OPTIMAL FAULT-TOLERANT EVENT DETECTION IN WIRELESS SENSOR NETWORKS

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ABSTRACT

In this paper, we propose an optimal fault-tolerant algorithm for distributed event detection in wireless sensor networks. Two important problems are addressed: 1. How to handle both the noise-related measurement error and sensor fault simultaneously in fault-tolerant detection? 2. How to choose a proper neighborhood size n for a sensor node in fault correction such that the maximum energy could be conserved? Both theoretical analysis and experimental results confirm the effectiveness and efficiency of the proposed algorithm.

1. INTRODUCTION

Recent advancement in wireless communications and electronics has enabled the development of low-cost wireless sensor networks. One of the important sensor network applications is for event detection in inaccessible environments. For example, sensor networks can be used to detect foreign chemical agents in the air and the water and the event could be unusual high chemical concentration that generates a lot of safety and health concerns for the public.

In general, there are two fundamental challenges in the event detection for a wireless sensor network: 1) the detection accuracy is limited by the amount of noise associated with the measurement process and the reliability of sensor nodes because of the low-end inexpensive devices; 2) the source of energy for a sensor node is most often an attached battery cell. Centralized event detection algorithms, which require all sensor nodes to transmit their individual sensor measurements and their geographical locations directly to a central monitoring node, are not suitable for a wireless sensor network due to the energy constraints. A localized and distributed detection algorithm is highly preferred for wireless sensor networks.

The basic idea of distributed detection [1] is to have a number of independent sensors each makes a local decision (typically a binary one) and then to combine these decisions at a fusion sensor to generate a global decision. Statistically the distributed event detection could be modelled as a hypothesis test problem. n sensors observe an unknown hypothesis. The sensor observations are independent and identically distributed given the unknown hypothesis. Each sensor transmits its decision over a multiple access channel to a fusion sensor. Based on the received sensor decision, the fusion sensor makes the final decision regarding the unknown hypothesis.

In this paper, we propose an optimal fault-tolerant detection algorithm that 1) considers both measurement error and sensor fault in fault correction and 2) is able to achieve better balance between detection accuracy and energy usage. The remaining of the paper is organized as follows. In Section 2, we briefly review some related work of distributed detection in wireless sensor networks. Theoretical analysis and the detail of the proposed algorithm is presented in Section 3. In Section 4, We present our experiments and results. We give our conclusions in Section 5.

2. STATEMENT OF THE PROBLEM AND RELATED WORK

N sensor nodes are deployed over a interested region to perform event detection. All sensors have the same modalities and ability to communicate with each other. Each sensor node has n neighbor sensors. A sensor node could make its binary decision independently based on its own measurement from the noisy environment. The network considered is also likely to contain faulty sensor nodes due to the harsh environment and manufacturing reasons. The faulty behavior we consider includes that normally an event, if happens, should be detected as "event" by sensors at the location, but the detection decision is converted to "no-event" due to the sensor fault, or vice versa.

Let us consider a two layer detection system that consists of a fusion sensor and its n neighbor sensors. The fusion sensor makes a final decision whether an unknown hypothesis is H_0 or H_1 based on the decision from the n sensors. Let q_0 and q_1 denote the prior probabilities of H_0 and H_1 respectively, X_i denote the observation of the ith sensor, $i = 1,...,n$. When H_j is true, X_i follows the probability distribution function $p(x_i|H_j)$, $j = 0,1$. Let u_i denote the binary decision (0 or 1) of the ith sensor, which is the output of likelihood ratio threshold test [2],

$$\frac{p(x_i|H_1)}{p(x_i|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \tag{1}$$

where λ is the common threshold used for all sensor nodes.

The sensor nodes transmit their decisions to the fusion sensor. Based on the received sensor decisions, the fusion sensor makes the final decision u_0 with the optimum fusion rule which, in this case, is a k-out-of-n rule [3]. Let $u_0 = 0$ if the fusion sensor decides H_0 and let $u_0 = 1$ if the fusion sensor decides H_1, we have,

$$u_0 = \begin{cases} 1, & u_1 + \cdots + u_n \geq k \\ 0, & u_1 + \cdots + u_n < k \end{cases} \tag{2}$$

where k is an integer between 1 and n.

For all the sensors, let P_F denote the identical false alarm probability and P_D the identical detection probability, we have

$$P_F = P(u_i = 1|H_0) \tag{3}$$

$$P_D = P(u_i = 1|H_1) \tag{4}$$

The quality of the fusion sensor decision u_0 is measured by the system false alarm probability Q_F and the system detection probability Q_D,

$$Q_F = \sum_{i=k}^{n} \binom{n}{i} P_F^i (1-P_F)^{n-i} \tag{5}$$

$$Q_D = \sum_{i=k}^{n} \binom{n}{i} P_D^i (1-P_D)^{n-i} \tag{6}$$

Under Bayesian detection framework, the probability of detection error with n sensor nodes is given by,

$$P_e^n = q_0 Q_F + q_1(1-Q_D) \tag{7}$$

Zhang et al. show that given k, the probability of detection error is a quasiconvex function of λ and has a single minimum that is achieved by the unique optimal λ_{opt} [3]. The overall optimal solution is obtained by optimizing (k,λ) pair via the SECANT algorithm.

Recently Krishnamachari and Iyengar introduce a fault-tolerant event detection method for wireless sensor networks [4]. Based on the observation that the sensor faults are likely to be stochastically uncorrelated, while event measurements are likely to be spatially correlated, they propose to let an individual sensor node communicate with its n neighbors and use their binary decisions to correct its own decision. A majority voting scheme is shown to be the optimal decision scheme for fault correction in their work. In this paper, we propose to improve effectiveness (in terms of fault tolerance) and the efficiency (in terms of energy consumption) of their approach by considering two additional important questions:

1. How to include the decision error caused by the noisy measurement into consideration during fault-tolerant event detection? In Krishnamachari and Iyengar's work [4], they only consider the sensor fault problem. The measurement error is not discussed by assuming that a preset threshold enables each sensor node to make its own binary decision. However, the measurement inaccuracy has direct impact on the effect of fault correction.

2. How to decide a proper neighborhood size n for a sensor node? To our knowledge, this problem has not been studied at all in the literatures. Currently the neighborhood size n is typically determined by the maximum communication radius of a sensor node, which is not energy-efficient. A large n will result in extensive communication within the network and thus consume great amount of energy. In general, an energy-efficient detection scheme should be able to make accurate detection while keeping the energy dissipation at its minimum.

These two problems are theoretically analyzed in the following section.

3. THEORETICAL ANALYSIS

3.1 Optimization under both noisy measurement and sensor fault

Define two situations in event detection,

$$H_0: \quad normal$$
$$H_1: \quad event$$

The objective of the distributed detection is to choose the optimal threshold λ at each sensor, as well as the optimal k at the fusion sensor, given q_0, q_1 and the neighborhood size n (usually determined by the maximum communication radius of a sensor node). Zhang et al. studied this problem and their work can be summarized as the following theorem without proof (refer to [3] for the detail of the proof).

Theorem 1 *For fixed n and k, the probability of detection error P_e^n is a quasiconvex function of λ and has a single minimum that is achieved by the unique optimal λ_{opt}. λ minimizes P_e^n if is satisfies*

$$ln\frac{q_1}{q_0} + ln\lambda + (k-1)ln\frac{P_D}{P_F} + (n-k)ln\frac{1-P_D}{1-P_F} = 0 \tag{8}$$

The optimal (τ(=$ln\lambda$), k) pair could be obtained based on Equation (8) via an optimization algorithm. Then each neighborhood sensor makes its own decision with the τ and the fusion sensor makes its final decision based on the k-out-of-n rule.

To consider sensor fault, let $P_f = \beta + \gamma$ be the probability of the sensor fault. β denotes the probability of type I sensor fault: originally an event is not detected and the decision is converted to event detected due to the sensor fault. γ denotes the probability of type II sensor fault: an event is detected originally and the decision is converted to event undetected by the sensor fault. Let $\widetilde{P}_F$ and $\widetilde{P}_D$ be false alarm and detection probability of each sensor, and $\widetilde{Q}_F$, $\widetilde{Q}_D$ are system false alarm and detection probability after the consideration of sensor fault. Without loss of generality, let's assume that these two type of sensor faults are symmetric, i.e., $\beta = \gamma = \frac{1}{2}P_f$.

The probability of detection error $\widetilde{P}_e^n$ needs to be minimized

$$\widetilde{P}_e^n = q_0\widetilde{Q}_F + q_1(1-\widetilde{Q}_D) \tag{9}$$

Since P_f is a constant, according to Theorem 1, when n is fixed, for a given k, λ minimizes $\widetilde{P}_e^n$ if it satisfies

$$\begin{aligned} &ln\frac{q_1}{q_0} + ln\lambda + (k-1)ln\frac{P_D(1-P_f)+\frac{1}{2}P_f}{P_F(1-P_f)+\frac{1}{2}P_f} \\ &+(n-k)ln\frac{1-P_D(1-P_f)-\frac{1}{2}P_f}{1-P_F(1-P_f)-\frac{1}{2}P_f} = 0 \end{aligned} \tag{10}$$

The optimal threshold pair (τ(=$ln\lambda$), k) for fault-tolerant distributed detection could be obtained based on Equation (10) via an optimization algorithm.

3.2 How to choose n?

Assume that each sensor node, when activated, consumes the same amount of energy. If we could choose a minimum neighborhood size n for a given detection error bound, the energy consumption during detection will be minimized.

Under the Bayesian framework, the detection scheme that optimizes (λ, k) pair gives us the minimum detection error $\widetilde{P}^n_{e,min} = min(\widetilde{P}^n_e)$. We claim,

Lemma 1 *Given sensor fault probability P_f, the minimum probability of detection error $\widetilde{P}^n_{e,min}$ approaches 0 exponentially with infinite neighborhood size n. for all is.*

Essentially it can be shown that $\widetilde{P}^n_{e,min}$ is bounded between 0 and the Chernoff bound [5], which approaches 0 exponentially as n approaches ∞. By squeeze theorem, Lemma 1 is proved. Space constraints preclude us from providing the detail of the proof.

Remark 1 *With Lemma 1, it is straightforward that, for a given upper bound of the detection error, the minimal neighborhood size n_{min} exists.*

With Remark 1, better balance between detection accuracy and energy consumption is able to be achieved.

3.3 The Algorithm

We propose a two-loop search algorithm to find the optimal solutions for a given bound of detection error $P_{e,bound}$ and a sensor fault probability P_f. In the inner loop, the optimal (τ, k) pair is obtained through numerical optimization for a fixed n. In the outer loop, a binary search is employed to find the minimum n that satisfy the given error bound.

Once we obtain the optimal τ, k, and n, the detection could be done as follows,

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1. Set (preset before the deployment or through online broadcasting after the deployment) τ, k, and n in each sensor node.
2. Each sensor obtains its binary decision u_i based on its measurement and $\tau(= ln\lambda)$ with threshold test.
3. Each sensor obtains the binary decisions of its n neighbors $u_1, u_2, ..., u_n$ (randomly selected within the communication radius) and compute $u_1 + \cdots + u_n$.
4. Each sensor makes its final fault-tolerant decision based on the k-out-of-n rule.

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4. EXPERIMENTS AND DISCUSSION

We consider the detection of known signals in Gaussian noise. The event to be observed by the sensor is $x_i = s_i + z_i$, where $s_i = \pm d$ is the interested signal and z_i is a Gaussian random variable with zero mean and unit variance. Define

$$\begin{aligned} H_0 &: \quad s_i = -d \\ H_1 &: \quad s_i = d \end{aligned}$$

The log-likelihood ratio τ_i for this problem is $\tau_i = 2dx_i$. The sensor false alarm and detection probabilities are given by,

$$P_F = Q(\frac{\tau}{2d} + d) \tag{11}$$

$$P_D = Q(\frac{\tau}{2d} - d) \tag{12}$$

where τ is the log-likelihood ratio threshold and

$$Q(z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \tag{13}$$

Our objective is to find the optimal τ, k, and n, given the prior probabilities q_0, q_1, a detection error bound $P_{e,bound}$, sensor fault probability P_f, and the maximum neighborhood size n_{max} (determined by the maximum communication radius of a sensor node).

4.1 Interesting Properties on $\widetilde{P}^n_e$, $\widetilde{P}^n_{e,min}$, n and τ

Fig. 1 shows the probability of detection error against log-likelihood ratio threshold τ with $q_0 = 0.75$, $q_1 = 0.25$, $d = 0.5$, $P_f = 12\%$ and different n. The (n, k) pairs in the figure indicate the optimal fusion rule for the given n. "*" indicates the minimum probability of detection error for the corresponding (n, k) pair. The top pair ($n = 3, k = 3$) corresponds to the curve with the highest "*" position; the second pair ($n = 5, k = 4$) corresponds to the curve with a "*" in the second highest position, and so on. From Fig. 1, we can see clearly that the minimum of detection error $\widetilde{P}^n_{e,min}$ decreases when n increases, which confirms Lemma 1.

However, Lemma 1 only holds for the optimal solution solved from Equation (8) and some sub-optimal solutions nearby as shown in Fig. 1. In general, the probability of detection error $\widetilde{P}^n_e$ is NOT always a decreasing function of n if a log-likelihood threshold τ is chosen outside the small interval around optimal τ.

Our experiment shows that the absolute difference between the two prior probabilities $|q_0 - q_1|$ is proportional to the reduction rate of $\widetilde{P}^n_{e,min}$. In other words, $\widetilde{P}^n_{e,min}$ decreases more quickly against n with a larger $|q_0 - q_1|$. This result indicates that if we have prior knowledge about the events in advance, less energy (smaller n) is required to maintain the same level of detection accuracy.

It is also interesting to notice that in Fig. 1 the optimal k (i.e. k_{min}) $= 3, 4, 5, 6, 7, 8$ for $n = 3, 5, 7, 9, 11, 13$ respectively. The majority voting rule ($k_{min} = \frac{1}{2}n$) is NOT the optimal fusion rule in this case any more. Our experiment also shows that in the particular case of $q_0 = q_1$, majority voting rule is always the optimal fusion rule. In other words, Theorem 2 in [4] is only a special case for $q_0 = q_1$ when both measurement error and sense fault are considered in fault corrections.

4.2 Detection Performance

In all experiments, we let $d = 1$, $q_0 = 0.75$, $q_1 = 0.25$ and $P_f = 10\%$. We also assume that the event region is in the 10 by 10 area at the bottom-left corner of the operation zone.

Fig. 2 shows the initial detection results (step 2 of the proposed detection scheme) with 400 sensors in total and 100 sensors in the event region. The log-likelihood ratio threshold τ is set at $\tau = 0.5964$ based on Equation (10). If the event is detected by a sensor node, it is marked by a circle "$\circ$" in the corresponding location. Otherwise the location is marked by a dot "$\cdot$". We simulated 10% sensor faults by reversing the sensor decisions. A "+" indicates that originally an event is detected at the location and the decision is converted to no-event due to sensor fault, while a "$\times$" represents the case of no-event detected originally. The probability of detection error is 19.75% with measurement error and sensor fault. The

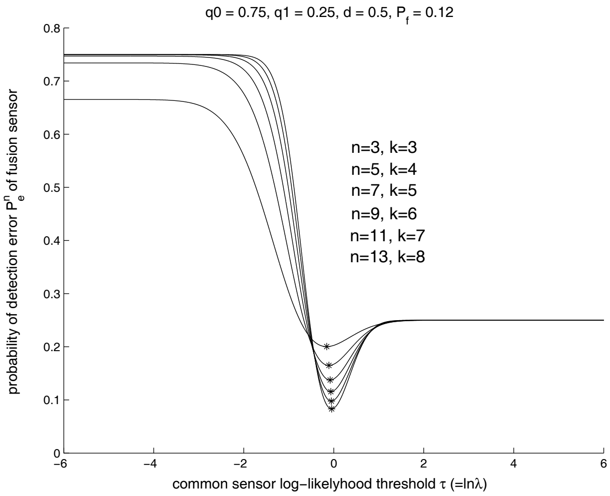

Figure 1: Probability of detection error $\widetilde{P}_e^n$ v.s. the neighborhood size n for the case of $q_0 = 0.75$, $q_1 = 0.25$, $d = 0.5$ and $P_f = 0.12$.

final detection result (step 3 and 4 of the proposed detection scheme) is presented in Fig. 3. After applying k-out-of-n rule ($n = 3$ and $k = 2$ computed based on Equation (10)), many incorrect detections (including false alarm and missing detection) have been corrected. As indicated in Fig. 1, the minimum probability of detection error is reduced more as n increases. However we could still offer 97% detection accuracy by choosing $n = 3$ if energy conservation is our top priority.

Notice that the detection accuracy in our simulation is lower than the theoretical estimation, which is mainly caused by the confusions along the boundary of the event region (see Fig. 3). How to identify the sensor nodes near the boundary and process their information accordingly are still challenging problems in event detections. We leave it for the future research.

We design two sensor networks, one with 20×20 sensors and the other with 100×100 sensors. We repeat the experiment 100 times. The average of detection errors of initial and final detection are shown in Table 1. In all cases, the detection errors have been greatly reduced. The effectiveness of the proposed algorithm is obvious. Notice that more energy will be saved by choosing a smaller n for a larger sensor network.

Table 1: Average initial and final detection errors for sensor networks with 400 and $10,000$ sensor nodes for the case of $P_f = 10\%$, $q_0 = 0.75$, $q_1 = 0.25$ and $d = 1$.

$\#of Sensors N$	τ	n,k	Ini. Err.	Final Err.
400	0.5964	3,2	20.78%	6.75%
400	0.2482	9,5	21.65%	4.31%
10,000	0.5964	3,2	20.70%	5.32%
10,000	0.2482	9,5	21.62%	3.21%

5. CONCLUSIONS

In this paper, we propose a fault-tolerant distributed detection scheme for wireless sensor networks, in which both measurement error and sensor fault are tightly coupled to achieve better detection. In the proposed scheme, the neighbor size n of fault correction is chosen based on the given detection error bound, such that better balance between detection accuracy and energy usage is obtained. Our experiment confirms the effectiveness of the proposed algorithm. Our work makes it possible to perform energy-efficient fault-tolerant event detections in a wireless sensor network.

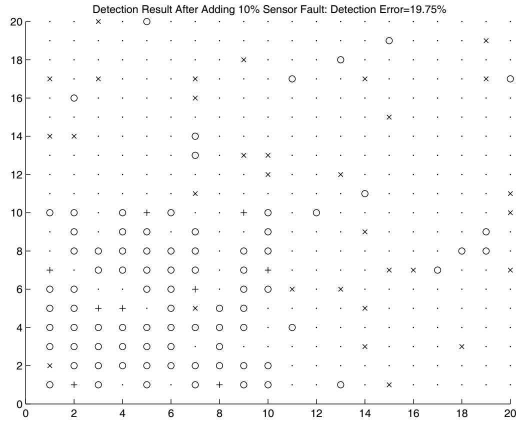

Figure 2: Initial detection results with 10% sensor fault ($P_f = 10\%$).

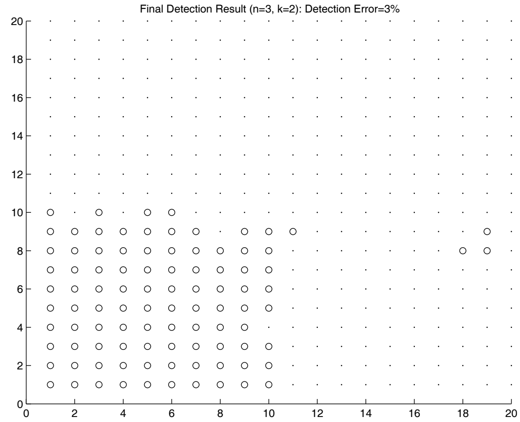

Figure 3: Final detection results with k-out-of-n rule ($n = 3$, $k = 2$).

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