CONSTANT NORM ALGORITHMS FOR MIMO COMMUNICATION SYSTEMS

Aïssa Ikhlef, Daniel Le Guennec, and Jacques Palicot

IETR / Supelec-Campus de Rennes
Av. de la Boulaie, BP 81127
35511 Cesson-Sévigné, France
phone: + (33) 2 99 84 45 00, fax: + (33) 2 99 84 45 99, email: {aissa.ikhlef, daniel.leguennec, jacques.palicot}@supelec.fr
web: www.supelec.fr

ABSTRACT

In this paper we present a new algorithm for blind source separation (BSS) based on the Constant Norm (CN) criterion for Multiple-Input Multiple-Output (MIMO) communication systems. The treated problem consists in blindly recovering (i.e. without the use of training sequences) the signals transmitted over a linear MIMO memoryless system, which introduces only Inter Stream Interference (ISI). From the proposed algorithm, we deduce two other new algorithms designed especially for QAM signals. The first one is named Constant sQuare Algorithm (CQA) and the second one, which is a weighting between the Constant Modulus Algorithm (CMA) and the CQA to get the advantages of both, is named Constant Dynamic Norm Algorithm (CDNA). At each iteration, the algorithms combine a stochastic gradient update and a Gram-Schmidt orthogonalization procedure. The simulation results show that the proposed algorithms have better performances compared to CMA and Multiuser Kurtosis Algorithm (MUK) with comparable complexity.

1. INTRODUCTION

The BSS problem emerged two decades ago. It arises in a wide variety of signal processing applications, as for example speech enhancement, seismic analysis, medical applications (EEG) [1] and others. Since ten years, the application of this technique (BSS) in digital communications has received increased interest, as in [2] for multiuser communications (CDMA) modeled as a MIMO system. In literature, we find also the kurtosis-based algorithm (MUK) [3], which represents an extension of the kurtosis-based algorithm developed first for blind equalization in SISO systems by [4]. More recently, a BSS technique has been applied to Bell Labs Layered Space-Time (BLAST) communication system based on the multimodulus algorithm MMA [5].

In this paper we propose a new BSS technique named CNA-MIMO algorithm, which represents an extension to MIMO systems of the CNA introduced recently by [6], combined with Gram Schmidt orthogonalization procedure. This class of algorithms contains the well-known CMA. Then, we present two new algorithms derived from CNA. The first, named Constant sQuare Algorithm (CQA) better adapted for QAM modulation than the classical CMA. It results in lower algorithm noise and comparable complexity compared to CMA. The second is a weighting between the CMA and CQA to get the advantages of both. The weighting coefficient is dynamically driven and justifies the name of Constant Dynamic Norm Algorithm (CDNA).

The basic idea of the proposed algorithms is to minimize CNA-MIMO cost functions by the gradient stochastic algorithm then project the updated parameters to the orthogonality constraints, which ensures the independence among the equalizer outputs at each iteration.

The paper is organized as follows. In section 2, the problem formulation and assumptions are introduced. In section 3, we present our new CNA algorithm. Then we drive two other algorithms named CQA and CDNA in section 4 and 5 respectively. The performances of the proposed algorithms are compared with CMA and MUK algorithms in section 6. Finally a conclusion is given in section 7.

2. PROBLEM FORMULATION

We assume that n_T signals are transmitted through a MIMO system with n_T transmitter and n_R receiver antennas. The channel is assumed to be linear memoryless (frequency flat) that introduces only ISI. We assume also for simplicity that the received signals are sampled at the symbol rate 1/T. The received signal model then takes the familiar form:

$$\mathrm{Y}(k) = \mathbf{H}\,\mathrm{A}(k) + \mathrm{n}(k) \qquad (1)$$

where:

$\mathrm{A}(k) = [a_1(k),...,a_{n_T}(k)]^T$ is the $(n_T \times 1)$ vector of the transmitted signal,

$\mathbf{H}$ represents the $(n_R \times n_T)$ instantaneous channel matrix,

$\mathrm{Y}(k)$ is the $(n_R \times 1)$ vector of the received signal,

$\mathrm{n}(k)$ is the $(n_R \times 1)$ noise vector.

We work under the following assumptions :

1) $n_R \geq n_T$

2) $\mathbf{H}$ has full rank n_T, independent and identically distributed (i.i.d.) complex, zero-mean and unit variance entries.

3) The noise is additive white Gaussian, zero mean, independent from the source signals, with covariance $\mathrm{R_n} = E[\mathrm{n}\,\mathrm{n}^H] = \sigma_\mathrm{n}^2 \mathbf{I}_{n_R}$.

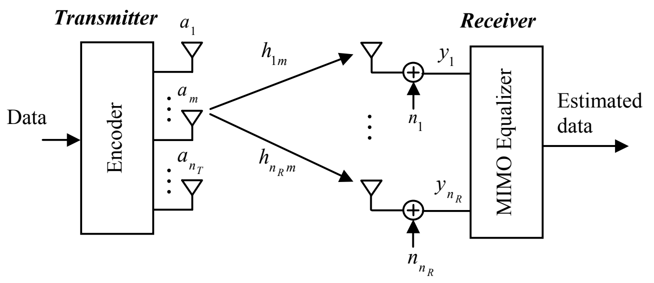

Figure 1. Model of a MIMO system

4) The source signals are i.i.d, mutually independent and zero mean discrete-time sequences that share the same statistical properties, with covariance $\mathrm{R_A} = E[\mathrm{A\,A}^H] = \mathbf{I}_{n_T}$

In this paper we use the following notation: $(.)^*$ denotes complex conjugation, $(.)^T$ is the vector or matrix transpose, $(.)^H$ is the vector or matrix complex conjugate transpose and $\mathbf{I}_p$ is the $(p \times p)$ identity matrix.

The MIMO system is depicted on figure 1.

In order to recover the transmitted signals, the received signal Y is processed by a $(n_R \times n_T)$ matrix equalizer $\mathbf{W}(k)$. The receiver output can be written as:

$$\begin{aligned} \mathrm{Z}(k) &= \mathbf{W}^T(k)\,\mathrm{Y}(k) = \mathbf{W}^T(k)\,\mathbf{H}\,\mathrm{A}(k) + \bar{\mathrm{n}}(k) \\ &= \mathbf{G}(k)\,\mathrm{A}(k) + \bar{\mathrm{n}}(k) \end{aligned} \tag{2}$$

Where:

$\mathrm{Z}(k)$ is the $(n_T \times 1)$ vector of output signal;

$\mathbf{G}(k) = \mathbf{W}^T(k)\,\mathbf{H}$ is the $(n_T \times n_T)$ global system matrix and $\bar{\mathrm{n}}(k)$ is the filtered noise at the receiver output.

The matrix $\mathbf{W}$ is feasible to separate source signals, except for a possible permutation and up to a unitary scalar rotation for each source signal [3].

In the next section, we present our new algorithm to blind separate the MIMO signal.

3. MIMO-CNA ALGORITHM

The proposed algorithm is based on the Constant Norm (CN) criterion, recently developed by [6] in single user context. In MIMO context, the criterion to minimize, named MIMO-CNA, can be formulated as follows:

$$J_{MIMO-CNA}(\mathbf{W}) = \sum_{i=1}^{n_T} E[\mathcal{N}(z_i)^P - R]^q \tag{3}$$

where $\mathcal{N}(.)$ is a norm on $\Re^2$ and R is a constant, given by:

$$R = \frac{E[\mathcal{N}^{2p}(a)]}{E[\mathcal{N}^{p}(a)]}, \text{ for } q = 2 .$$

In what follows, we consider only the case where : $p = q = 2$.

Clearly, in the particular case where $\mathcal{N}(.) = |.|$, i.e. the norm is the modulus, we find the CMA algorithm.

In order to avoid convergence to sets of signals that contain multiple times the same signal while missing other signals, the criterion MIMO-CNA will be modified. To be done, the approach of interest in this paper is Gram-Schmidt orthogonalization procedure [3]. An other cross-correlation based approach has been proposed in literature as in [2].

The problem is reformulated as a constrained optimization problem, equation (3) can be written as:

$$\min_{\mathbf{W}} \; J_{MIMO-CNA}(\mathbf{W}) = \sum_{i=1}^{n_T} E[\mathcal{N}(z_i)^2 - R]^2 \quad \text{Subject to} : \mathbf{G}^H\mathbf{G} = \mathbf{I}_{n_T} \tag{4}$$

The constraint comes from the fact that $E[\mathrm{Z\,Z}^H] = \mathbf{I}_{n_T}$, which is the condition that penalizes the extraction of the same signal on many outputs.

To satisfy the constraint given in equation (4), the channel matrix $\mathbf{H}$ is assumed to be unitary in order that the constraint can be reduced to:

$$\mathbf{W}^H(k+1)\,\mathbf{W}(k+1) = \mathbf{I}_{n_T}$$

When the channel matrix is not unitary, a stage of prewhitening is necessary. It can be performed simply by the use of the well-known eigen-decomposition. In general, the main purpose of the prewhitening is to reduce the data vector dimension from n_R to n_T, which is the number of source. In this paper the purpose is to whiten the data covariance matrix. In what follows we assume that this operation is realized either by supposing a unitary channel matrix or by application of prewhitening by means of eigenvalues decomposition.

The simplest way of minimizing (4) is by means of a conventional stochastic gradient algorithm and to satisfy the constraint we perform a Gram-Schmidt orthogonalization, as we will show in the next paragraph.

We have noticed that, when the norm represents the modulus, then, we find the CMA algorithm, which is first conceived for PSK signals. The use of CMA for QAM signals is possible, but, in this case, the descent algorithm generates a significant amount of noise. For that purpose, in the next section we propose a new algorithm designed especially for QAM signals.

4. MIMO-CQA ALGORITHM

Here we present an extension of the CQA algorithm [6] to MIMO context. The main reason for introducing this algorithm is that the QAM modulation is rather "square" than "round" (see figure 2). The idea is that, in place of constraining the equalizer outputs to be on a circle as in CMA, we consider a square. The residual noise would be lower since the average distance between the symbols of constellation and the square (ℓ_{CQA}) is shorter than that between the symbols and the circle (ℓ_{CMA}) [6], this concept is well illustrated on figure 2.

In order to better derive the square, we resort to the so-called infinite norm, it's given by:

$$\|z_i\|_\infty = Max\left(|\Re z_i|, |\Im z_i|\right)$$

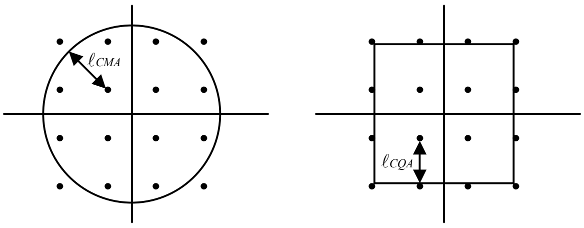

Figure 2. Principle of CMA and CQA

where $\Re z_i$ and $\Im z_i$ are the real and imaginary part of z_i respectively.

The cost function (4) is modified as:

$$\min_{\mathbf{W}} \ J_{MIMO-CQA}(\mathbf{W}) = \sum_{i=1}^{n_T} E\left[\left\| z_i \right\|_\infty^2 - R\right]^2 \qquad (5)$$
$$\text{Subject to} : \mathbf{G}^H \mathbf{G} = \mathbf{I}_{n_T}$$

where R is given by: $R = \dfrac{E\left[\|z\|_\infty^4\right]}{E\left[\|z\|_\infty^2\right]}$.

The stochastic gradient algorithm used for minimizing (5), is written as:

$$\overline{\mathbf{W}}(k+1) = \mathbf{W}(k) - \mu \, \nabla_{\mathbf{W}}(J_{MIMO-CQA})$$

By a straightforward operation (calculus of gradient) we find:

$$\overline{\mathbf{W}}(k+1) = \mathbf{W}(k) - \mu \, [\Delta_1(k) \ \ldots \ \Delta_{n_T}(k)] \, \mathrm{Y}^*(k)$$

where $\Delta_i(k) = \left(\| z_i(k) \|_\infty^2 - R\right) \| z_i(k) \|_\infty \, F(z_i(k))$

$$F(z_i) = \begin{cases} \operatorname{sgn}(\Re z_i), & \text{if } |\Re z_i| > |\Im z_i| \\ j.\operatorname{sgn}(\Im z_i), & \text{otherwise} \end{cases}$$

where sgn is the sign function and $j = \sqrt{-1}$.

In order to satisfy the constraint, we perform the Gram-Schmidt orthogonalization on $\mathbf{W}$ defined by [3]:

$$\begin{cases} W_1(k+1) = \dfrac{\overline{W}_1(k+1)}{\left\| \overline{W}_1(k+1) \right\|} \\ W_p(k+1) = \dfrac{\overline{W}_p(k+1) - Q}{\left\| \overline{W}_p(k+1) - Q \right\|}, \ p = 2, \ldots, n_T \\ Q = \displaystyle\sum_{l=1}^{p-1} [W_l^H(k+1) \, \overline{W}_p(k+1)] \, W_l(k+1) \end{cases} \qquad (6)$$

where: W_i and $\overline{W}_i$ are the *i*-th column vectors of $\mathbf{W}$ and $\overline{\mathbf{W}}$ respectively.

The main advantage of Gram-Schmidt orthogonalization procedure is that it results in an algorithm updating procedure having a deflation structure, which turns out to be the key in its convergence behavior.

5. MIMO-CDNA ALGORITHM

As we take the infinite norm definition into account, it is obvious, contrary to CMA, that the CQA algorithm is sensitive to carrier residue because it recovers the phase. To alleviate this drawback, we present an extension of CDNA [6] to MIMO system. The CDNA is a particular case of CNA class algorithms, it consists in combining the advantages of both CMA and CQA, i.e. uses the CMA in transient phase followed by CQA for his better steady state. The used norm is defined by:

$$\| z_i \|_\lambda = \alpha \, \lambda \, \| z_i \|_\infty + (1-\lambda) |z_i|$$

this norm depends on the weighting parameter λ, thus, for $\lambda = 0$ and $\lambda = 1$ we find the modulus and the infinite norm respectively. The constant $\alpha > 0$ gives an additional degree of freedom. So as to make the best of this definition of norm, the weighting parameter λ may be adaptively updated by means of stochastic gradient algorithm.

The cost function is modified as:

$$\min_{\mathbf{W}} \ J_{MIMO-CDNA}(\mathbf{W}) = \sum_{i=1}^{n_T} E\left[\| z_i \|_\lambda^2 - R(\lambda)\right]^2 \qquad (7)$$
$$\text{Subject to} : \mathbf{G}^H \mathbf{G} = \mathbf{I}_{n_T}$$

where $R(\lambda) = \dfrac{E\left[\|z\|_\lambda^4\right]}{E\left[\|z\|_\lambda^2\right]}$

and the updating algorithm is given by:

$$\begin{cases} \overline{\mathbf{W}}(k+1) = \mathbf{W}(k) - \mu_{\mathbf{W}} \, [\Delta_1(k) \ \ldots \ \Delta_{n_T}(k)] \, \mathrm{Y}^*(k) \\ \lambda_{k+1} = \lambda_k - \mu_\lambda \displaystyle\sum_{i=1}^{n_T} \left[\| z_i(k) \|_{\lambda_k}^2 - R(\lambda_k)\right] \left[2 \| z_i(k) \|_{\lambda_k} . \right. \\ \qquad \left. \left(\alpha \| z_i(k) \|_\infty - |z_i(k)|\right) - R'(\lambda_k)\right] \end{cases}$$

where:

$$\Delta_i(k) = \left[\| z_i(k) \|_{\lambda_k}^2 - R(\lambda_k)\right] \| z_i(k) \|_{\lambda_k} \left[\alpha \, \lambda_k \, F(z_i(k)) + (1-\lambda_k) \, z_i(k) / |z_i(k)| \right]$$

and $R'(\lambda)$ is the derivative of $R(\lambda)$ with respect to λ.

The constraint is satisfied by application of Gram-Schmidt orthogonalization procedure at each iteration as in MIMO-CQA algorithm (see equation 6).

6. SIMULATION RESULTS

In this section, the results of computer simulations are presented to illustrate the behavior of the proposed algorithms. To measure the algorithm performance we consider the Inter Stream Interference (ISI) of the *i*-th signal source at the *k*-th output, defined by:

$$ISI = \frac{\sum_{i=1}^{n_T} |g_{ki}|^2 - Max_i |g_{ki}|^2}{Max_i |g_{ki}|^2}.$$

where $g_{ki} = [\mathbf{G}(k,i)] = W_k^T H_i$, W_k and H_i are the *k*-th and *i*-th column vectors of matrices **W** and **H** respectively.
In our simulations, the system inputs are i.i.d, mutually independent and drawn from 64-QAM constellation. We have considered 4 transmitting antennas and 4 receiving antennas, they are assumed uncorrelated (ideal). The (4×4) channel matrix is chosen randomly. The system noise is complex white Gaussian with zero mean and variance determined by the Signal to Noise Ratio (SNR) of system. In both experiments, the received signal is prewhitened before applying the algorithms.
In the first simulation experiment, we compare our proposed CQA algorithm with CMA [2] and MUK [3] algorithms. The step sizes were chosen to have sensibly the same convergence speed for all algorithms. We consider an average over 1000 independents runs. Figure 3 shows ISI at each output, in dB's for SNR=30 dB. It can be seen that the CQA reaches a better steady state than that of CMA and MUK, with a difference of 3 dB between CQA and CMA.

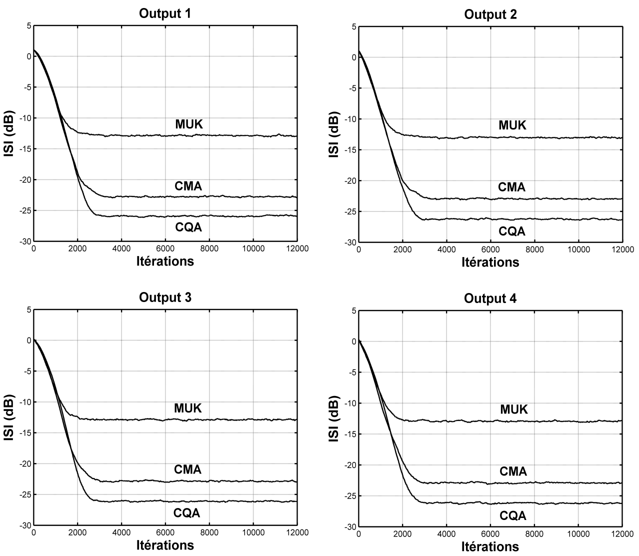

Figure 3. Average performance for CQA, CMA and MUK

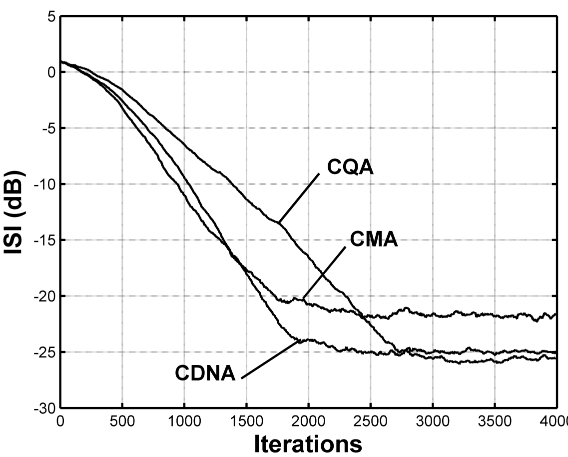

Figure 4. Average performance for CDNA, CQA and CMA

In the second experiment, we compare the CDNA with the CMA and CQA. The parameter λ is initialized to zero (so the CDNA starts off like a CMA), the coefficient α is set to 1; and the step size μ_λ is fixed at 1.10^{-3}. The step size for all algorithms was fixed at 8.10^{-3}. The performance comparison is given for one output, the other outputs have the same behavior (figure 4). Clearly at the beginning the CDNA behaves as the CMA (exactly for the 300 iterations and approximatively until 1500 iterations). After 2500 iterations the CQA becomes better than CMA, and consequently the CDNA tends to the CQA. This result confirms that the CDNA tends to choose the best algorithm between the CMA and CQA.

7. CONCLUSION

In this paper, we have proposed a novel algorithm (CNA) for the MIMO communication systems, from which we have deduced the CQA and CDNA algorithms for blind separation of mutually independent i.i.d source signals that have the same probability distribution and are received in the presence of linear interference. The criteria consists of minimizing a constrained optimization problem that contains a sum of Constant Norm (CN) criterion and a constraint to prevent the extraction of the same signal at several outputs simultaneously. The proposed algorithms have shown their better performances compared to CMA and MUK algorithms with comparable complexity. We have found that the CQA reaches a better steady state than CMA (gain of 3 dB), and the CDNA tends to the best algorithm between CQA and CMA. Future work will target toward the extension of the proposed algorithms to convolutive mixing channels and to other situations (CDMA, OFDM) modeled like MIMO systems.

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