Frequency Offset Estimation Based on Phase Offsets Between Sample Correlations

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***Abstract*—This paper deals with frequency offset estimation algorithms that exploit sample correlations of the examined sequence as described in [1], [2]. While previous work has only considered the phase offsets between succeeding correlation values, we want to demonstrate that it is possible to obtain better results by considering phase offsets between more correlation lags. The improvement is especially notable for low signal-to-noise ratios (SNR). Further, we will show, that it is possible to either improve performance, by restricting the estimation range (with no additional complexity) or by using more complex algorithms without having to sacrifice estimation range.**

I. Introduction

Frequency offset estimation has been discussed in many previous publications. Most of these publications exploit sample correlations to generate the frequency offset estimate. There are, however, many different ways to obtain this estimate from the sample correlations.

[3] and [4] describe simple estimators that are efficient and have a very low computational complexity. It is shown that it does make a difference whether we use phase-unwrapping and operate on the phase-angles only, or whether we obtain phase differences by complex multiplications. In [5] the power consumption in circuits for these two approaches was compared. Both do not perform well for low signal-to-noise ratios (SNR), however, because they exploit one correlation lag, only. A small improvement could be obtained by using greater delays (and thus sacrificing estimation range).

The schemes proposed in [6] and [7] do improve performance by using more correlation lags. The drawback is, that these approaches suffer from a significantly reduced estimation range.

The approach presented in [1] and [2] also exploits several correlation lags. However, it does not suffer from a reduced estimation range, because it evaluates the phase differences between succeeding sample correlation values, only. While the estimation range is very large in this case, most authors assume that common frequency offsets are about 10% of the symbol rate $r = \frac{1}{T}$ or less (where T is the symbol-time).

Hence, one possibility to improve performance is to simply restrict the estimation range of [1] and [2]. A reduced range would still be sufficient for most applications, while the performance is improved without increasing complexity.

The second approach is also based on the schemes from [1], [2]. In this case, we evaluate more than just one phase rotation between two correlation lags. That is, we consider several of them to further improve performance. The drawback of this approach is, that the computational complexity is increased.

II. System Model

We assume that we receive a single-frequency complex tone, that is disturbed by additive white Gaussian noise (AWGN). This means that we assume, that any modulation has been undone. This could be achieved either by using known symbols (e.g. a preamble) or by using an M-th power synchronizer for $2\pi/M$-rotationally symmetric signal constellations.

Sampling the signal at rate T yields $r(k)$

$$r(k) = \sqrt{E_s} e^{j(2\pi \Delta f k T + \Phi)} + n(k), \quad 0 \leq k \leq L-1. \tag{1}$$

Here E_s is the energy of the each symbol, Δf is the frequency offset, k is the time-index, Φ is the phase-offset, $n(k)$ is a noise-sample, and L is the length of the received sequence.

Note, that Δf and Φ are deterministic, unknown constants and the noise-samples are zero mean, complex, white Gaussian random variables with variance σ_n^2 in real and imaginary part.

For such a model the Cramer-Rao lower bound (CRB), which describes the smallest estimation variance that could be achieved by any estimator, can be derived [9]. It is given by:

$$\sigma_{CR}^2 = \frac{3}{2\pi^2 T^2} \frac{1}{\rho L (L^2 - 1)}, \tag{2}$$

where ρ indicates the SNR, which is defined as

$$\rho = \frac{E_s}{2\sigma_n^2}. \tag{3}$$

Several estimators do approach the CRB at moderate to high SNR. Usually, a threshold effect can be observed at lower SNR regions. That means, that the performance of estimators starts to degrade rapidly below a certain SNR. The focus of this work is, to present estimators that perform better in this low-SNR region.

A. Frequency offset estimation schemes using sample-wise phase-increments

We would like to compare our algorithm to schemes that rely on the sample autocorrelation function, which is given by

$$R(l) = \frac{1}{L-l} \sum_{k=l}^{L-1} r(k) r^*(k-l). \quad (4)$$

Another option would be to evaluate the angle of each multiplication result in (4) and take the average over the angles. These different approaches have been compared in [3], where we can see that the form in (4) might offer an advantage at lower signal-to-noise ratios (SNR), while the phase-averaging is preferred for high SNR-values.
The estimator in [7], which we call the L&R-estimator, is given by

$$\Delta\hat{f} = \frac{1}{\pi T(N+1)} arg\left\{\sum_{l=1}^{N} R(l)\right\}, \quad (5)$$

where N is the maximum delay considered. It can be seen as design parameter. Higher values do improve estimator performance at lower frequency offsets, but they reduce the estimation range. The problem is that only phase rotations within $\pm\pi$ can be used for estimates, since otherwise ambiguities would arise.
The Fitz-estimator is very similar, but performs phase unwrapping:

$$\Delta\hat{f} = \frac{1}{\pi TN(N+1)} \sum_{l=1}^{N} arg\left\{R(l)\right\}. \quad (6)$$

This estimator obviously suffers from a reduced estimation range, too.
[1] suggested a simple scheme that can handle multiple symbols, while still maintaining a large estimation range. His idea was to evaluate the phase offsets between autocorrelation-values with succeeding delays:

$$\varphi(l) = \left[arg\{R(l)\} - arg\{R(l-1)\}\right]_{-\pi}^{\pi}. \quad (7)$$

Here, we would also like to examine the effect of using

$$R_1(l) = R(l)R^*(l-1) \quad (8)$$

instead. While it is very similar to the previously described scheme, it does offer some advantages concerning the estimation range as we shall see later. Both schemes only evaluate the phase-offsets between autocorrelation values with a single lag. This ensures that the phase rotation is kept as small as possible, which explains the large estimation range.
Using $\varphi(l)$ we obtain the frequency offset estimate in flat fading channels according to [1] as

$$\Delta\hat{f} = \frac{1}{2\pi T} \sum_{l=1}^{N} \beta(l)\varphi(l). \quad (9)$$

Here, $\beta(l)$ are weight factors, that can be obtained from the covariance matrix of $\varphi = [\varphi(1), \varphi(2), \ldots, \varphi(N)]^T$. In the following, we will refer to this as the M&M-estimator.
[8] suggested to use a uniform weights in slow and moderate fading conditions. This leads to a small performance degradation, only, and it is not necessary to estimate any covariance matrix.
The other possibility would be to use (8), which gives us

$$\Delta\hat{f} = \frac{1}{2\pi T} arg\left\{\sum_{l=1}^{N} R_1(l)\right\}. \quad (10)$$

B. Improving the M&M-Estimator by Estimating the Offset in the Sample Correlation Function

There is a simple method to improve the estimates that are obtained by the previously described estimators. It is based on the fact, that we can view the sample autocorrelation function as a new sequence in which we can perform another frequency offset estimation. This new sequence is shorter than the original one, but the samples have a much higher SNR, with the phase rotations between the samples having the same mean. The original M&M-estimator can therefore be seen as a simple estimation scheme that has been applied to the sample correlation function. This point should be emphasized here. We principally still apply well known standard estimators, but we do not do it on the original sequence, but the sample correlation function instead.

1) Improving the M&M-Estimator by Limiting the Estimation Range:
It is well known, that for simple estimation schemes (like the estimators in [3] or [4]) can offer a better performance, if the estimation range is reduced. In this case, different correlation lags are chosen for computation, which means, that the estimation range is affected, but the complexity remains the same.
Here, we want to apply the same idea to the M&M-estimator. In its original form, only sample correlations with distance one are considered (see (7)). Instead, we could use a different distance D:

$$\varphi(l) = \left[arg\{R(l)\} - arg\{R(l-D)\}\right]_{-\pi}^{\pi} \quad (11)$$

or

$$R_1(l) = R(l)R^*(l-D). \quad (12)$$

$\varphi(l)$ and $R_1(l)$ can now simply be inserted into (13) and (14), where we have adjusted (9) and (10) to the new estimation range:

$$\Delta\hat{f} = \frac{1}{2\pi TD} \sum_{l=1}^{N} \beta(l)\varphi(l) \quad (13)$$

and

$$\Delta\hat{f} = \frac{1}{2\pi TD} arg\left\{\sum_{l=1}^{N} R_1(l)\right\}. \quad (14)$$

2) Improving the M&M-Estimator by Using Several Phase Offsets between Correlation Values:
A further improvement can be achieved, if several correlation lags are used like in [6] or [7]. The proposed estimator

would still suffer from a reduced estimation range, but would be expected to perform better than the previously described scheme. A drawback of this approach is that the computational complexity is considerably increased.

3) Improving the M&M-Estimator by Iterative Estimation:
Using the M&M-estimator again on the sample correlation values is another option, which has the advantage that the estimation range is not reduced. Further, we might try to do this in several iterations, since we always evaluate a sample correlation function, which can then be used for a further estimation. In every iteration, we would obtain a shorter sequence than before, but the SNR of the samples keeps increasing.
Like in the previously described scheme, here we also have to consider that there is a significant increase in computational complexity.

III. Simulation Results

In this section we want to compare the simulation results in AWGN environments.

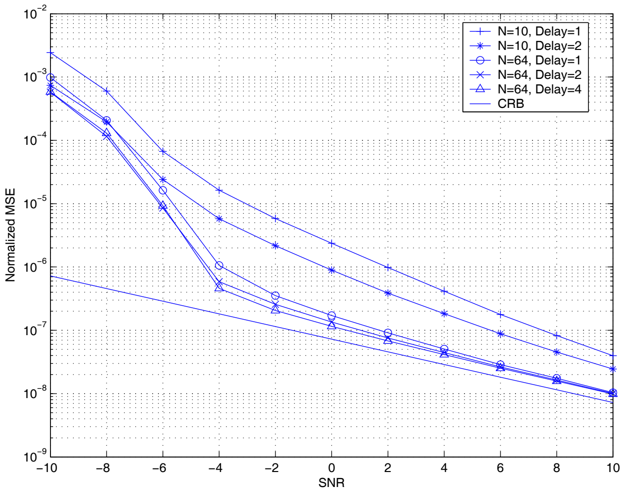

Fig. 1. Normalized MSE, Sequence Length $L = 128$, $\Delta fT = 0.05$. Comparison of the effect of different delays in the M&M-estimator.

Figure 1 shows the impact of using higher delays in the M&M-estimator. We chose a frequency-offset $\Delta fT = 0.05$ and we vary the number of autocorrelation samples N that are used in the M&M-estimator. We can see that for smaller numbers of N the improvement by using sample correlation values with larger delays D is greater. For $N = 10$ we gain about 2 dB in most SNR-regions. For larger numbers of N (here we chose $N = \frac{L}{2} = 64$) there is still some improvement in the lower SNR-range, whereas in the higher SNR-region the performance remains almost the same. The threshold effect can still be observed at lower SNR-values, however. We can conclude that in systems, where the maximum frequency offsets are limited, it would be a good idea to use greater lags D for the phase difference evaluation, since we do not encounter any increase in computational complexity, while getting an improvement – especially for small values of N.

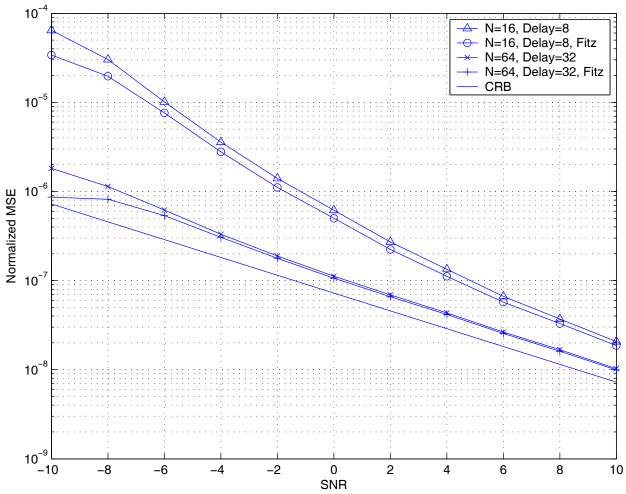

Fig. 2. Normalized MSE, Sequence Length $L = 128$, $\Delta f = 0$. Effect of using several correlations lags in the second stage of the M&M-estimator (in contrast to just using different delays).

The next step is to exploit the results of multiple correlations. In the previous example, we saw that with small delays we could still obtain a reasonable estimation range. To demonstrate the effect of using several correlation lags we set the frequency offset to $\Delta f = 0$. This way, we can purely examine the effect of the two schemes without any influence from frequency offsets. Otherwise, large delays would not be possible and the differences between different schemes would not be that apparent. Again, we show the results for two different cases. In the first we chose $N = 10$ samples, while in the second we use the maximum number of samples $N = \frac{L}{2}$. The second parameter in this case is, the delay D up to which the phase differences between the previously obtained autocorrelation samples are exploited. If we simply use the Fitz-Estimator, then the maximum delay is still the parameter which limits the estimation range. As we can see from the results in Figure 2, the improvement compared to estimators, that simply operate on one specific delay, is quite small. The problem is that all the correlation lags greater than the maximum delay are still not exploited here. The estimation range is slightly increased by the multiple correlation-lag approach, since here phase offsets with lower lags, which have a greater estimation range, are used as well.
The next step is to use a M&M-estimator in the second iteration as well. The advantage is that the estimation range does not depend on the maximum delay that is used. This time the frequency offset is $\Delta fT = 0.05$, again. In Figure 3 we see the results in two scenarios. In the first we chose $N = 16$ and the delay $D = 4$ in order to maintain a reasonable estimation range. In the estimator, that uses the M&M scheme in the second stage as well, a maximum delay $D = 8$ was chosen. Here, we can see that simply choosing a greater delay seems to offer the better results. In the second case, we have $N = 64$ and the delay is kept

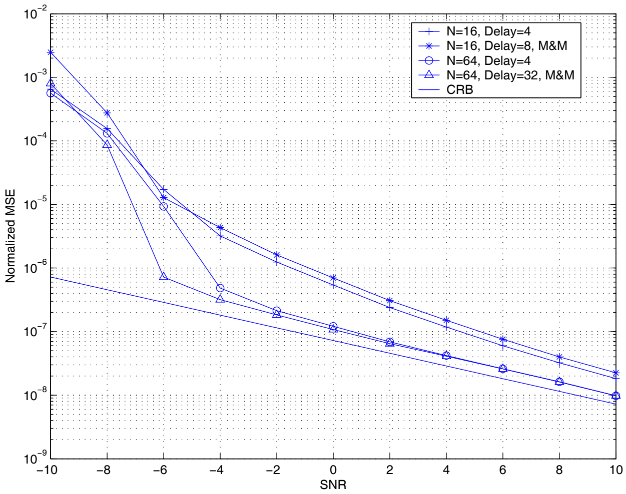

Fig. 3. Normalized MSE, Sequence Length $L = 128$, $\Delta fT = 0.05$. The use of a M&M-estimator in two stages.

at $D = 4$ (again, to ensure a reasonable estimation range). For the second stage in M&M-estimation, $D = 32$ was chosen. Here, we can see a small improvement in the very low-SNR region. It is not very big, though. But we still have the advantage that the estimation range is not reduced by applying the M&M-scheme twice.

One problem has not been discussed so far. That is the evaluation of phases according to the original M&M-scheme. If we have phase-values close to π, it can easily happen that phase-ambiguities occur (i.e. the result might be either close to $+\pi$ or close to $-\pi$). These phase ambiguities cause problems, because values close to $\pm\pi$ might cancel out when taking the sum. While it is not a real problem, when applying the traditional M&M-estimator, it does become a problem when greater correlation lags are exploited in the two-stage M&M-estimator. The solution could be to use (10). Since here we are dealing with complex numbers, phase-ambiguities may still occur after the summation, but the summation-process is not disturbed. In any case it is just important to get values close to either $+\pi$ or $-\pi$ (while the original M&M-scheme could produce 0). The ambiguity does not influence the result, because this phase is compared to other phases and thus just a relative measure is evaluated at this point (that is why values around 0 do cause problems).

In Figure 4 we compare estimation according to (9) and (10) for $\Delta fT = 0.05$ and M&M-estimators with $N = 64$. We can see that the differences are not huge, although the phase-averaging according to (7) does perform better at low SNR, while the linear estimator according to (8) is slightly better at high SNR-values. Overall, the differences are not huge, however, and it depends on the context which approach is better suited.

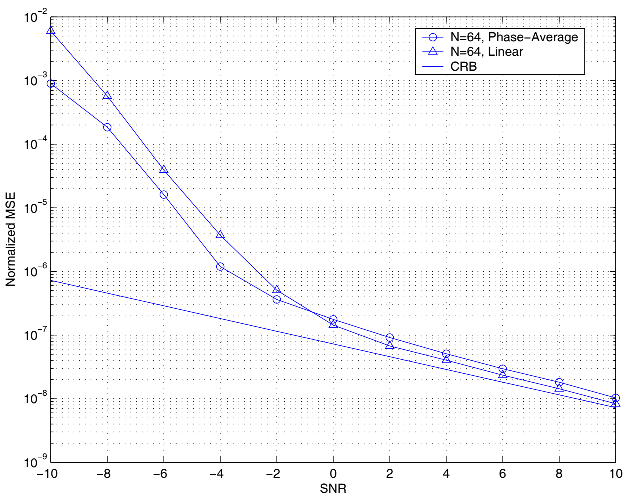

Fig. 4. Normalized MSE, Sequence Length $L = 128$, $\Delta fT = 0.05$. Comparison of different ways to evaluate phase-offsets.

IV. Conclusion

We have proposed a few methods to improve the performance of a well-known frequency offset estimation algorithm [1]. The advantage of [1] over other commonly used estimators is the large estimation range. This is much more than what is required in many systems. Therefore, some of the presented schemes aim at improving the estimation accuracy and sacrificing range (without any additional computational complexity). Another option is to treat the sample autocorrelation, which is used for estimation, as a new sequence with a certain frequency offset. That means that any of the well known estimation algorithms could be applied to it.

From the results we can see, that an improved performance can be achieved – especially in the low-SNR region.

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