inequality of (5) does not indicate where $x(n)$ and $X(k)$ are non-zero: these may be intervals or any other sets [7] (i.e. this differs from the idea of bandwidth and duration for the Heisenberg uncertainty principle).

The periodic spike train $\mathbf{III}_N^T$ defined in (4) is a special signal in relation to this uncertainty principle as it provides the minimum uncertainty [7]. Also of importance is the DFT of $x(n) = \mathbf{III}_N^T$ which is $X(k) = K \cdot \mathbf{III}_N^\Omega(k)$. This Fourier sequence $X(k)$ has N_f non-zero points, the number of samples between two non-zero points is $\Omega = N/N_f$, and [7]

$$K = \sqrt{N_t/N_f} \tag{6}$$

Using the discrete uncertainty principle in (5), we can show that MP will represent $\mathbf{III}_N^T$ with spike atoms only if $T > \sqrt{N}$ and with Fourier atoms if $T < \sqrt{N}$ [7]. If a signal $\mathbf{III}_N^T$ has $T > \sqrt{N}$, then $N_t < \sqrt{N}$ and according to (5) $N_f > N_t$. Using (6) we see that $K < 1$, and that in the first MP iteration the spike atoms with coefficient of 1 from the Fourier/Spike dictionary will contain more signal energy than the Fourier atoms with coefficient K. Therefore a spike

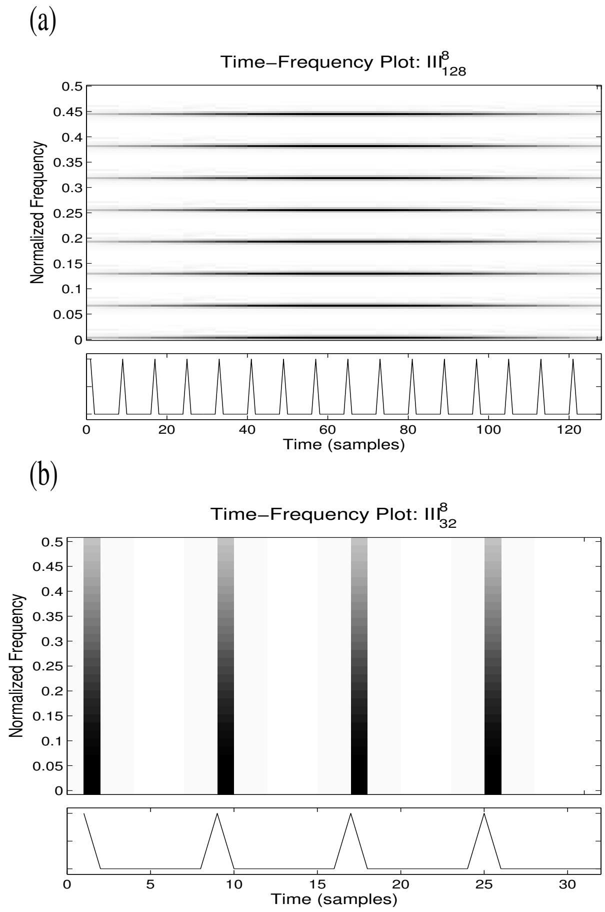

Figure 1: Time-frequency representations of synthetic spike trains with spike intervals of 8 samples and signal lengths of (a) 128 samples and (b) 32 samples

atom is chosen in the first iteration and the residual sequence becomes

$$R^1x = \begin{cases} 0 : n = 0,1,\ldots,T-1 \\ \mathbf{III}_{N-T}^T(n-T) : \text{else} \end{cases} \tag{7}$$

For the sequence R^1x in (7), there are only $N_{t1} = N_t - 1$ non-zero points. Therefore the number of non-zero points for the DFT is $N_{f1} > N_f$ using (5). Also the DFT of R^1x has N_f maximums. This means that in the second MP iteration, the spike atoms with coefficient 1 will contain more energy than any Fourier atoms as the maximum coefficient will be less than K. This means that a spike atom will be selected in the second iteration and MP will continue to select spike atoms in this way.

If $x = \mathbf{III}_N^T$ has $T < \sqrt{N}$, then using (5) $N_t > N_f$ and from (6) $K > 1$. This infers that any Fourier atom from the Fourier/spike dictionary with a non-zero coefficient will represent more signal energy than any spike atom as the maximum coefficient will be 1. Therefore, MP will select a Fourier atom in its first iteration. Removing the selected Fourier atom, we are left with a residual R^1x which has a

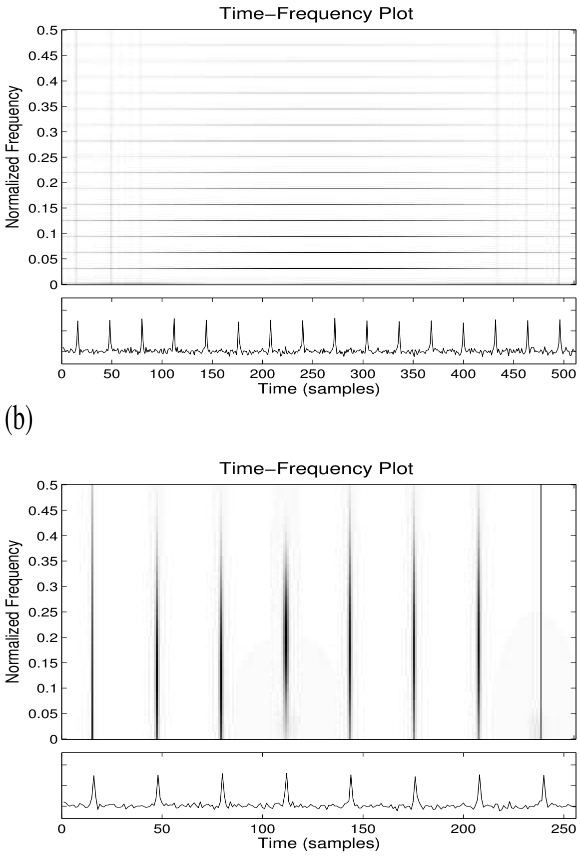

Figure 2: Time-frequency representations of synthetic spike trains with spike intervals of 32 samples and signal lengths of (a) 512 samples and (b) 256 samples

discrete Fourier transform

$$\text{DFT}\{R^1 x\} = \begin{cases} 0 : k = 0, 1, \ldots, f-1 \\ K \cdot \mathbf{III}^{T}_{N-f}(k-f) : \text{else} \end{cases} \tag{8}$$

It is known that if the $\text{DFT}\{x(n)\} = X(k)$, then according to the duality property, $\text{DFT}\{X(n)\} = \tilde{x}(-k)$, where $\tilde{x}(-k)$ is $x(k)$ index reversed [8]. Using the duality property and the results of the previous case when $T > \sqrt{N}$, we find that MP continues to select Fourier atoms to represent the synthetic spike train.

This result is demonstrated in the TF representations in Figure 1. It can be seen in Figure 1(a) that MP cannot represent individual spikes with spike atoms as a result of the signal, $\mathbf{III}^{8}_{128}$, having $T < \sqrt{N}$. However, when a smaller epoch of the signal, $\mathbf{III}^{8}_{128}$, is taken, for example $\mathbf{III}^{8}_{32}$ in Figure 1(b), the spikes are represented with spike atoms due to the spike interval being $T > \sqrt{N}$. This initial demonstration indicates that for this dictionary there is a strict relationship between the period and signal length for MP to detect the periodic spikes.

3.2 Gabor Dictionary

The above results are a start in the assessment of MP for detecting periodic spikes. However, the TF dictionary used

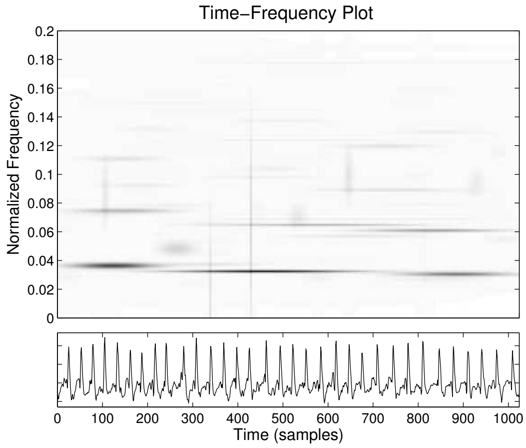

Figure 3: Time-frequency representations of real EEG spike sequence

is very basic and not often used in practice. The most frequently used redundant TF dictionary used with MP is a Gabor dictionary, Φ_G, and was initially proposed for MP [3]. The Gabor dictionary consists of translated (μ), modulated (ξ) and dilated (s) Gaussian windows $g(t)$ such that the Gabor atoms are

$$g_\gamma(t) = \frac{1}{\sqrt{s}} g\left(\frac{t-\mu}{s}\right) e^{j\xi t}$$

where $\gamma = [s, \mu, \xi]$ and $\gamma \in \mathbb{R}^+ \times \mathbb{R}^2$

Using the Gabor dictionary we find similar results to that for the Fourier/Spike dictionary occurs for the decomposition of repetitive spike sequences. Figure 2(a) shows a TF representation of a spike sequence signal with length 512 samples and periodic spiking activity. The spikes have a duration of 3 samples and are separated by 32 samples. The spike sequence also has white Gaussian noise added to it with a signal to noise ratio (SNR) of 10dB. From Figure 2(a) we can see that a majority of the spikes are not represented with spike atoms, which are shown in the TF representation as vertical lines. The few that are represented result from the end effects of the MP decomposition.

In Figure 2(b), we have taken an epoch of the signal in Figure 2(a) using the first 256 samples. In the TF representation of Figure 2(b) it is clear that MP can represent the signal spikes with spike atoms, allowing for spike detection. This again indicates the ability of MP to detect repetitive spikes is highly dependent on the relationship between the period of successive spikes and the signal length.

4. NEWBORN EEG SPIKE SEQUENCE REPRESENTATION USING MP

The newborn EEG seizure has been previously characterized in the time domain signal as displaying sharp repetitive waveforms [9]. Focal trains of sharp repetitive waveforms are a major form of ictal discharge in the newborn [10]. Spike events have been defined in [11] as having a time duration of between 20-70msec with significant amplitude. Sharp waves have duration that is slightly longer than spike waveforms.

An example of newborn EEG seizure containing repetitive spikes is shown in Figure 3. The MP decomposition of

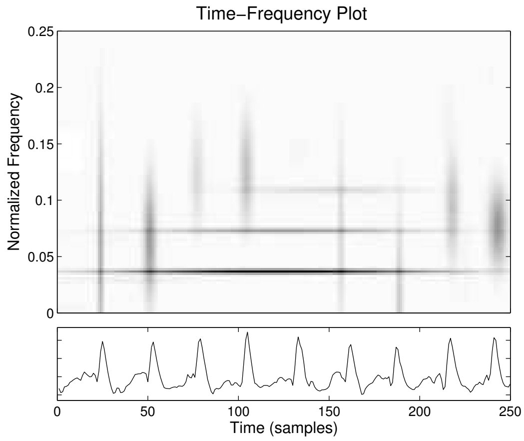

Figure 4: Time-frequency representation of an epoch at the start of the real EEG spike sequence in Figure 3

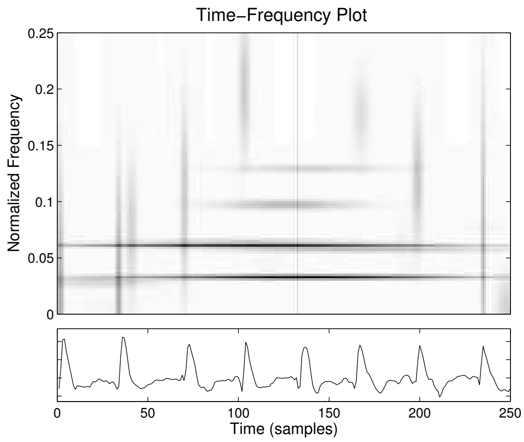

Figure 5: Time-frequency representation of an epoch at the end of the real EEG spike sequence in Figure 3

the EEG signal, of length 1024 samples, was performed with MP iterations continuing until the energy in the approximation error $R^i x$ was less than 5% of the energy in the original signal x. The TF representation using MP is also presented in Figure 3. From the TF representation we can see that MP fails to represent the repetitive spikes with vertical line patterns which are representative of spike waveforms. Instead, MP TF representation only shows the harmonic relationship between EEG seizure spikes, illustrated by horizontal lines.

The spike sequence in Figure 3 contains 33 individual spike events. Of these individual spike events, the MP TF representation only shows 4 clear spike events, indicated by vertical lines in the TF representation. However, the results of the previous section demonstrate that MP may be able to detect the repetitive spikes if smaller epochs are chosen.

Figures 4 and 5 are epochs of length 250 samples taken at the start and end of the signal shown in Figure 3. The TF representations in Figures 4 and 5 clearly illustrate the successive spike events, which are displayed with vertical lines. There is a total of 17 spikes in these two epochs for which MP clearly indicates 16. This is a dramatic improvement in the true spike detection rate of the same signal.

5. CONCLUSION

The detection of transient signals or spikes is an important application in signal processing, especially in the newborn EEG where abnormal brain functioning can be represented as spikes in the EEG. Detection of spikes in EEG, which may characterise seizures, is significant. In this paper we have investigated the ability of MP in detecting repetitive spike sequences using synthetic and real signals. We have shown that the ability of MP to detect repetitive spikes depends significantly on the relationship between signal length and the interval between successive spikes. Generally, we find shorter epochs are better for spike detection. Incorporating this finding can increase the ability of MP in detecting spikes.

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