ASYMPTOTIC NORMALITY OF STATISTICAL-FUNCTION ESTIMATORS FOR GENERALIZED ALMOST-CYCLOSTATIONARY PROCESSES

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ABSTRACT

The problem of estimating second-order statistical functions of generalized almost-cyclostationary (GACS) processes is addressed. The class of such nonstationary processes includes, as a special case, the almost-cyclostationary (ACS) processes. ACS processes filtered by Doppler channels and communications signals with time-varying parameters are further examples. It is shown that, for GACS processes, the cyclic correlogram is an asymptotically Normal mean-square consistent estimator of the cyclic autocorrelation function. Thus, well-known results for ACS processes can be obtained as a special case of the results of this paper.

1. INTRODUCTION

In the last two decades, a big effort was devoted to analysis and exploitation of the properties of the almost-cyclostationary (ACS) processes. In fact, almost-all modulated signals adopted in communications can be modelled as ACS [3], [16]. For ACS processes, multivariate statistical functions are almost-periodic functions of time and can be expressed by (generalized) Fourier series expansions whose frequencies, referred to as cycle frequencies, do not depend on the lag shifts of the processes.

More recently, wider classes of nonstationary processes have been considered in [7]–[12]. In particular, in [7], the class of the generalized almost-cyclostationary (GACS) processes has been introduced and characterized. Processes belonging to this class exhibit multivariate statistical functions that are almost-periodic functions of time whose Fourier series expansions have coefficients and frequencies, referred to as lag-dependent cycle frequencies, that can depend on the lag shifts of the processes. The class of the GACS processes includes, as a special case, the class of the ACS processes. Moreover, chirp signals and several angle-modulated and time-warped communication signals are GACS processes. In [8] and [9], it is shown that several time variant channels of interest in communications transform a transmitted ACS signal into a GACS one. In particular, in [9] it is shown that the GACS model is appropriate to describe the output signal of Doppler channels when the input signal is ACS and the product transmitted-signal-bandwidth times data-record-length is not too small. Thus, the GACS model turns out to be useful in modern mobile communication systems where wider and wider bandwidths are required to get higher and higher bit rates and, moreover, large data-record lengths are necessary for blind channel identification techniques or detection algorithms in highly noise- and interference-corrupted environments. In [7], [8], and [9], it is also shown that communications signals with slowly time-varying parameters, such as carrier frequency or baud rate, should be modelled as GACS, rather than ACS, if the data-record length is such that the parameter time variations can be appreciated.

The autocorrelation function of GACS processes is completely described by the cyclic autocorrelation function as a function of the two variables cycle frequency and lag shift [11]. Such a function is defined analogously to the case of ACS processes, but is non zero in a more than countable set of values of cycle frequency.

In this paper, the cyclic correlogram is proposed as an estimator of the cyclic autocorrelation function of GACS processes. It is shown that, for GACS stochastic processes satisfying some mixing conditions expressed in terms of summability of their second- and fourth-order cumulants, the cyclic correlogram, as a function of the two variables lag shift and cycle frequency, is a mean-square consistent and asymptotically Normal estimator of the cyclic autocorrelation function. Furthermore, in the limit as the data-record length approaches infinite, the region of the cycle-frequency lag-shift plane where the cyclic correlogram is significantly different form zero becomes a thin strip around the support curves of the cyclic autocorrelation function, that is, around the lag-dependent cycle frequency curves. Thus, the proved asymptotic Normality result can be used to establish statistical tests for presence of generalized almost-cyclostationarity. Finally, it is shown that the well-known result for ACS processes that the cyclic correlogram is a mean-square consistent and asymptotically Normal estimator of the cyclic autocorrelation function (see [1], [2], [4], [5], [6]) can be obtained as a special case of the results established in this paper.

2. GACS STOCHASTIC PROCESSES

A finite-power complex-valued continuous-time stochastic process $x(t)$ is said *second-order GACS in the wide sense* [7], [8] if its autocorrelation function is almost-periodic in t:

$$\mathcal{R}_{xx^*}(t, \ ) \triangleq \mathrm{E}\{x(t+ \ )\, x^*(t)\} = \sum_{\ \in A} R_{xx^*}(\ , \ )\, e^{j2 \quad t} \tag{1}$$

where

$$R_{xx^*}(\ , \ ) \triangleq \lim_{T\to} \frac{1}{T}\int_{-T/2}^{T/2} \mathcal{R}_{xx^*}(t, \ )\, e^{-j2 \quad t}\, \mathrm{d}t \tag{2}$$

is the *cyclic autocorrelation function* at *cycle frequency* . Moreover,

$$A \ \triangleq \{ \ \in \mathbb{R} : R_{xx^*}(\ , \ ) \neq 0\} \tag{3}$$

is a countable set which, in general, depends on .

Note that, even if the set A is always countable, the set

$$A \triangleq \bigcup_{\ \in \mathbb{R}} A \tag{4}$$

is not necessarily countable. Thus, the class of the second-order wide-sense GACS processes extends that of the wide-sense ACS which are obtained as a special case of GACS processes when the set A is countable [2].

A useful characterization of wide sense GACS processes can be obtained by observing that the set A can be expressed as [7], [8]

$$A \ = \bigcup_{n\in\mathbb{I}} \{ \ \in \mathbb{R} : \ = \ _n(\ )\} \tag{5}$$

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where $\mathbb{I}$ is a countable set and the functions $\ _n(\ )$, referred to as *lag dependent cycle frequencies*, are such that, for each and , there exists at most one $n \in \mathbb{I}$ such that $\ = \ _n(\ )$. Thus, the autocorrelation function $\mathcal{R}_{xx^*}(t,\ )$ of a second-order wide-sense GACS process can be expressed as [7], [8]

$$\mathcal{R}_{xx^*}(t,\ ) = \sum_{n\in\mathbb{I}} R^{(n)}_{xx^*}(\ )\, e^{j2\ \ _n(\ )t} \tag{6}$$

where the functions $R^{(n)}_{xx^*}(\ )$, referred to as *generalized cyclic autocorrelation functions*, are defined as

$$R^{(n)}_{xx^*}(\ ) \triangleq \begin{cases} \lim\limits_{T\to\ } \dfrac{1}{T}\displaystyle\int_{-T/2}^{T/2} \mathcal{R}_{xx^*}(t,\ )\, e^{-j2\ \ _n(\ )t}\, \mathrm{d}t\,, & \ \in \mathcal{T}^{(n)} \\ 0\,, & \ \in \mathbb{R}-\mathcal{T}^{(n)} \end{cases} \tag{7}$$

$$\mathcal{T}^{(n)} \triangleq \{\ \in \mathbb{R} :\ \ _n(\ ) \text{ is defined}\}$$

Note that, in (6) the sum ranges over a set not depending on as, on the contrary, it occurs in (1). Moreover, unlike the case of second-order ACS processes, both coefficients and frequencies of the Fourier series in (6) depend on the lag parameter . Thus, the wide-sense ACS processes are obtained as a special case of GACS processes when the lag-dependent cycle frequencies are constant with respect to and, hence, are coincident with the cycle frequencies [7].

In [7], [8], it is shown that, by properly defining the functions $R^{(n)}_{xx^*}(\ )$ in the discontinuity points, the cyclic autocorrelation function and the generalized cyclic autocorrelation functions are related by the relationship

$$R_{xx^*}(\ ,\ ) = \sum_{n\in\mathbb{I}} R^{(n)}_{xx^*}(\ )\ \ _{-\ _n(\ )} \tag{8}$$

where denotes Kronecker delta, that is, $\ = 1$ for $\ = 0$ and $\ = 0$ otherwise.

In the special case of ACS processes, the lag dependent cycle frequencies are constant and coincident with the cycle frequencies, only one term is present in the sum in (8) and, consequently, the generalized cyclic autocorrelation functions are coincident with the cyclic autocorrelation functions.

In Figure 1, the support in the $(\ ,\ )$ plane of the cyclic autocorrelation function $R_{xx^*}(\ ,\ )$ is reported for (a) an ACS signal and (b) a GACS signal. For an ACS signal, such a support is constituted by lines parallel to the axis in correspondence of the cycle frequencies. For a GACS signal, the support is constituted by the curves $\ = \ _n(\ )$, $n \in \mathbb{I}$ (see (8)).

Examples of GACS signals are nonuniformly sampled signals [7] and modulated signals with sinusoidally varying carrier frequency. The former can be expressed as

$$x(t) \triangleq w(t) \sum_{k\in\mathbb{Z}} \ (t - kT_p(t)) \tag{9}$$

where $\ (\cdot)$ denotes Dirac's delta and $T_p(t)$ is a slowly time varying sampling period, and the latter can be written as

$$x(t) = w(t)\cos(2\ (f_0 + \ \cos(2\ f_m t))t) \tag{10}$$

where $\ \ll 1$ and, in both examples, $w(t)$ is a stationary or ACS signal. Moreover, the output $y(t)$ of the Doppler channel existing between a transmitter and a receiver with nonzero relative radial acceleration is GACS when the input signal $x(t)$ is ACS [9]. Such a channel is characterized by the input-output relationship

$$y(t) = a\,x(t - D(t)) \tag{11}$$

where a is attenuation and

$$D(t) \triangleq d_0 + d_1 t + d_2 t^2 \quad d_2 \neq 0 \tag{12}$$

is a quadratically time-varying delay. Further examples where the GACS model turns out to be appropriate in mobile communications systems can be found in [8], [9].

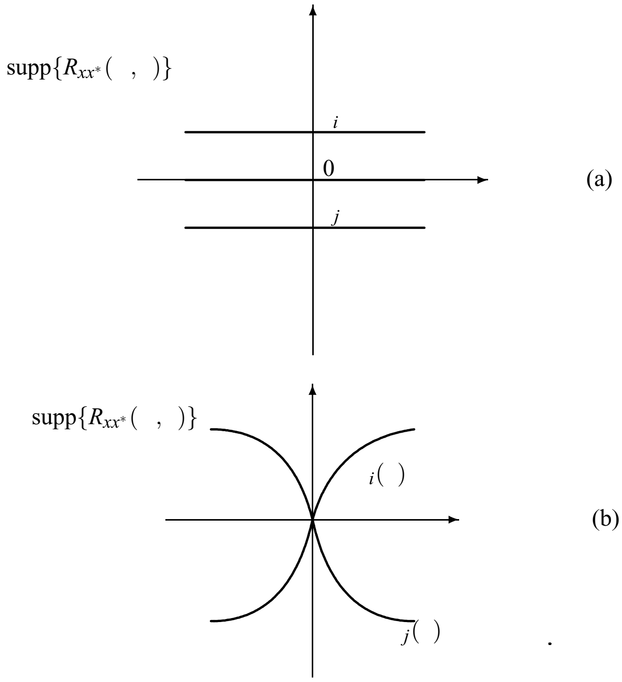

Figure 1: Support in the $(\ ,\ )$ plane of the cyclic autocorrelation function $R_{xx^*}(\ ,\ )$ of (a) an ACS signal and (b) a GACS signal.

3. MEAN-SQUARE CONSISTENCY OF THE CYCLIC CORRELOGRAM

The *cyclic correlogram* is defined as

$$R_{xx^*}(\ ,\ ;t_0,T) \triangleq \int_{\mathbb{R}} w_T(t-t_0)\,x(t+\ )\,x^*(t)\,e^{-j2\ \ t}\,\mathrm{d}t \tag{13}$$

where $w_T(t)$ is a unit-area data-window nonzero in $(-T/2, T/2)$.

In order to prove the asymptotic properties of the cyclic correlogram, the following assumptions should be made.

Assumptions

1) The stochastic process $x(t)$ is (second-order) GACS in the wide sense, that is, for any choice of z_1 and z_2 in $\{x, x^*\}$,

$$\mathrm{E}\{z_1(t+\ _1)\,z_2(t)\} = \sum_n R^{(n)}_{z_1 z_2}(\ _1)\,e^{j2\ \ ^{(n)}_{z_1,z_2}(\ _1)t}\,. \tag{14}$$

2) For any choice of z_1 and z_2 in $\{x, x^*\}$, the fourth-order cumulant $\mathrm{cum}\{x(t+\ _1), x^*(t+\ _2), z_1(t+\ _3), z_2(t)\}$ can be expressed as

$$\begin{aligned}&\mathrm{cum}\{x(t+\ _1), x^*(t+\ _2), z_1(t+\ _3), z_2(t)\} \\ &\quad = \sum_n C^{(n)}_{xx^*z_1z_2}(\ _1,\ _2,\ _3)\,e^{j2\ \ _n(\ _1,\ _2,\ _3)t}\end{aligned} \tag{15}$$

where cumulants of complex processes are defined according to [16, App. A].

3) For any choice of z_1 and z_2 in $\{x, x^*\}$ it results

$$\sum_n \|R^{(n)}_{z_1z_2}\|\ \ < \tag{16}$$

where $\|R\|\ \ \triangleq \mathrm{ess\,sup}_{\ \in\mathbb{R}} |R(\ )|$ is the essential supremum of $R(\ )$.

4) For any choice of z_1 and z_2 in $\{x,x^*\}$ it results

$$\sum_{n} \|C^{(n)}_{xx^* z_1 z_2}\| \quad < \quad . \tag{17}$$

5) There exists a positive number M_4 such that

$$\mathrm{E}\left\{|x(t)|^4\right\} \leqslant M_4 < \qquad \forall t \in \mathbb{R}\,. \tag{18}$$

6) $w_T(t)$ is a T-duration data-tapering window that can be expressed as

$$w_T(t) = \frac{1}{T}\, a(t/T) \tag{19}$$

with $a(t) \in L^1(\mathbb{R}) \cap L \;\; (\mathbb{R})$,

$$\int_{\mathbb{R}} a(t)\, \mathrm{d}t = 1 \tag{20}$$

$$\lim_{T\to} a(t/T) = 1 \quad \forall t \in \mathbb{R}\,. \tag{21}$$

7) For any choice of z_1 and z_2 in $\{x,x^*\}$ it results

$$\sum_{n} \int_{\mathbb{R}} \left| R^{(n)}_{z_1 z_2}(s) \right| \mathrm{d}s < \quad . \tag{22}$$

8) For any choice of z_1 and z_2 in $\{x,x^*\}$ and $\forall \;_1, \;_2 \in \mathbb{R}$ it results

$$\sum_{n} \int_{\mathbb{R}} \left| C^{(n)}_{xx^* z_1 z_2}(s + \;_1, s, \;_2) \right| \mathrm{d}s < \quad . \tag{23}$$

Under the above assumptions we have [13]

$$\lim_{T\to} \mathrm{E}\{R_{xx^*}(\;,\;;t_0,T)\} = R_{xx^*}(\;,\;) \tag{24}$$

$$\lim_{T\to} T \operatorname{cov}\{R_{xx^*}(\;_1,\;_1;t_1,T), R_{xx^*}(\;_2,\;_2;t_2,T)\} = \mathcal{O}(1)\,. \tag{25}$$

That is, the cyclic correlogram is a mean-square consistent estimator of the cyclic autocorrelation function.

4. ASYMPTOTIC NORMALITY OF THE CYCLIC CORRELOGRAM

Let

$$z_i(t) \triangleq \left[x(t + \;_i)\, x^*(t)\right]^{[*]_i}, \quad i = 1, \dots, k \tag{26}$$

be second-order lag-product waveforms with optional complex conjugations $[*]_i$, $i = 1,\dots,k$, and let us make the following assumptions:

Assumptions

9) The stochastic processes $z_i(t)$, $i = 1,\dots,k$ are jointly kth-order GACS; that is

$$\begin{aligned} &\operatorname{cum}\{z_k(t), z_i(t+s_i),\; i = 1,\dots,k-1\} \\ &\quad = \sum_{n} C^{(n)}_{z_1\cdots z_k}(s_1,\dots,s_{k-1})\, e^{j2 \;\; ^{(n)}_{z_1\cdots z_k}(s_1,\dots,s_{k-1})t}\,. \end{aligned} \tag{27}$$

10) For every $\;_i$, $i = 1,\dots,k$ and every conjugation configuration $[*]_1,\dots,[*]_k$, it results that

$$\sum_{n} \int_{\mathbb{R}^{k-1}} \left| C^{(n)}_{z_1\cdots z_k}(s_1,\dots,s_{k-1}) \right| \mathrm{d}s_1 \cdots \mathrm{d}s_{k-1} < \quad . \tag{28}$$

11) For every $k \in \mathbb{N}$ and every $\{\ell_1,\dots,\ell_n\} \subseteq \{1,\dots,k\}$, there exists a positive number $M_{\ell_1\cdots\ell_n}$ such that

$$\mathrm{E}\left\{|z_{\ell_1}(t_1)\cdots z_{\ell_n}(t_n)|\right\} \leqslant M_{\ell_1\cdots\ell_n} < \qquad \forall t_1,\dots,t_n \in \mathbb{R}\,. \tag{29}$$

Under Assumptions 1)–11), the following result can be proved [14], where the made assumptions allow the interchange of cum$\{\cdot\}$, sum, and integral operations.

Lemma 4.1 For any $k \geqslant 2$ and $\;\; > 0$ it results that

$$\lim_{T\to} T^{k-1-} \operatorname{cum}\left\{R^{[*]_1}_{xx^*}(\;_1,\;_1;t_1,T),\dots,R^{[*]_k}_{xx^*}(\;_k,\;_k;t_k,T)\right\} = 0\,. \tag{30}$$

Proof:

$$\begin{aligned} &\operatorname{cum}\left\{R^{[*]_1}_{xx^*}(\;_1,\;_1;t_1,T),\dots,R^{[*]_k}_{xx^*}(\;_k,\;_k;t_k,T)\right\} \\ &= \operatorname{cum}\left\{\int_{\mathbb{R}} w^{[*]_1}_T(u_1-t_1)\, z_1(u_1)\, e^{-j2 \;[-]_1 \;_1 u_1}\, \mathrm{d}u_1,\right. \\ &\qquad \left.\dots, \int_{\mathbb{R}} w^{[*]_k}_T(u_k-t_k)\, z_k(u_k)\, e^{-j2 \;[-]_k \;_k u_k}\, \mathrm{d}u_k\right\} \\ &= \int_{\mathbb{R}} \cdots \int_{\mathbb{R}} \operatorname{cum}\{z_1(u_1),\dots,z_k(u_k)\} \\ &\qquad w^{[*]_1}_T(u_1-t_1)\cdots w^{[*]_k}_T(u_k-t_k) \\ &\qquad e^{-j2 \;[-]_1 \;_1 u_1}\cdots e^{-j2 \;[-]_k \;_k u_k}\, \mathrm{d}u_1\cdots \mathrm{d}u_k \\ &= \int_{\mathbb{R}^k} \operatorname{cum}\{z_k(u), z_i(u+s_i),\; i=1,\dots,k-1\} \\ &\qquad \prod_{i=1}^{k-1} \frac{1}{T}\, a^{[*]_i}\left(\frac{u+s_i-t_i}{T}\right) e^{-j2 \;[-]_i \;_i\cdot(u+s_i)} \\ &\qquad \frac{1}{T}\, a^{[*]_k}\left(\frac{u-t_k}{T}\right) e^{-j2 \;[-]_k \;_k u}\, \mathrm{d}s_1\cdots \mathrm{d}s_{k-1}\, \mathrm{d}u \\ &= \int_{\mathbb{R}^k} \sum_{n} C^{(n)}_{z_1\cdots z_k}(s_1,\dots,s_{k-1})\, e^{j2 \;\; ^{(n)}_{z_1\cdots z_k}(s_1,\dots,s_{k-1})u} \\ &\qquad \prod_{i=1}^{k-1} \frac{1}{T}\, a^{[*]_i}\left(\frac{u+s_i-t_i}{T}\right) e^{-j2 \;[-]_i \;_i\cdot(u+s_i)} \\ &\qquad \frac{1}{T}\, a^{[*]_k}\left(\frac{u-t_k}{T}\right) e^{-j2 \;[-]_k \;_k u}\, \mathrm{d}s_1\cdots \mathrm{d}s_{k-1}\, \mathrm{d}u \end{aligned} \tag{31}$$

where $[-]_i$ is an optional minus sign which is present if the optional conjugation $[*]_i$ is present, in the third equality the variable changes $u_k = u$, $u_i = u + s_i$, $i = 1,\dots,k-1$ are made, and in the fourth equality Assumption 9) is used. Thus,

$$\begin{aligned} &\left|\operatorname{cum}\left\{R^{[*]_1}_{xx^*}(\;_1,\;_1;t_1,T),\dots,R^{[*]_k}_{xx^*}(\;_k,\;_k;t_k,T)\right\}\right| \\ &\leqslant \frac{\|a\|^{k-1}}{T^{k-1}} \sum_{n} \int_{\mathbb{R}^{k-1}} \left|C^{(n)}_{z_1\cdots z_k}(s_1,\dots,s_{k-1})\right| \mathrm{d}s_1\cdots \mathrm{d}s_{k-1} \\ &\qquad \int_{\mathbb{R}} |a(s)|\, \mathrm{d}s \end{aligned} \tag{32}$$

where the variable change $s = (u - t_k)/T$ is made. Therefore, from (32), accounting for Assumptions 6) and 10), it immediately follows that, for every $k \geqslant 2$ and every $\;\; > 0$, (30) holds.

Theorem 4.1 For every fixed $\;_i$, $\;_i$, t_i, $i = 1,\dots,k$, the random variables $\sqrt{T}\, R_{xx^*}(\;_i,\;_i;t_i,T)$ are jointly complex Normal.

Proof: From (25) we have that

$$\lim_{T\to} \text{cum}\left\{\sqrt{T}\, R_{xx^*}(\ _1,\ _1;t_1,T), \sqrt{T}\, R^*_{xx^*}(\ _2,\ _2;t_2,T)\right\}$$
$$= \lim_{T\to} \text{cov}\left\{\sqrt{T}\, R_{xx^*}(\ _1,\ _1;t_1,T), \sqrt{T}\, R_{xx^*}(\ _2,\ _2;t_2,T)\right\} \quad (33)$$

is finite and its expression is given in [13]. Moreover, from Lemma 4.1 with $k \geqslant 3$ and $\ = \frac{k}{2} - 1$ in (30), we have

$$\lim_{T\to} \text{cum}\left\{\sqrt{T}\, R^{[*]_1}_{xx^*}(\ _1,\ _1;t_1,T), \ldots, \sqrt{T}\, R^{[*]_k}_{xx^*}(\ _k,\ _k;t_k,T)\right\} = 0\,. \quad (34)$$

That is, for every fixed $\ _i,\ _i,t_i$, the random variables $\sqrt{T}\, R_{xx^*}(\ _i,\ _i;t_i,T)$, $i = 1,\ldots,k$ are asymptotically ($T \to\ $) jointly complex Normal [15].

5. DISCUSSION

For complex processes, a complete second order characterization requires the knowledge of both the autocorrelation function and the conjugate autocorrelation function [15]. In [14], it is shown that the conjugate autocorrelation function of GACS processes is completely characterized by the conjugate cyclic autocorrelation function. Moreover, it is proved that, under Assumptions 1)–11), the conjugate cyclic correlogram is an asymptotically Normal and mean-square consistent estimator of the conjugate cyclic autocorrelation function.

From (24), (25), and Theorem 4.1 we have that the well-known results for ACS processes that the cyclic correlogram is a mean-square consistent and asymptotically Normal estimator of the cyclic autocorrelation function (see [1], [2], [4]) can be extended to a wider class of nonstationary signals, that is, the GACS signals.

Note that, as it is well known, for ACS processes, if the estimation of the cyclic autocorrelation function is performed at a fixed cycle frequency, say $\ _0$, then the not exact knowledge of the value of $\ _0$ leads to a biased estimate. Moreover, an analogous result can be found for GACS processes if the estimation is performed along a fixed lag-dependent cycle frequency curve $\ = \ _n(\ )$. However, if the estimation of the cyclic correlogram $R_{xx^*}(\ ,\ ;t_0,T)$ as a function of the two variables $(\ ,\ )$ is performed, then, in the limit as $T \to\ $, the regions of the $(\ ,\ )$ plane where $R_{xx^*}(\ ,\ ;t_0,T)$ is significantly different from zero tend to the support curves of the cyclic autocorrelation function, that is, the curves $\ = \ _n(\ )$, $n \in \mathbb{I}$ (see (8)).

If the lag-dependent cycle frequencies are unknown, a statistical test for presence of generalized almost-cyclostationarity can be performed to estimate the unknown functions $\ _n(\ )$ by exploiting the asymptotic complex Normality of the cyclic correlogram. A point in the $(\ ,\ )$ plane belongs to the estimated support curve if the magnitude of the cyclic correlogram exceeds a threshold whose value has to be fixed in order to get assigned probabilities of false alarm or missed detection.

A different behavior of statistical-function estimators is found for the class of the spectrally correlated stochastic processes [12] that also extend the class of the ACS processes. Spectrally correlated processes have the Loève bifrequency spectrum with spectral masses concentrated on a countable set of curves in the bifrequency plane. The support curves of the Loève bifrequency spectrum play, for spectrally correlated processes, in the frequency domain, a role analogous to that played for GACS processes, in the time domain, by the lag-dependent cycle frequencies. The ACS processes are obtained as a special case of spectrally correlated processes when the separation between correlated spectral components can assume values only in a countable set (which is set of the cycle frequencies). In such a case, the support curves of the Loève bifrequency spectrum are lines with unit slope. In [12] it is shown that, for spectrally correlated processes, when the location of the spectral masses is unknown, time- or frequency-smoothed versions of the periodogram do not provide estimates of the bifrequency spectral correlation density function that are asymptotically unbiased and with zero asymptotic variance. Moreover, there exists a tradeoff between the departure of the spectral-correlation-type nonstationarity from the almost cyclostationarity and the reliability of spectral correlation measurements obtainable by a single sample path.

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