# LOCALIZATION OF SOURCES RADIATING ON A LARGE ANTENNA

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## ABSTRACT

In this paper, we present a subspace-based method for direction finding for radiolocation of localization of mobile station. Indeed, an original partition of the data cross-spectral matrix is proposed in order to use a small part of the observations to estimate the noise subspace projector without eigendecomposition, when the number of sensors of antennas is very larger than the number of sources. Once, at least two angles of arrival of the mobile station are estimated by two different antennas, the position of the mobile is then calculated. Finally, to demonstrate the efficiency of the proposed method, performance results are presented.

## 1. INTRODUCTION

Recently, there has been a great deal of interest in developing mobile location systems for wireless communication systems [1, 2]. The motivation is the interest in applications for indoor/outdoor geolocation systems [1] or the localization of mobile stations.

Most conventional location techniques use the signal being transmitted by a mobile to determine its location [2]. Generally, the mobile's signal is received at several receivers with known positions. After reception, some characteristics of that signal are combined with the known positions of the receivers and used to solve the mobile's position. This could be the angle of arrival, or the time of arrival of the signal. For practical reasons, the reception points are usually existing base stations. This minimizes the extra equipment that has to be added to the network to implement location [2]. One method for locating a mobile station is the measurement of the line-of-sight (LOS) distance between the mobile and at least three participating base stations [2]. Each distance measurements generates a circle which is centered at the measuring base station and which has a radius equal to the distance between the mobile and a base station. In the absence of any measurement error, the intersection of the three circles unambiguously determines the location of the mobile.

Other method uses the estimates of angle of arrival from at least two participating base stations. To estimate the direction of arrival of a mobile, the line of sight (LOS) is required. In this paper, a direction finding method is presented. Indeed, in the last decades, sources localization has received great attention because of its potential applications in such as in radar, sonar, communication, seismic, radiolocation and medical signal processing. It has been shown that the second order statistics contains sufficient information for the estimation of the directions of arrival of existing sources. The second-order statistics based approach is attractive and simpler because it requires much less computation effort than the high order statistics approach. Many moment-based channel estimators belong to two different categories. The first includes those derived by matching the moments or the power spectra in some optimal way. The second exploits the eigenstructures of the second-order moments to obtain angles.

In order to estimate the mobile location with reducing the computational load of finding of directions of arrival of a mobile station with two antennas of very large number of sensors, we propose in this paper a new method for the estimation of the direction of arrival using subspace techniques without eigendecomposition. To avoid eigendecomposition calculations, a fast non-eigenvector algorithm is described in this contribution. This method provides a substantial saving in terms of computational load. Here, we make use of a new version of the propagator [4, 5], an operator that uses the linear dependence between the columns of the channel matrix, so that the noise subspace can be determined without eigendecomposition of the cross-spectral matrix of data.

## 2. PROBLEM FORMULATION

Consider an array of $N$ sensors which received the wavefield generated by $P$ narrow-band sources in the presence of an additive noise.

The signal received by the antenna array, in time domain, can be written as:

$$\mathbf{r}(t) = \sum_{k=1}^{P} \mathbf{h}(\ _k)\mathbf{s}_k(t) + \mathbf{n}(t) = \mathbf{H}(\ )\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

The received signal, in the frequency domain, is given by:

$$\mathbf{r}(f) = \mathbf{H}(f,\ )\mathbf{s}(f) + \mathbf{n}(f), \quad (2)$$

where, $\mathbf{r}(f)$ is the Fourier Transform of the array output vector, $\mathbf{s}(f)$ is the $(P \times 1)$ vector of complex signals of $P$ wavefronts, $\mathbf{s}(f) = [s_1(f) \quad s_2(f) \quad \ldots \quad s_P(f)]^T$. $\mathbf{n}(f)$ is the $(N \times 1)$ vector of additive noise in sensors, $\mathbf{n}(f) = [n_1(f) \quad n_2(f) \quad \ldots \quad n_N(f)]$, and $\mathbf{H}(f,\ )$ is the $(N \times P)$ transfer matrix of the source-sensor array systems with respect to some chosen reference point:

$\mathbf{H}(f,\ ) = [\mathbf{h}(f,\ _1), \ldots, \mathbf{h}(f,\ _P)]$ and $\ = [\ _1,\ _2, \ldots,\ _P]^T$.

$\mathbf{h}(f,\ _i)$ is the steering vector of the array toward the direction $\ _i$ at the frequency $f$. For example, the steering vector of a linear uniform array with $N$ sensors is given by:

$\mathbf{h}(f,\ _i) = \frac{1}{\sqrt{N}}\left[1, e^{j\ _i}, e^{2j\ _i}, \ldots, e^{(N-1)j\ _i}\right]^T$, where $\ _i = 2\ f\frac{d}{c}sin(\ _i)$; $d$ is the sensor spacing; $\ _i$ is the direction of arrival (DOA) of the $i^{th}$ source as measured from broadside; $c$ is the velocity wave propagation and $f$ is the center frequency of the narrow-band source. Assume that the signals and the additive noises are stationary and ergodic zero mean complex valued random processes. In addition, the noises are assumed to be uncorrelated between sensors, and to have identical variance $\ ^2$ in each sensor. It follows from these assumptions that the spatial $(N \times N)$ cross-spectral matrix of the observation vector at the frequency $f$ is given by: $\mathbf{\Gamma}(f) = E\left[\mathbf{r}(f)\mathbf{r}^+(f)\right] = \mathbf{H}(f, \ )\mathbf{\Gamma}_s(f)\mathbf{H}^+(f, \ ) + \mathbf{\Gamma}_n$, where $E[.]$ denotes the expectation operator, the superscript $.^+$ represents conjugate transpose, $\mathbf{\Gamma}_s(f) = E\left[\mathbf{s}(f)\mathbf{s}^+(f)\right]$ is the $(P \times P)$ sources cross-spectral matrix, and $\mathbf{I}$ is the identity matrix. $\mathbf{H}(f, \ ) = \left[\mathbf{h}(f, \ _1), \mathbf{h}(f, \ _2), ..., \mathbf{h}(f, \ _P)\right]^T$.

$$\mathbf{H}(f, \ ) = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & \cdots & 1 \\ e^{j \ _1} & e^{j \ _2} & \cdots & e^{j \ _P} \\ \vdots & \vdots & \cdots & \vdots \\ e^{(N-1)j \ _1} & e^{(N-1)j \ _2} & \cdots & e^{(N-1)j \ _P} \end{pmatrix}$$

The eigendecomposition of the cross-spectral matrix of the data at frequency $f$ is given by:

$$\mathbf{\Gamma}(f) = \sum_{i=1}^{N} \ _i(f)\mathbf{u}_i(f)\mathbf{u}_i^+(f) \qquad (3)$$

where $\ _i(f), i = 1, ..., N$, $( \ _1 \geq \ _2 \geq ... \ _P > \ _{P+1} \simeq \ _{P+2} \simeq ... \simeq \ _N = \ ^2)$, and $\mathbf{u}_i(f)$ are the $i^{th}$ eigenvalue and its corresponding eigenvector. It is well known that the eigenvectors corresponding to the minimum eigenvalues are orthogonal to the columns of the matrix $\mathbf{H}(f, \ )$.
Let $\mathbf{U}_n(f) = [\mathbf{u}_{P+1}(f) \quad \mathbf{u}_{P+2}(f) \quad ... \quad \mathbf{u}_N(f)]$ is the $(N \times N - P)$ matrix constructed with the $(N-P)$ last eigenvectors, which is called the noise subspace.
In order to avoid the eigendecomposition, an operator called the "Propagator"[4, 5] was proposed based on the partition of the cross-spectral matrix. In the following, a new partition of this operator is proposed in the case of a large number of sensors compared to the number of existing sources. Indeed, solely a small part of the cross-spectral matrix of data is used without ignoring the advantages of using a very long antenna.

## 3. CLASSICAL SUBSPACE METHOD

In this section, a class of direction-of-arrival estimators are briefly recalled, the so-called the subspace method. Indeed, this approach is based on the eigendecomposition of the data cross-spectral matrix: $\mathbf{\Gamma} = [\mathbf{U}_s \quad \mathbf{U}_n] \begin{bmatrix} \mathbf{\Lambda}_s & \\ & \mathbf{\Lambda}_n \end{bmatrix} [\mathbf{U}_s \quad \mathbf{U}_n]^+$, where the different matrices are defined as above. The subspace method yields an estimate $\widehat{\mathbf{H}}$ of $\mathbf{H}$ by solving the equation [3]: $\mathbf{U}_n^+\widehat{\mathbf{H}} = \mathbf{0}$, in a least square sense (where $\widehat{\mathbf{H}}$ is subject to the same structure as $\mathbf{H}$), or $\mathbf{h}^+\mathbf{U}_n = \mathbf{0}$
The MUltiple Signal Classification [3] null-spectrum (MUSIC) is given by: $S(f, \ ) = \left\{\mathbf{h}^+(f, \ )\mathbf{U}_n\mathbf{U}_n^+\mathbf{h}(f, \ )\right\}^{-1}$.
It has been shown that $S(f, \ )$ has maximum points at round $\ $ in $\{ \ _1, \ _2, ..., \ _P\}$ ($P$ is the number of sources). Therefore we can estimate the $P$ directions by taking the local maximum points of $S(f, \ )$.

## 4. THE PROPAGATOR OPERATOR

### 4.1 Definition of the propagator

The rank of the channel matrix $\mathbf{H}$ plays an important role in the construction of the noise subspace, consequently in the localization of sources. The definition of the propagator is based on the partitioning of the channel matrix $\mathbf{H}$ into two submatrices, indicated as follows [5]:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_a \\ --- \\ \mathbf{H}_b \end{bmatrix}, \qquad (4)$$

where $\mathbf{H}_a$ is a square matrix of dimension $(P \times P)$ and $\mathbf{H}_b$ is a matrix of dimension $(N - P \times P)$.
If we assume the propagation model is such as the rows (or columns) of $\mathbf{H}_a$ are linearly independents, then $\mathbf{H}_a$ is nonsingular matrix. In general $\mathbf{H}_a$ can not be of rank $P$. Then, an adequate permutation of rows (or columns) of $\mathbf{H}$ is necessary in order to obtain $\mathbf{H}_a$ of rank $P$. The $(N-P)$ rows of $\mathbf{H}_b$ are, then, linearly dependents of the $P$ first rows, we have: $\mathbf{H}_b = \mathbf{\Pi}^+\mathbf{H}_a$. The operator $\mathbf{\Pi}$ is called propagator of dimension $(P \times N - P)$.

### 4.2 The propagator operator in noiseless case

We consider the case of free-noise; the decomposition of the data cross-spectral matrix is as follows:

$$\mathbf{\Gamma} = \mathbf{H}\mathbf{H}^+ = \begin{bmatrix} \mathbf{\Gamma}_{11} & \mathbf{\Gamma}_{12} \\ \mathbf{\Gamma}_{21} & \mathbf{\Gamma}_{22} \end{bmatrix}, \qquad (5)$$

where $\mathbf{\Gamma}_{11}$, $\mathbf{\Gamma}_{12}$, $\mathbf{\Gamma}_{21}$ and $\mathbf{\Gamma}_{22}$ are the blocks matrices of dimension $(P \times P)$, $(N-P \times P)$, $(P \times N - P)$ and $(N-P \times N - P)$, respectively. We have:

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{\Gamma}_{11} & \mathbf{\Gamma}_{11}\mathbf{\Pi} \\ \mathbf{\Pi}^+\mathbf{\Gamma}_{11} & \mathbf{\Pi}^+\mathbf{\Gamma}_{11}\mathbf{\Pi} \end{bmatrix}, \qquad (6)$$

with, $\mathbf{\Gamma}_{11} = \mathbf{H}_a\mathbf{H}_a^+$. To reduce the computational load, we use the sub-matrices $\mathbf{\Gamma}_{11}$ and $\mathbf{\Gamma}_{12}$: $\mathbf{\Gamma}_{12} = \mathbf{\Gamma}_{11}\mathbf{\Pi}$.
The propagator operator is, then given by: $\mathbf{\Pi} = \mathbf{\Gamma}_{11}^{-1}\mathbf{\Gamma}_{12}$, or $\mathbf{\Pi}^+ = \mathbf{\Gamma}_{12}^+\mathbf{\Gamma}_{11}^{-1}$.
The matrix $\mathbf{\Pi}$ is obtained by inverse of a matrix of dimension $(P \times P)$. The estimation of the data blocks matrices from $K$ independents samples is given by:

$$\widehat{\mathbf{\Gamma}}_{11} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{r}_1(k)\mathbf{r}_1^+(k), \qquad \widehat{\mathbf{\Gamma}}_{12} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{r}_1(k)\mathbf{r}_2^+(k) \qquad (7)$$

where $\mathbf{r}_1(k) = \mathbf{H}_a\mathbf{s}$, and $\mathbf{r}_2(k) = \mathbf{H}_b\mathbf{s}$ are two sub-vectors of the data vector $\mathbf{r}(k)$.
In the presence of noise, the problem is more complicated, the relationship (6) is not verified, therefore an other estimator is necessary to decrease the noise influence.

### 4.3 The propagator in noisy case

We assume that the noise cross-spectral matrix is diagonal with different values, the linear dependence between the $P$ first rows of the channel matrix $\mathbf{H}$ and the others rows, is always verified. We introduce a new partitioning of the channel matrix to isolate the affected blocks matrices.

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_a \\ --- \\ \mathbf{H}_{b1} \\ --- \\ \mathbf{H}_{b2} \end{bmatrix}, \qquad (8)$$

where $\mathbf{H}_{b1}$ and $\mathbf{H}_{b2}$ are of dimensions $(1 \times P)$ and $(N-P-1 \times P)$, respectively.
Assume $(N \geq P+1)$, let the partitioning of the propagator $\mathbf{\Pi}$:

$$\mathbf{\Pi}^{+} = \begin{bmatrix} \mathbf{\Pi}_1^{+} \\ --- \\ \mathbf{\Pi}_2^{+} \end{bmatrix}, \tag{9}$$

where, $\mathbf{\Pi}_1$ is a vector of dimension $(P \times 1)$ and $\mathbf{\Pi}_2$ is a matrix $(P \times N-P-1)$, let,

$$\begin{cases} \mathbf{H}_{b1} &= \mathbf{\Pi}_1^{+}\mathbf{H}_a \\ \mathbf{H}_{b2} &= \mathbf{\Pi}_2^{+}\mathbf{H}_a \end{cases} \tag{10}$$

the following partitioning shows the affected blocks matrices by the additive noise, $\widetilde{\mathbf{\Gamma}}_{22}$ and $\widetilde{\mathbf{\Gamma}}_{11}$,

$$\mathbf{\Gamma} = \begin{bmatrix} \widetilde{\mathbf{\Gamma}}_{11} & \mathbf{\Gamma}_{12} \\ \mathbf{\Gamma}_{21} & \widetilde{\mathbf{\Gamma}}_{22} \end{bmatrix}. \tag{11}$$

The blocks matrices of $\mathbf{\Gamma}$ are given by:

$$\begin{cases} \widetilde{\mathbf{\Gamma}}_{11} &= \mathbf{\Gamma}_{11} + \mathbf{\Gamma}_n^{P}, \\ \mathbf{\Gamma}_{12} &= [\mathbf{\Gamma}_{11}\mathbf{\Pi}_1 \mid \mathbf{\Gamma}_{11}\mathbf{\Pi}_2], \\ \mathbf{\Gamma}_{12} &= \left[\mathbf{\Gamma}_{12}^{1} \mid \mathbf{\Gamma}_{12}^{2}\right], \\ \mathbf{\Gamma}_{21} &= \mathbf{\Gamma}_{12}^{+} \end{cases} \tag{12}$$

$$\widetilde{\mathbf{\Gamma}}_{22} = \begin{bmatrix} \mathbf{\Pi}_1^{+}\mathbf{\Gamma}_{11}\mathbf{\Pi}_1 + {}^{2}_{P+1} & \mathbf{\Pi}_1^{+}\mathbf{\Gamma}_{11}\mathbf{\Pi}_2 \\ \mathbf{\Pi}_2^{+}\mathbf{\Gamma}_{11}\mathbf{\Pi}_1 & \mathbf{\Pi}_2^{+}\mathbf{\Gamma}_{11}\mathbf{\Pi}_2 + \mathbf{\Gamma}_n^{N-P-1} \end{bmatrix}, \tag{13}$$

where ${}^{2}_{P+1}$ is the noise power of $(P+1)^{th}$ element, and $\mathbf{\Gamma}_{11}$ is the data cross-spectral matrix of the signal vector without noise ($\mathbf{\Gamma}_{11} = \mathbf{H}_a\mathbf{s}$). The matrices $\mathbf{\Gamma}_n^{P}$ and $\mathbf{\Gamma}_n^{N-P-1}$, are the noise diagonals matrices, of dimensions $(P \times P)$ and $(N-P-1 \times N-P-1)$, respectively.
Let the following partitioning of the matrix $\widetilde{\mathbf{\Gamma}}_{22}$:

$$\widetilde{\mathbf{\Gamma}}_{22} = \begin{bmatrix} \widetilde{\mathbf{\Gamma}}_{22}^{11} & \mathbf{\Gamma}_{22}^{12} \\ \mathbf{\Gamma}_{22}^{21} & \widetilde{\mathbf{\Gamma}}_{22}^{22} \end{bmatrix}. \tag{14}$$

From the expressions (12), (12), (13) and (14), we have:

$$\mathbf{\Gamma}_{22}^{12} = \mathbf{\Pi}_1^{+}\mathbf{\Gamma}_{12}^{2}, \quad or \quad \mathbf{\Pi}_1^{+} = \mathbf{\Gamma}_{22}^{12}\mathbf{\Gamma}_{12}^{2\dagger} \tag{15}$$

with, $\mathbf{\Gamma}_{12}^{2\dagger} = \mathbf{\Gamma}_{12}^{2+}\left(\mathbf{\Gamma}_{12}^{2}\mathbf{\Gamma}_{12}^{2+}\right)^{-1}$, and,

$$\begin{cases} \mathbf{\Gamma}_{22}^{21} &= \mathbf{\Pi}_2^{+}\mathbf{\Gamma}_{12}^{1}, \\ \mathbf{\Pi}_2^{+} &= \mathbf{\Gamma}_{22}^{21}\mathbf{\Gamma}_{12}^{1\dagger}. \end{cases} \tag{16}$$

The expressions (15) and (16), give the propagator operator. This operator is estimated from the non-affected elements by noise of the data cross-spectral matrix.

### 4.4 The Propagator for a large number of sensors

We introduce a new partitioning of the channel matrix to isolate the affected blocks matrices:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ --- \\ \mathbf{H}_2 \\ --- \\ \vdots \\ --- \\ \mathbf{H}_n \end{bmatrix} \tag{17}$$

$\mathbf{H}_1$, $\mathbf{H}_2$ and $\mathbf{H}_i$ are the same dimensions: $(P \times P)$.
As we assume that the matrix $\mathbf{H}$ is of full-rank. The $P$ rows of $\mathbf{H}_i$ are, then, linearly dependents of the $P$ rows of $\mathbf{H}_j$, we can write any submatrice $\mathbf{H}_i$ function of $\mathbf{H}_j$ of $\mathbf{H}$, with $n$ is an even number and $i-j = \frac{n}{2}$. Then, we have:

$$\mathbf{H}_j = \mathbf{\Pi}_{i,j}^{+}\mathbf{H}_i \tag{18}$$

with $i-j = n/2$ and $n$ is assumed to be an even number.
This technique allows to use a small part of the cross-spectral matrix to build the noise subspace.

From (18), $\mathbf{\Pi}_{i,j}^{+}\mathbf{H}_i - \mathbf{H}_j = \mathbf{0}$, then $\left[\mathbf{\Pi}_{i,j}^{+} \mid -\mathbf{I}_P\right]\begin{bmatrix} \mathbf{H}_i \\ -- \\ \mathbf{H}_j \end{bmatrix} = \mathbf{0}$.

Let $\mathbf{Q}^{+} = [\mathbf{\Pi}^{+} \mid -\mathbf{I}_P]$, and $\overline{\mathbf{H}} = \begin{bmatrix} \mathbf{H}_i \\ -- \\ \mathbf{H}_j \end{bmatrix}$, then we have:

$$\mathbf{Q}^{+}\overline{\mathbf{H}} = \mathbf{0} \tag{19}$$

The expression (19) shows that the $P$ columns of $\mathbf{Q}$ are linearly independents and orthogonals to the channel vectors $\overline{\mathbf{H}}$.
This means that the columns of $\mathbf{Q}$ span the noise subspace.
In the following, we estimate $\mathbf{Q}$ from the data cross-spectral matrix.

### 4.5 Extraction of the propagator from data

After the presentation of different approaches of the propagator, here, we present its corresponding noise subspace estimated from the data received on an antenna with a large number of sensors.
This method is called DESO for Direct Extraction of the noise Subspace from the Observations.
We assume that the antenna has a large number of sensors compared to the number of sources (the application is in the radiolocation domain with the presence of few mobile stations). We divide the data cross-spectral matrix $\mathbf{\Gamma}$ into $n$ submatrices. Where $N = nP$ and $N \gg P$, with $n$ is an even number $n > 1$. The propagator operator can be estimated from the data cross-spectral matrix.
Rule: let the data cross-spectral matrix, $\mathbf{\Gamma} = \mathbf{H}\mathbf{\Gamma}_s\mathbf{H}^{+} + \mathbf{\Gamma}_n$. As we assume that the matrix $\mathbf{H}$ is of full-rank. The $P$ rows of $\mathbf{H}_i$ are, then, linearly dependents of the $P$ rows of $\mathbf{H}_j$. Let the following partitioning of $\mathbf{\Gamma}$: $\mathbf{\Gamma} = [\mathbf{A}_1 \mid ... \mid \mathbf{A}_i \mid ... \mid \mathbf{A}_j \mid ... \mid \mathbf{A}_n]$. where $\mathbf{A}_i$, $\mathbf{A}_j$, ..., $\mathbf{A}_n$ are matrices of dimensions $(N \times P)$, respectively. We can write any submatrice $\mathbf{A}_i$ function of $\mathbf{A}_j$ of $\mathbf{\Gamma}$, with $j-i = \frac{n}{2}$ and $n$ is an even number. Then, we have: $\mathbf{A}_j = \mathbf{\Pi}_{i,j}^{+}\mathbf{A}_i$.

$\mathbf{\Pi}_{i,j}^{+}\mathbf{A}_i - \mathbf{A}_j = \mathbf{0}$, then $\left[\mathbf{\Pi}_{i,j}^{+} \mid -\mathbf{I}_P\right]\begin{bmatrix} \mathbf{A}_i \\ -- \\ \mathbf{A}_j \end{bmatrix} = \mathbf{0}$.

Let $\mathbf{Q}^{+} = \left[\mathbf{\Pi}_{i,j}^{+} \mid -\mathbf{I}_P\right]$, and $\overline{\mathbf{A}} = \begin{bmatrix} \mathbf{A}_i \\ -- \\ \mathbf{A}_j \end{bmatrix}$ then, we have:

$$\mathbf{Q}^{+}\overline{\mathbf{A}} = \mathbf{0} \tag{20}$$

The expression (20) shows that the $P$ columns of $\mathbf{Q}$ are linearly independents and orthogonals to $\overline{\mathbf{A}}$.
To estimate the DESO operator, the optimization criterion is:

$$\widehat{\mathbf{\Pi}}_{i,j} = \arg\underset{\mathbf{\Pi_{i,j}}}{min} \parallel \mathbf{A}_j - \mathbf{A}_i\mathbf{\Pi} \parallel_F, \tag{21}$$

where $\parallel . \parallel_F$ is the Frobenius norm, we get: $\mathbf{\Pi}_{i,j} = \mathbf{A}_i^{\dagger}\mathbf{A}_j$.

In the case of an additive white noise , i.e., $\boldsymbol{\Gamma}_n = \ ^2\mathbf{I}_N$, it can be shown that:

$$\mathbf{Q}^+ = \left[\boldsymbol{\Pi}_{i,j}^+ \mid -\mathbf{I}_P\right], \tag{22}$$

$$\text{verifies} \quad \mathbf{Q}^+\overline{\mathbf{H}} = \boldsymbol{\Pi}_{i,j}^+\mathbf{H}_i - \mathbf{H}_j = \mathbf{0}, \tag{23}$$

The expression (23) shows that the $P$ columns of $\mathbf{Q}$ are linearly independents and orthogonals to the source vectors $\overline{\mathbf{H}}$. This means that the columns of $\mathbf{Q}$ span the noise subspace. The DESO null-spectrum $P(f, \ )$ is given by:

$$P(f, \ ) = \left\{\mathbf{h}^+(f, \ )\mathbf{Q}\mathbf{Q}^+\mathbf{h}(f, \ )\right\}^{-1}$$

In the performance simulations, we compare the two null-spectrums of MUSIC and the Propagator, $S(f, \ )$ and $P(f, \ )$ respectively .

### 4.6 DESO's noise projector vs. noise eigenvectors

Let the eigendecomposition of the data cross-spectral matrix: $\boldsymbol{\Gamma} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^+ = \mathbf{U}_s\boldsymbol{\Lambda}_s\mathbf{U}_s^+ + \mathbf{U}_n\boldsymbol{\Lambda}_n\mathbf{U}_n^+$, where $\mathbf{U} = [\mathbf{U}_s \mid \mathbf{U}_n]$, $\boldsymbol{\Lambda} = diag(\boldsymbol{\Lambda}_s, \boldsymbol{\Lambda}_n)$; $\boldsymbol{\Lambda}_s = diag(\ _1, ..., \ _P)$ contains the $P$ largest eigenvalues of $\boldsymbol{\Gamma}$ in descending order and $\mathbf{U}_s = [\mathbf{u}_1, ..., \mathbf{u}_P]$ contains the corresponding orthonormal eigenvectors; $\boldsymbol{\Lambda}_n = \ ^2\mathbf{I}_{N-P}$ and $\mathbf{U}_n = [\mathbf{u}_{P+1}, ..., \mathbf{u}_N]$ contain the $(N-P)$ orthonormal eigenvectors that correspond to the eigenvalue $\ ^2$, i.e., the noise eigenvectors.

In the following, we establish the relationship between $\mathbf{Q}$ and $\mathbf{U}_n$. Define the partition of $\mathbf{U}_n$: $\mathbf{U}_n = \begin{bmatrix} \mathbf{U}_{na} \\ --- \\ \mathbf{U}_{nb} \end{bmatrix}$, where $\mathbf{U}_{na}$ and $\mathbf{U}_{nb}$ are two matrices of dimensions $(M+N \times N-P)$ and $(N-P \times N-P)$, respectively. Since,

$$\mathbf{H}^+\mathbf{U}_n = \mathbf{H}_a^+\mathbf{U}_{na} + \mathbf{H}_b^+\mathbf{U}_{nb} = \mathbf{0}, \tag{24}$$

we have, $\boldsymbol{\Pi} = -\mathbf{U}_{na}\mathbf{U}_{nb}^{-1}, \qquad \mathbf{Q} = -\mathbf{U}_n\mathbf{U}_{na}^{-1}$.

The orthonormal matrix $\mathbf{Q}_o$ (normalized version of $\mathbf{Q}$) is equal to $-\mathbf{U}_n$. This expression shows that the columns of $\mathbf{Q}_o$ are equal to the smallest eigenvectors of the observation cross-spectral matrix.

## 5. PERFORMANCE SIMULATIONS

To demonstrate the efficiency of DESO method, some computer simulations have been conducted. For all these simulations, the number $K$ of data samples used to estimate each is equal to 1000 samples. The normalized root mean-square error $(NRMSE)$ defined in the following is employed as a performance criterion of the input estimates: $NRMSE = \frac{1}{\| \ \|}\left(\frac{1}{K}\sum_{k=1}^{K}\| \ _k - \ \|^2\right)^{\frac{1}{2}}$, $\ _k$ is the estimate of the inputs from the $k$th trial.

In this simulation, 2 sources are received by an antenna of 20 sensors. In this case, the computational load is reduced 10 times by using the DESO algorithm.

Based on the NRMSE, we compare the classical subspace method [5] and DESO algorithm . The figure 1 demonstrates that the subspace identification method based on the eigendecomposition of the data cross-spectral matrix shows significantly less normalized mean square estimation error than the DESO method. The NRMSE tends to zero when the SNR value increases for both methods and when the SNR value is greater that $10dB$, the two methods give the same performances.

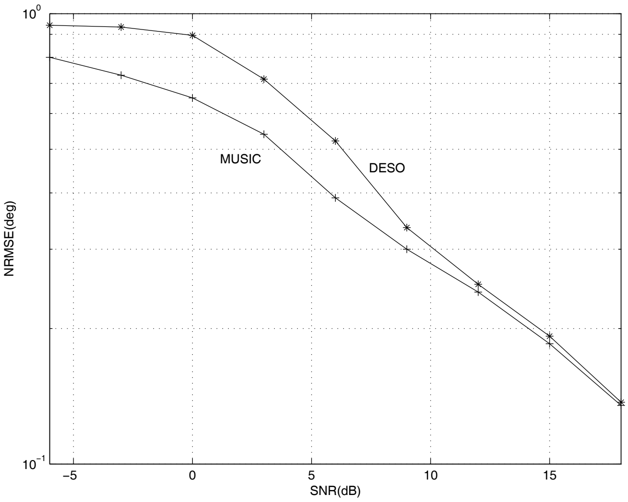


Figure 1: NRMSE of the parameters estimates versus SNR ($K = 1000$).

## 6. CONCLUSION

This paper presents a fast method for direction of arrival estimation for radiolocation. The DESO method is computationally more efficient, because it doesn't need any eigendecomposition of the data spectral matrix to estimate the noise subspace. For high value of the SNR, DESO method has, almost, the same performances as the classical subspace method.

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