UWB PRECISION GEOLOCATION USING FFT INTERPOLATION

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ABSTRACT

A precision positioning algorithm employing low data rate ultra wideband (UWB) technology is proposed. The positioning algorithm utilizes cross-correlation for estimating time difference of arrival (TDOA) between UWB pulses received at multiple receivers and provides a precise location estimate of the signal source thanks to FFT interpolation. The geolocation estimate is obtained from an iterative maximum likelihood estimator implemented as a Gauss-Newton algorithm. The effectiveness of the proposed precision positioning algorithm is verified by way of computer simulations.

1. INTRODUCTION

Precision geolocation and tracking of emitters is an important application of UWB technology [1]. The unique advantages of low-rate UWB in combating multipath propagation make it well-suited for accurate localization of sources by employing TDOA techniques. The basic idea behind UWB is to excite a wide-band electromagnetic radiator with very short and rapid rising pulses, approximating an impulse function, which in turn generates high frequency wideband signals. The benefits of increased RF bandwidth are well understood. From a communications point of view, UWB offers the following advantages:

- immunity to multi-path cancellation,
- low interference to existing systems,
- secure communications,
- increased time resolution,
- low-power consumption.

According to the FCC definition, a UWB signal must have a 10 dB bandwidth of at least 500 MHz and the fractional bandwidth must be at least 0.2, as determined by the -10 dB band-edge frequencies [2].

Taking advantage of precise time of arrival (TOA) measurements, precision geolocation techniques have been developed based on TOA measurements of short-duration UWB pulses [3]. In large-noise environments, time delay estimation based on generalized cross-correlation [4] can provide better performance than simple TOA estimation. The sampling of the analog UWB waves can be challenging. This is because of the large Nyquist rate. In [5] a subband channelization technique was proposed to reduce the required sampling rate. In this paper, we present a bandpass sampling technique to reduce the sampling rate. In geolocation applications, reduced sampling results in decreased range resolution. To restore the range resolution, we develop a novel high-resolution cross-correlation technique incorporating FFT interpolation.

2. GEOLOCATION BY TDOA

Geolocation by TDOA is a passive localization method that employs TDOA estimates of signals arriving at spatially distributed receivers at known locations. Central to geolocation by TDOA is the availability of time delay estimates at multiple pairs of receivers. For the sake of simplicity we will restrict our attention to 2-D localization of a stationary source using stationary receivers. Extension to 3-D localization is straightforward.

The UWB signal transmitted by the source arrives at the receivers with time delays proportional to the range. The receiver locations are given by the two-dimensional column vectors $\boldsymbol{r}_i = [x_i, y_i]^T$, $i = 1, \ldots, N$, and the source location to be estimated by $\boldsymbol{p} = [x_p, y_p]^T$. The range vectors are $\boldsymbol{d}_i = \boldsymbol{p} - \boldsymbol{r}_i$. For N receivers ($N \geq 3$), the TDOA estimates between pairs of receivers can be utilized to estimate the location of the emitter. Each receiver picks up a delayed, attenuated and possibly corrupted version of the UWB signal transmitted by the source, denoted $s_i(t)$. The $s_i(t)$ are fed to the TDOA estimator. The geolocation algorithm uses the TDOA estimates and the receiver locations $\boldsymbol{r}_i$ to estimate the UWB source location. Under the Gaussian distribution assumption for the TDOA estimates, a maximum likelihood (ML) estimate of the source location can be obtained by maximizing the joint probability density function of the TDOA estimates. The ML estimate can be written as

$$\hat{\boldsymbol{p}}_{\mathrm{ML}} = \arg\min_{\boldsymbol{p}} J_{\mathrm{ML}}(\boldsymbol{p}) \tag{1}$$

where $J_{\mathrm{ML}}(\boldsymbol{p})$ is the ML cost function

$$J_{\mathrm{ML}}(\boldsymbol{p}) = \boldsymbol{e}^T(\boldsymbol{p})\boldsymbol{\Sigma}^{-1}\boldsymbol{e}(\boldsymbol{p}) \tag{2}$$

and $\boldsymbol{e}(\boldsymbol{p})$ is the TDOA error vector

$$\boldsymbol{e}(\boldsymbol{p}) = \begin{bmatrix} \|\boldsymbol{p} - \boldsymbol{r}_2\| - \|\boldsymbol{p} - \boldsymbol{r}_1\| - \hat{g}_{12} \\ \|\boldsymbol{p} - \boldsymbol{r}_3\| - \|\boldsymbol{p} - \boldsymbol{r}_1\| - \hat{g}_{13} \\ \vdots \\ \|\boldsymbol{p} - \boldsymbol{r}_N\| - \|\boldsymbol{p} - \boldsymbol{r}_1\| - \hat{g}_{1N} \end{bmatrix}_{(N-1)\times 1}. \tag{3}$$

Here $\hat{g}_{1i}$ is the range difference of arrival (RDOA) estimate between receiver 1 and receiver i, given by

$$\hat{g}_{1i} = g_{1i} + n_{1i} = c\hat{\tau}_{1i} \tag{4}$$

where $g_{1i} = \|\boldsymbol{d}_i\| - \|\boldsymbol{d}_1\|$, n_{1i} is the i.i.d. zero-mean Gaussian RDOA noise, c is the speed of electromagnetic

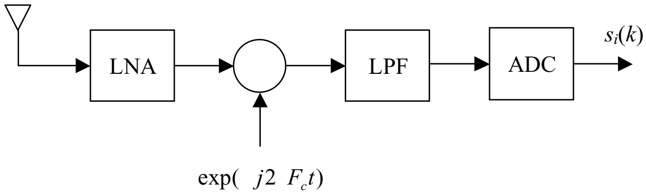

Figure 1: UWB receiver employing bandpass sampling.

wave propagation in open space and $\hat{\tau}_{1i}$ is the TDOA estimate between receiver 1 and receiver i. In (2) $\boldsymbol{\Sigma}$ is the covariance matrix of the RDOA noise:

$$\boldsymbol{\Sigma} = \sigma_n^2 \begin{bmatrix} 1 & 1/2 & \cdots & 1/2 \\ 1/2 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1/2 \\ 1/2 & \cdots & 1/2 & 1 \end{bmatrix} \tag{5}$$

where σ_n^2 is the RDOA noise variance.

Equation (1) does not have a closed-form solution. An iterative numerical solution can be formulated by using the Gauss-Newton (GN) algorithm—also known as the Taylor series method:

$$\hat{\boldsymbol{p}}_{k+1} = \hat{\boldsymbol{p}}_k - (\boldsymbol{J}_k^T \boldsymbol{\Sigma}^{-1} \boldsymbol{J}_k)^{-1} \boldsymbol{J}_k^T \boldsymbol{\Sigma}^{-1} \boldsymbol{e}(\hat{\boldsymbol{p}}_k), \quad k = 0, 1, \ldots \tag{6}$$

where $\boldsymbol{J}_k$ is the $(N-1) \times 2$ Jacobian matrix of $\boldsymbol{e}(\boldsymbol{p})$ evaluated at $\boldsymbol{p} = \hat{\boldsymbol{p}}_k$:

$$\boldsymbol{J}_k = \begin{bmatrix} \frac{(\hat{\boldsymbol{p}}_k - \boldsymbol{r}_2)^T}{\|\hat{\boldsymbol{p}}_k - \boldsymbol{r}_2\|} - \frac{(\hat{\boldsymbol{p}}_k - \boldsymbol{r}_1)^T}{\|\hat{\boldsymbol{p}}_k - \boldsymbol{r}_1\|} \\ \vdots \\ \frac{(\hat{\boldsymbol{p}}_k - \boldsymbol{r}_N)^T}{\|\hat{\boldsymbol{p}}_k - \boldsymbol{r}_N\|} - \frac{(\hat{\boldsymbol{p}}_k - \boldsymbol{r}_1)^T}{\|\hat{\boldsymbol{p}}_k - \boldsymbol{r}_1\|} \end{bmatrix}. \tag{7}$$

The GN iterations are stopped when the update term satisfies

$$\|\hat{\boldsymbol{p}}_{k+1} - \hat{\boldsymbol{p}}_k\|_2^2 < \eta \tag{8}$$

where η is a threshold that is used as a stopping criterion.

Despite its almost quadratic convergence, the GN algorithm requires an initial guess $\hat{\boldsymbol{p}}_0$ that is sufficiently close to the ML solution in order to avoid divergence. An initial guess can be obtained from another closed-form estimate (see e.g. [6, 7, 8, 9, 10, 11, 12]) or geometric insight into the localization scenario. The ML estimator enjoys certain desirable properties such as asymptotic unbiasedness and asymptotic efficiency.

3. UWB TDOA ESTIMATION

Suppose that a UWB signal source transmits pulses at regular intervals. Because of the multipath effects, the transmitted pulse arrives at the receivers with multiple delayed and attenuated replicas. Given the short pulse duration of a UWB pulse, the receiver will be capable of discriminating between direct pulse and multi-path reflected replicas. Only the first arriving pulse is used for TDOA estimation. This also provides immunity to multipath effects.

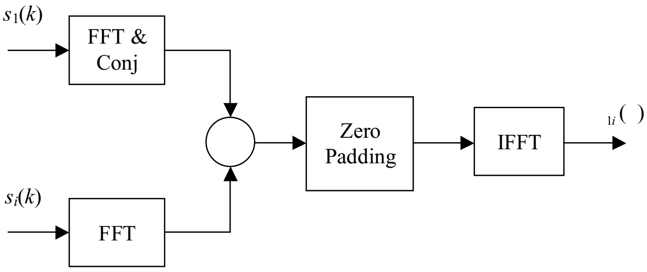

Figure 2: Implementation of cross-correlation for precision TDOA estimation.

A simplified block diagram of the proposed UWB receiver for geolocation purposes is shown in Fig. 1. The receiver comprises a low noise amplifier (LNA), a mixer and an analog-to-digital converter (ADC). The UWB receiver avoids Nyquist rate sampling of the pulse by employing a mixer circuit followed by a lowpass filter that generates an equivalent lowpass signal. Given the typically large fractional bandwidth of UWB pulses, the equivalent lowpass signal can be further bandlimited by choosing an appropriate LPF cut-off frequency at the expense of spreading the received pulse. This spreading has no undesirable effect on pulse detection as long as the first pulse arrival detection is carried out before digitization, e.g., by using a tunnel diode detector. The advantage of lowering the LPF cut-off frequency is to achieve further reduction in the required sampling rate.

The sampled pulses are next applied to a cross-correlator in a central hub for TDOA estimation. The bandpass sampling permits a significant reduction in sampling frequency. For example, for centre frequency $F_c = 5$ GHz and 10 dB bandwidth $W = 1$ GHz, the Nyquist rate will be 12 GHz and the bandpass sampling frequency 1 GHz. In this case, the Nyquist rate is prohibitively large. The range resolution that can be achieved at the bandpass sampling frequency is $c/F_s = 3 \times 10^8/10^9 = 0.3$ m. If the sampling rate is further reduced by way of lowpass filtering the signal to a smaller bandwidth, the range resolution becomes larger.

A disadvantage of bandpass sampling is poor range resolution. For bandpass sampled signals the range resolution can be improved by interpolation. Interpolation can be implemented by upsampling and lowpass filtering or by employing FFT, zero padding and IFFT [13]. We propose to incorporate the latter interpolation method into cross-correlation estimation as shown in Fig. 2, resulting in a cost-effective precision TDOA estimation method. The TDOA estimation and geolocation algorithms are implemented in a central hub that interfaces to the receivers through the ADC outputs. The P-point FFT of $s_i(k)$ is

$$S_i(n) = \sum_{k=0}^{P-1} s_i(k) e^{-j2\pi kn/P}, \quad n = 0, 1, \ldots, P-1.$$

To avoid distortion due to circular convolution the $s_i(k)$ may need to be zero padded before taking the FFT. For TDOA estimation between $s_1(k)$ and $s_i(k)$, $i =$

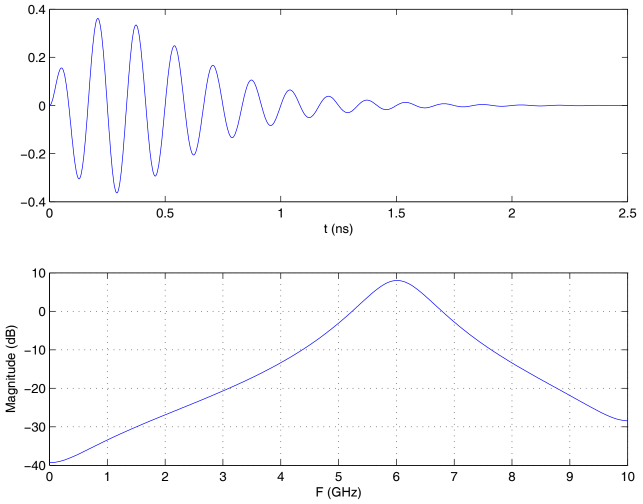

Figure 3: Simulated UWB wave and its amplitude spectrum.

$2, \ldots, N$, we first compute

$$V(n) = S_1^*(n)S_i(n)$$

where * denotes complex conjugate. Assuming that P is even, interpolation by a factor of L is simply achieved by zero padding $V(n)$ to a total length of LP before taking the IFFT. Zero padding in the FFT domain is done by inserting $(L-1)P$ zeros between $V(n_c)$ and $V(n_c+1)$ where n_c is the highest-frequency FFT bin defined by $n_c = \lfloor P/2 \rfloor$. Thus, the zero-padded $V(n)$ is given by

$$V_z(n) = [V(0), \ldots, V(n_c), \underbrace{0, 0, \ldots, 0}_{(L-1)P \text{ times}}, V(n_c+1), \ldots, V(P-1)]. \tag{9}$$

The LP-point IFFT of $V_z(n)$ gives the cross-correlation function interpolated by a factor of L:

$$\rho_{1i}(\tau) = \frac{1}{LP} \sum_{n=0}^{LP-1} V_z(n) e^{j2\pi\tau n/(LP)}. \tag{10}$$

Interpolation by zero padding avoids computationally demanding upsampling and lowpass filtering operations. For large L, the IFFT operation becomes computationally expensive. However, a significant reduction in complexity is achievable by discarding the zero terms in the IFFT expression in (10) and by restricting the IFFT computation to the lags of interest only using the Goertzel algorithm [14]. The lags of interest represent the range of possible time delays $|\tau| \leq \tau_{\max}$. The values of $\rho_{1i}(\tau)$ for negative lags are easily obtained by exploiting the periodicity of $V_z(n)$. An estimate of RDOA is given by

$$\hat{g}_{1i} = c\frac{T_s}{L} \arg\max_{\tau} |\rho_{1i}(\tau)|. \tag{11}$$

4. SIMULATION STUDIES

A UWB wave has been simulated with centre frequency $F_c = 6$ GHz and 10 dB bandwidth $W = 2$ GHz. The fractional bandwidth of the wave is $1/3$, which complies with the FCC requirements [2]. The simulated UWB pulse and its spectrum are shown in Fig. 3. The GN algorithm has been simulated with RDOA estimates obtained from the interpolated cross-correlation method developed in Section 3. The initial guess of the GN algorithm was set to the geometric centre of the receiver locations. The cross-correlation parameters were $P = 500$ and $L = 10$. The LPF at the mixer output was implemented as a Butterworth filter of order 10 and cut-off frequency 1 GHz. The sampling frequency of the ADC was set to $F_s = 2$ GHz, which gives a range resolution of $c/F_s = 3 \times 10^8 / 2 \times 10^9 = 0.15$ m. Interpolation by $L = 10$ reduces the range resolution to 1.5 cm.

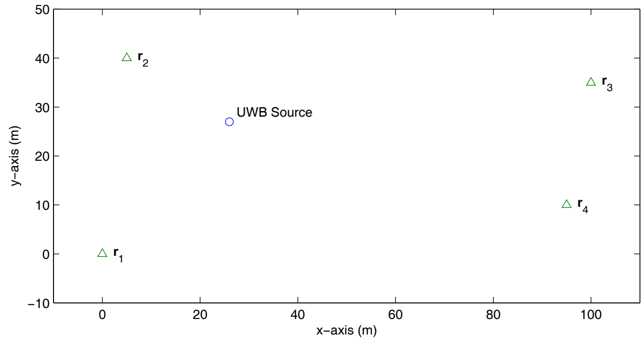

Figure 4: Localization geometry.

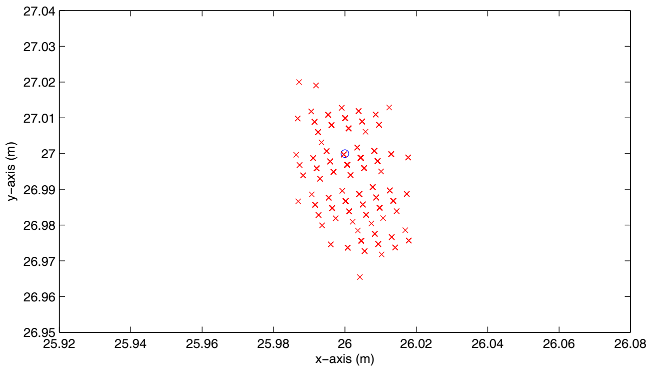

Figure 5: Geolocation estimates at -30 dBW noise level for $L = 10$.

Noise Power (dBW)	Bias Norm (m)	MSE
-40	0.0045	3.85×10^{-5}
-35	0.0059	7.29×10^{-5}
-30	0.0061	1.62×10^{-4}
-29	0.0059	2.16×10^{-4}
-28	0.0064	3.20×10^{-4}
-27	0.0073	4.67×10^{-4}
-26	0.2520	10.40

Table 1: Bias and MSE performance for $L = 10$.

The simulated localization geometry with four receivers is shown in Fig. 4. The received UWB pulses were assumed to be corrupted by additive white

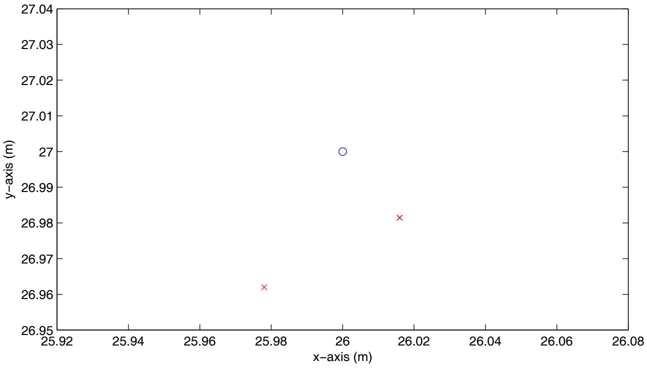

Figure 6: Geolocation estimates at −30 dBW noise level for $L = 1$.

Gaussian noise. The bias and mean-squared error (MSE) performance of the GN algorithm was estimated using 1000 Monte Carlo simulations. The results are given in Table 1 for several noise power values. The GN algorithm performs extremely well until the noise power exceeds −26 dBW (approx. 2.5 mW). The resulting geolocation accuracy is better than 2 cm with a bias of approx. 0.5 cm. The geolocation estimates for noise power of −30 dBW are shown in Fig. 5. Note the discrete nature of the geolocation estimates arising from sampling of the received signals.

Noise Power (dBW)	Bias Norm (m)	MSE
−40	0.0245	5.99×10^{-4}
−35	0.0245	5.99×10^{-4}
−30	0.0244	6.04×10^{-4}
−29	0.0243	6.25×10^{-4}
−28	0.0239	6.59×10^{-4}
−27	0.0233	7.54×10^{-4}
−26	0.2544	13.24

Table 2: Bias and MSE performance for no interpolation.

The simulations were repeated for $L = 1$, i.e., no interpolation. The bias and MSE performance is given in Table 2. Compared with Table 1, the geolocation algorithm exhibits increased bias and MSE. The geolocation estimates for noise power of −30 dBW are shown in Fig. 6. Note that the quantization of the estimates due to sampling is quite coarse compared with the case of $L = 10$ in Fig. 5.

5. CONCLUSION

A UWB precision geolocation method utilizing bandpass sampling and interpolated cross-correlation was proposed. The effectiveness of the algorithm was demonstrated by way of computer simulations. A significant reduction in sampling rate was achieved by bandpass sampling the received UWB signals. The loss in range resolution due to bandpass sampling was recovered by a novel cross-correlation method incorporating FFT interpolation.

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