A ROBUST NON-UNIFORM LUT INDEXING METHOD IN DIGITAL PREDISTORTION LINEARIZATION OF RF POWER AMPLIFIERS

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ABSTRACT

A new monotonic companding function is proposed for non-uniform lookup table (LUT) indexation used in digital predistortion (PD) of Radio Frequency (RF) power amplifiers (PA) under wideband signals. This function, designed to be robust to the input signal statistics variations, overcomes the disadvantages of optimal indexation which depends on the input back-off (IBO) operation point of the PA. Simulations results show the robustness of the proposed companding function to IBO variations, comparing with conventional LUT indexing techniques, namely power, amplitude and optimal.

1. INTRODUCTION

In wideband digital mobile communications, linearization of RF power amplifiers is crucial. Digital baseband predistortion seems to be a promising linearization technique allowing better power efficiency and reduced implementation complexity. It is often used in wide band Code Division Multiple Access (WCDMA) applications

Fig. 1 shows the cascade of the baseband PD and the PA. The PD fed with the complex envelope $v_m(t)$ of the input modulated signal, produces the predistorted signal $v_d(t)$ according to $v_d = v_m f(r_m)$, where $r_m = |v_m(t)|$ and $f(r)$ is the complex gain of the predistorter. The amplifier output $v_a(t)$ is then $v_a = v_d g(r_d)$, where $r_d = |v_d(t)|$ and $g(r)$ is the complex gain of the amplifier.

Ideally, the PD is optimized to produce a linear gain K, so that

$$v_a(t) = K v_m(t) \tag{1}$$

The condition to be satisfied by the optimum PD is then

$$f_o(r_m) g(r_m |f_o(r_m)|) = K \tag{2}$$

The PD may be implemented as LUT tables in which suitable values are stored as a discretization of f_o. The table is indexed as a function of the amplitude of the baseband input signal.

Nonuniform spacing of the table is investigated in order to provide better performances in terms of intermodulation (IM) power reduction. Cavers in [3] computed a nonuniform companding function named optimum, that minimizes the IM distortion power due to LUT's quantification errors. This function leads to the lowest IM power for a predetermined power backoff operation point of the PA. This dependency on the IBO level of the optimum indexation method reduces its robustness.

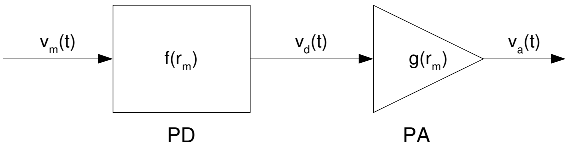


Figure 1: Predistorter (PD) / Power amplifier (PA) scheme.

To overcome this drawback, we proposed in [4] a non uniform indexing method which is robust to IBO variations. In this paper, a theoretical analysis is developed for the Saleh model of the PA. In the first part of the present paper, a robustness measure for a given LUT based predistortion scheme is presented. In the second part, the non robustness of the optimum companding function is analyzed. The third part reports the new proposed companding function independent on the input signal statistics. Simulations enhance the performances of this new indexing method through a comparison with three other companding functions (optimum, amplitude and power).

2. ROBUSTNESS MEASURE FOR A GIVEN LUT BASED PREDISTORTION SCHEME

As described in section 1, the ideal PD is evaluated for a given PA and a given input signal. An error $v_{ae} = v_a - K v_m$ occurs at the PA output in two cases:

- a variation of the PA characteristics which can occur due to temperature drift, aging or biasing point drifting;
- a variation in the input signal statistics which can occur due to network load and traffic conditions.

These variations modify the PD/PA behavior as described in the following.

2.1 Sensitivity of the ideal PD to the PA parameter's variations: case of Saleh's model

The PA exhibits two nonlinear distortions, which are amplitude distortion (AM/AM) and phase distortion (AM/PM). In this study, we adopt for the PA the Saleh model considering only the AM/AM distortion. The PA complex gain is then given by $g(r) = \dfrac{\alpha_0}{1+\beta_0 r^2}$, where r is the input amplifier am-

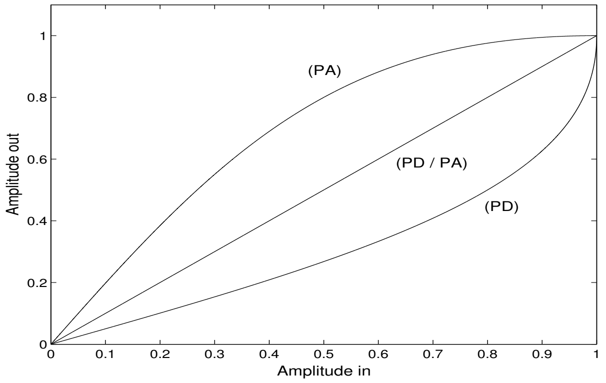


Figure 2: PA gain, optimal PD gain and ideal PD/PA gain.

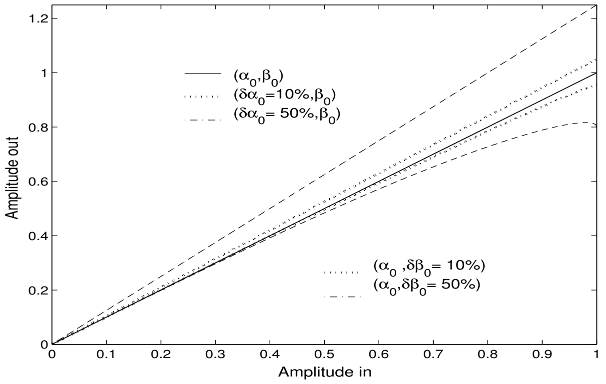


Figure 3: Sensitivity of the PD/PA gain to the PA ($\delta\alpha_0$,$\delta\beta_0$) parameters variations.

plitude.

Equation (2) can be solved assuming that the ideal linear gain is $K = 1$, without loss of generality. The optimum PD is then

$$f_o(r_m) = \frac{\alpha_0 - \sqrt{\alpha_0^2 - 4\beta_0 r_m^2}}{2\beta_0 r_m^2} \tag{3}$$

In all simulations, we choose ($\alpha_0 = 2.1517$, $\beta_0 = 1.1517$). The input r_m is normalized in the range $[0,1]$.

Fig. (2) shows the gain of the PA, the associated ideal PD gain given by f_o and the final linear gain ($K = 1$) obtained through the cascade PD/PA. The PD is evaluated for the set (α_0,β_0). Fig. 3 shows the sensitivity of the PA parameters variations on the linearity of the PD/PA system.

2.2 Sensitivity of the implemented LUT based PD to the statistical input variations

The PD is implemented using an LUT table (fig. (4)). The inverse characteristic of the PA given by the optimum PD gain f_o, is discretized and stored into the LUT table. The entries of the LUT are equispaced in y, that is an index variable related to the amplitude of the baseband input signal r_m. The distorted signal $v_d(t)$ is then extracted.

The key of analysis of nonuniform spacing is the companding function $s(r)$ inserted between the input signal amplitude r_m and the LUT index $y = s(r_m)$. The amplifier model is normalized to saturate at unit amplitude, the companding function should be also defined over the same interval having

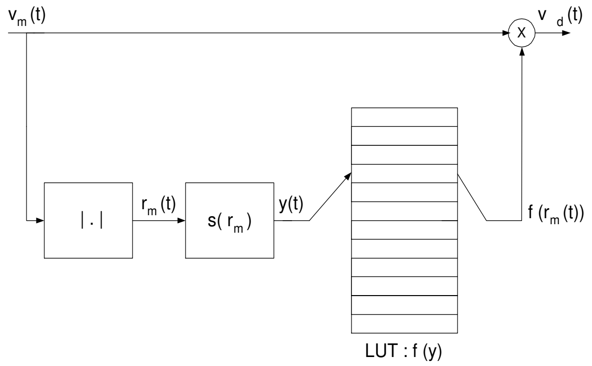


Figure 4: Internal structure of the predistorter.

$s(0) = 0$ and $s(1) = 1$, and is monotonically increasing, so that $s'(r) > 0$ for $0 \leq r \leq 1$.

Because the PD is defined only at table points, an error $v_{ae} = v_a - Kv_m$ occurs in the PA output. The IM power is then obtained by $P_{ae} = \int_0^1 E[|v_{ae}|^2]$.

Cavers in [3] proposed a simple way to compute the IM distortion power for any companding function. The total IM distortion power is given by

$$P_{ae} = \int_0^1 \frac{w(r_m)}{s'(r_m)^2} p_r(r_m) dr_m \tag{4}$$

where $s'(r_m)$ is the derivative of the companding function and $w(r_m)$ is the non negative function defined as

$$w(r_m) = \frac{r_m^2}{12N_t^2} \frac{|g'(r_m|f_o(r_m)|)|^2}{|g(r_m|f_o(r_m)|)|^4} \tag{5}$$

where N_t is the table length and $p_r(r_m)$ is the input amplitude signal pdf (probability density function).

In the case of WCDMA applications, the complex input has white gaussian real and imaginary parts, the r_m pdf is truncated Rayleigh as follows

$$p_r(r_m) = \frac{2r}{IBO} exp\left(\frac{-r^2}{IBO}\right)\left(1 - exp\left(\frac{-1}{IBO}\right)\right)^{-1} \tag{6}$$

where the IBO is defined as the ratio of actual input power to input power required for saturation.

2.3 Notion of robustness of a given predistortion scheme

A predistorter is considered to be robust, if its performances are independent on the input signal statistics variations. Note that, the statistics can vary both through pdf variations (Gaussian, Laplacian, ...) and IBO variations. Only robustness to IBO variations will be considered in this paper.

3. NON ROBUSTNESS OF THE OPTIMUM INDEXING APPROACH

The minimization of the IM distortion power given by (4), leads to optimal companding function [3]

$$s_o'(r_m) = w(r_m)^{\frac{1}{3}} p_r(r_m)^{\frac{1}{3}} \left(\int_0^1 w(r_m)^{\frac{1}{3}} p_r(r_m)^{\frac{1}{3}} dr_m\right)^{-1} \tag{7}$$

and the corresponding minimum IM distortion power is

$$P_{ae_o} = \left(\int_0^1 w(r_m)^{\frac{1}{3}} p_r(r_m)^{\frac{1}{3}} dr_m \right)^3 \tag{8}$$

This optimal companding function depends on the input signal pdf $p_r(r_m)$, and in particular on the input backoff (IBO). This means that a different function is required at every backoff value. This introduces the question of mismatch between optimization conditions and operating conditions.
In order to show the non robustness of the optimum companding functions, we compare the IM to signal power ratio resulting from the optimum companding function with equal spacing in amplitude ($s_a(r_m) = r_m$) and spacing in power ($s_p(rm) = r_m^2$). This ratio is defined as

$$\frac{P_s}{P_{ae}} = \frac{\displaystyle\int_0^1 r_m^2 p_r(r_m)}{\displaystyle\int_0^1 \frac{w(r_m)}{s'(r_m)^2} p_r(r_m) dr_m} \tag{9}$$

The LUT length adopted for simulations is $N_t = 200$.

On fig. (5), are shown the IM to signal power ratio for the optimum companding function optimized at all backoffs and the companding function optimized for one particular backoff value, (respectively -10 dB for fig. (5a) and 0 dB for fig. (5b)), but used at all backoffs. The main remarks are:

- The optimum function (optimized at each IBO) defines naturally the limit for all other spacing methods (amplitude and power).
- For the considered PA model, indexing by power leads to better results than amplitude almost everywhere.
- When the spacing is optimized for one backoff value but used at all backoffs, the results are disappointing. In this case, the simpler equispacing by power leads to better performances almost everywhere. This confirm the non robustness of optimum indexation.

These limitations of the optimal function motivated this study. The new non uniform indexation method proposed in the following is robust on the input signal statistics variations .

4. COMPANDING FUNCTION ROBUST TO THE INPUT SIGNAL STATISTICS VARIATIONS (IND. PDF

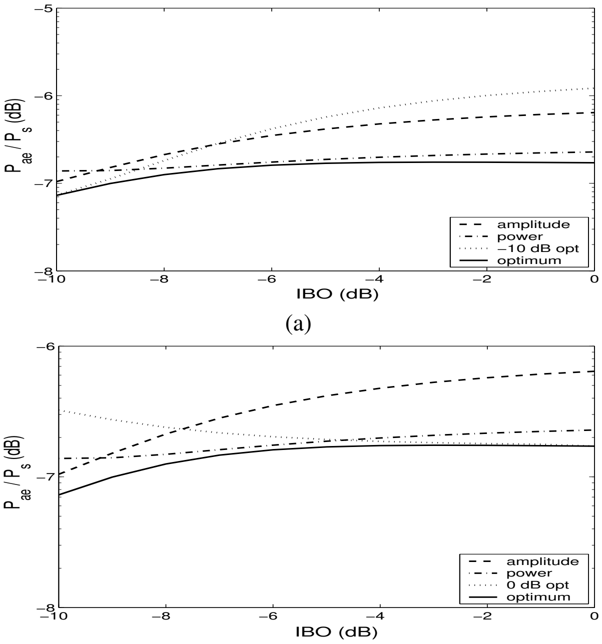
From relations (9) and (5), the IM to signal power ratio is

$$\frac{P_s}{P_{ae}} = \frac{\displaystyle\int_0^1 r_m^2 p_r(r_m)}{\displaystyle\frac{1}{12N_t^2}\int_0^1 r_m^2 \frac{|g'(r_m|f_o(r_m)|)|^2}{s'(r_m)^2 |g(r_m|f_o(r_m)|)|^4} p_r(r_m) dr_m} \tag{10}$$

This ratio is independent on the input signal pdf $p_r(r_m)$ if we impose the following condition on the companding derivative function

$$s'(r_m) = \frac{a|g'(r_m|f_o(r_m)|)|}{|g(r_m|f_o(r_m)|)|^2} \tag{11}$$

where a is a positive constant.
Under these conditions, the signal to intermodulation power rate for the proposed companding function is



(a)

(b)

Figure 5: Non robustness of the optimal companding function.

$$\frac{P_s}{P_{ae}} = 12 N_t^2 a^2 \tag{12}$$

The choice of the constant a is imposed by the constraints $s(0) = 0$ and $s(1) = 1$

$$a = \frac{1}{\displaystyle\int_0^1 \frac{|g'(r_m|f_o(r_m)|)|}{|g(r_m|f_o(r_m)|)|^2} dr_m} \tag{13}$$

For the considered PA, a numerical evaluation of relation (13) gives $a = 3.3165$. For ($N_t = 200$, IBO=-5 dB), fig. (6) shows the derivative of selected companding functions. For the considered PA model, we have $a = 3.3165$. At first sight, we remark the strong similarities between the three laws: ind. pdf, power and optimum.

Fig. (7) shows the IM to signal power ratio through backoff level variations. The main remarks are:

- the new indexing method (ind. pdf) gives a constant ratio which confirm the theoretical expected value of relation (12): $\frac{P_{ae}}{P_s} = \frac{1}{12 N_t^2 a^2} = 1.8941\ 10^{-7}$;
- the ind. pdf for the considered IBO range gives almost everywhere better performances than amplitude spacing and equivalent performances with power spacing;
- when the spacing is optimized for one backoff value (-10 dB on fig. (7a) and 0 dB on fig. (7b)), the ind. pdf spacing offers better performances almost everywhere and is considered to be more robust to IBO variations.

We focus through fig.(8) on the robustness of the considered companding functions on the change of the amplifier.

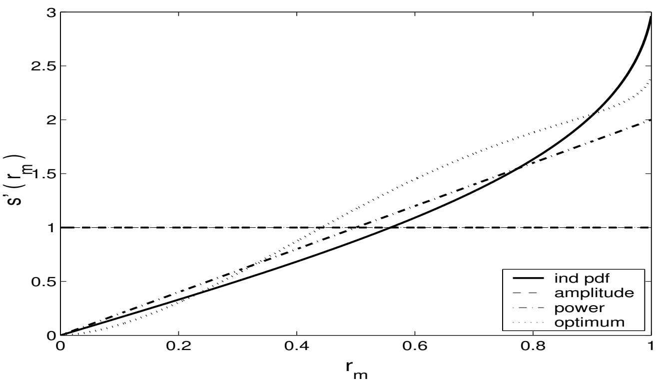


Figure 6: Derivative of companding functions.

For ($N_t = 200$, IBO=0 dB), the companding functions are evaluated for the parameter's set (α_0, β_0) of the PA model. The IM to signal power ratio is computed considering a changement into the PA's parameters. We remark that:

- For all indexing methods, the sensitivity seems to be more important to α variations.
- For α variation, the independent pdf indexing is more robust, since the degradation of the IM to signal power ratio is less important for this new indexation than the optimum one.

5. CONCLUSIONS

This paper has presented a new basic result: simple expression for the independent pdf nonuniform spacing of predistortion table entries and its performances. It also provided a numerical comparison of four candidate methods: equispacing by amplitude, power, both of which are fixed, optimum spacing, which depends on amplifier and backoff, and independent pdf spacing which only depends on the power amplifier.

The main result is that whether optimum spacing is strongly dependent on the changes in backoff level, independent pdf spacing offers robust performances on IBO variations.

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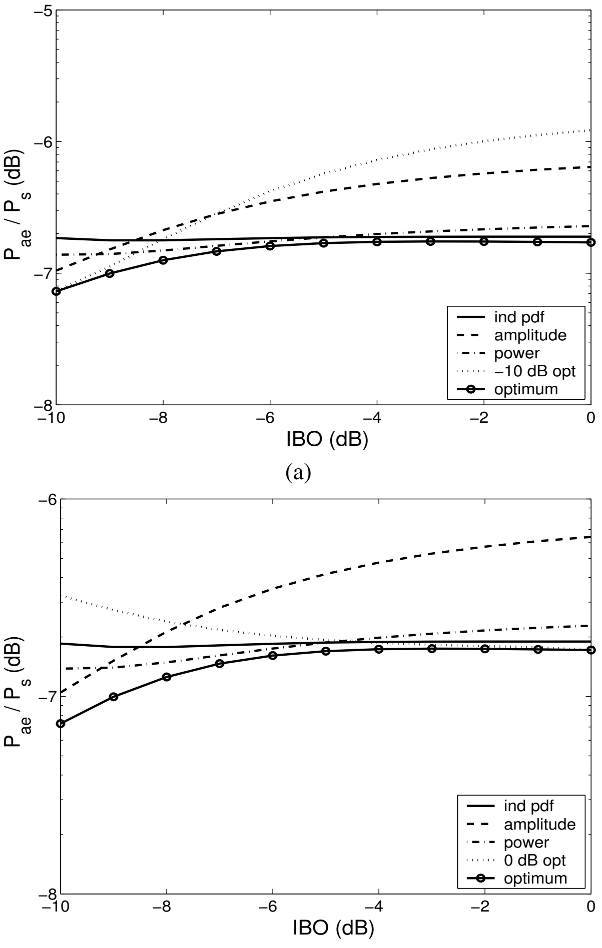

(a)

(b)

Figure 7: Robustness of the proposed independent pdf companding function.

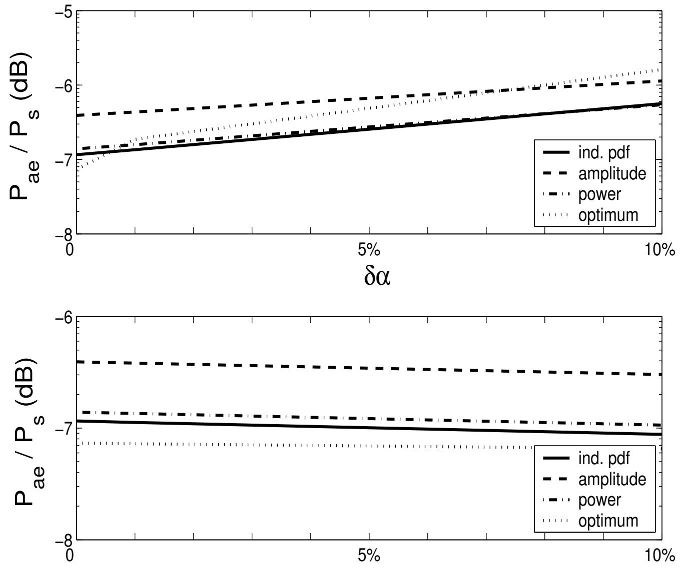

Figure 8: Sensitivity of indexation to the PA parameters variations.