GAUSSIAN CHANNEL MODEL FOR MACROCELLULAR MOBILE PROPAGATION

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ABSTRACT

A model of an angle-spread source is described, termed the 'Gaussian Channel Model' (GCM). This model is used to represent signals transmitted between a User Equipment and a cellular Base Station. The model assumes a Gaussian law of scatterer occurrence probability, depending upon the scatterer distance from the user. Analysis is presented to demonstrate that this model produces a better fit to measured data than some other widely-used scattering models in the literature.

1. INTRODUCTION

The implementation of smart antennas at macrocellular base stations is expected significantly to enhance the capacity of wireless networks [1][2]. Various algorithms for adaptive array signal processing have been proposed and investigated [2][3][4]. The effectiveness of these algorithms depends on the behaviour of the fading channel and in particular on the degree of azimuthal dispersion in the channel. Therefore, accurate statistical channel models are required for the testing of these adaptive algorithms. These models must be realistic and close to real-life channels, in order to replicate the Angle-of-Arrival (AoA) distribution of the multi-path components.

The propagation channel between the base station (BS) and the user equipment (UE) is generally held to be reciprocal in most respects. However the azimuthal angle dispersions seen at the BS and UE antenna differ significantly from each other. The classical Clarke channel model [5] assumes a uniform probability density function (PDF) of the incoming rays at the UE antenna. However, if the BS antenna array is elevated above the surrounding scatterers, then the rays incoming to the BS are concentrated in some smaller range of azimuth angles than those incoming to the UE. Note also that Clarke's model provides the well-known 'rabbit-ear' form of the classical Doppler spectrum, which often is used to characterise the fading of the signals seen both at the BS and at the UE. Some statistical propagation models which include the azimuthal dispersion at the BS have been developed in [6][7][8]. For example, the channel model proposed in [7] is geometrically based, and assumes that scatterers are uniformly distributed within the area of a circle centred at the UE antenna. This means that the AoA of the multipath components at the BS will be restricted to an angular region dependent both upon the circle radius and upon the distance between BS and user. However, in a real-life channel, the scatterer distribution around the UE can differ significantly from uniform. Therefore other researchers [9][10][11] have proposed other more realistic models based on a Gaussian distribution of scatterer location.

The goal of this paper is to analyse further the Gaussian proposal for the scatterer distribution. We assume that the scatterers can be situated in any point in the horizontal plane. In this model, the probability of occurrence of the scatterer location decreases in accordance with a Gaussian law when its distance from the UE antenna increases. Therefore we call this model the 'Gaussian Channel Model' (GCM). We provide here no strict physical justification for this model. However, we note that the Gaussian model intuitively seems more appropriate than the above models, because it places no strict upper bound on scatterer distance from the UE. We believe that such an assumption about the scatterer location is closer to the real-life environment than some of the other models mentioned above. Therefore, as we will demonstrate later, the comparison of the obtained PDF of AoA of the multipath for the GCM with the measured results presented in [8] gives very good agreement.

This provides good justification for adopting this model. Note also that, like Clarke's model, the proposed GCM also provides the classical Doppler signal spectrum.

2. GAUSSIAN CHANNEL MODEL AND THE PDF OF THE AOAS OF MULTIPATH COMPONENTS SEEN AT THE BASE STATION

The signal received by the BS is a sum of many signals, reflected from different scatterers randomly situated around the UE antenna. The AoAs of the multipath signal components are thus various and random. Therefore the set of the scatterers can be considered collectively as a spread source, and the angle spread is a measure used to determine the angular dispersion of the channel.

Here we present the details of the GCM, and derive an analytical expression for the PDF of the AoAs of multipath components as observed at the BS.

First of all we list the initial assumptions used for creating the channel model. We assume that:

- The scattered signals arrive at the BS in the horizontal plane - i.e. the proposed GCM is two-dimensional and the elevation angle is not taken into account.
- Each scatterer is an omni-directional reradiating element, and the plane wave is reflected directly to the BS without influence from other scatterers (i.e. we have only 'single-bounce' scattering paths).
- The direct path from the UE to BS antenna is infinitely attenuated.
- The reflection coefficient from each scatterer has unity amplitude and random phase.
- The probability of the (random) scatterer location is independent of azimuth angle (from the UE), and decreases if its distance from the UE antenna increases. This dependence has a Gaussian form.

The last of these assumptions distinguishes our channel model from many of the other known models [**5**][**6**][**7**].

Thus we can write that

$$p(r,\varphi) = \frac{1}{\pi r_{eff}^2} \exp\left(-\frac{r^2}{r_{eff}^2} \right) \qquad (1)$$

where (r,φ) is the polar coordinate system centred at the UE, r is the distance to a given scatterer from the UE antenna, and r_{eff} is the radius at which the PDF decreases by a factor of e, i.e. $p(r_{eff},\varphi) = e^{-1} p(0,\varphi)$. Figure 1 illustrates the GCM, where D is the distance between the BS and UE antennas, and (x,y) are the rectangular coordinates.

In [**7**], a uniform scatterer distribution within the circle of radius r_0 around the UE was assumed. So for the model of [**7**] this means that the AoAs of multipath components seen at the BS are limited to the angular region $[-\theta_{max}...\theta_{max}]$, where $\theta_{\max} = \sin^{-1}(r_0/D)$. However, for our GCM model the AoAs of scattered signals as received at the BS are *not* restricted to any constrained angular region.

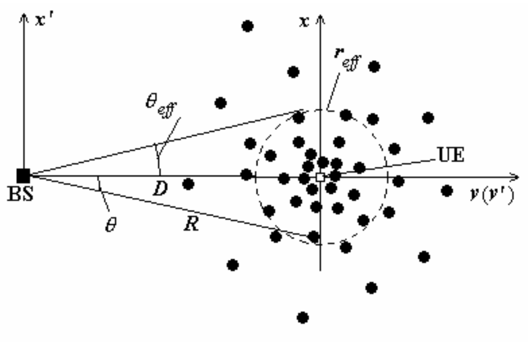

Figure 1 Illustration of the Gaussian channel model

In order to derive the ensemble PDF of the AoA for the GCM (i.e. averaged over many model realisations) we choose the origin of the system coordinates (x',y') to be the location of the BS. This means that $x'=x$ and $y'=y+D$. We then transform to the polar coordinates (R,θ), where $x'=R\sin\theta$, $y'=R\cos\theta$, and the angle θ is measured relative to the line joining the BS and UE antennas. It can be shown that the Jacobian of this transformation is equal to R. Furthermore, we have

$$\begin{aligned} r^2 &= x^2 + y^2 = x'^2 + (y'-D)^2 = \\ &R^2 - 2RD\cos\theta + D^2 \end{aligned} . \qquad (2)$$

As a result of substituting (2) into (1) we obtain that

$$\begin{aligned} &p(R,\theta) = \\ &\frac{R}{\pi r_{eff}^2} \cdot \exp\left(-\frac{D^2}{r_{eff}^2} \right) \cdot \exp\left(-\frac{R^2 - 2RD\cos\theta}{r_{eff}^2} \right) \cdot \end{aligned} \qquad (3)$$

In order to derive the one-dimensional PDF of the AoA (i.e. the Power Angle Density) of the multipath components as seen at the BS, an integration over the radius R must be carried out. Therefore the PDF is expressed as the following integral

$$\begin{aligned} &p(\theta) = \int_0^\infty p(R,\theta)dR = \\ &\frac{1}{\pi r_{eff}^2} \cdot \exp\left(-\frac{D^2}{r_{eff}^2} \right) \int_0^\infty \exp\left(-\frac{R^2 - 2RD\cos\theta}{r_{eff}^2} \right) R dR \end{aligned} \qquad (4)$$

This integral can be calculated analytically and a closed form solution obtained. To do this, take into account that ([12], № 3.462.1)

$$\begin{aligned} &\int_0^\infty x^{\nu-1} \exp\left(-\beta x^2 - \gamma x \right) dx = \\ &(2\beta)^{-\nu/2} \Gamma(\nu) \exp\left(\frac{\gamma^2}{8\beta} \right) C_{-\nu}\left(\frac{\gamma}{\sqrt{2\beta}} \right) \end{aligned} \qquad (5)$$

where $\mathrm{Re}(\nu,\beta)>0$, $\Gamma(\nu)$ is the gamma function, $C_p(z)$ is the function of the parabolic cylinder. In our case we have $\nu=2$,

$\beta = r_{eff}^{-2}$ and $\gamma = -2Dr_{eff}^{-2}\cos\theta$. If v=2 then the function $C_{-2}(z)$ can be expressed in terms of the probability integral $\Phi(z)$ ([12], № 9.254.2[1]), i.e.

$$C_{-2}(z) = -\exp\left(\frac{z^2}{4}\right)\sqrt{\frac{\pi}{2}}\left\{z\left[1-\Phi\left(\frac{z}{\sqrt{2}}\right)\right]-\sqrt{\frac{2}{\pi}}\exp\left(-\frac{z^2}{2}\right)\right\} \quad (6)$$

where the probability integral $\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)dt$.

Take into account that $z = -\sqrt{2}Dr_{eff}^{-1}\cos\theta$, $\Gamma(2)$=1 and $\Phi(z)$ is an odd function of its argument z. As a result of straightforward transformations we can obtain from (5) and (6) that the desired one-dimensional PDF $p(\theta)$ of AoA of the multipath components is given by

$$p(\theta) = \frac{1}{2\pi}\cdot\exp\left(-\frac{D^2}{r_{eff}^2}\right) \times\left\{1+\sqrt{\pi}\frac{D}{r_{eff}}\cos\theta\cdot\exp\left(\frac{D^2}{r_{eff}^2}\cos^2\theta\right)\left[1+\Phi\left(\frac{D}{r_{eff}}\cos\theta\right)\right]\right\} \quad (7)$$

It is convenient to introduce the angle $\theta_{eff} = \sin^{-1}(r_{eff}/D)$. Then (7) can be rewritten as

$$p(\theta) = \frac{1}{2\pi}\cdot\exp\left(-\frac{1}{\sin^2\theta_{eff}}\right) \times\left\{1+\sqrt{\pi}\frac{\cos\theta}{\sin\theta_{eff}}\cdot\exp\left(\frac{\cos^2\theta}{\sin^2\theta_{eff}}\right)\cdot\left[1+\Phi\left(\frac{\cos\theta}{\sin\theta_{eff}}\right)\right]\right\} \quad (8)$$

Thus the PDF $p(\theta)$ depends only upon $\cos\theta$. The effective angle spread for this PDF can be introduced as $\Delta=2\theta_{eff}$. The PDF $p(\theta)$ is an even function of its argument θ.

The expression (8) is true in the general case. However this formula takes a very simple form for the case of small angle spread $\theta_{eff}<<\pi$ when $\sin\theta\approx\theta$. In this case the PDF is approximately given by

$$p(\theta) \approx \frac{1}{\sqrt{\pi\theta_{eff}^2}}\cdot\exp\left(-\frac{\theta^2}{\theta_{eff}^2}\right) \quad (9)$$

and described by a (1-dimensional) Gaussian PDF with zero mean and variance $\sigma^2 = 0.5\theta_{eff}^2$.

Figure 2 shows the PDF $p(\theta)$ of the AoA of the multipath components for the different values θ_{eff}=10°; 30° and 50°. The solid and dashed curves correspond to the exact formula (8) and to its Gaussian approximation (9), respectively. We can see that the exact and Gaussian PDFs are very close to each other for a large interval of θ_{eff} up to $\theta_{eff}\leq 0.5$ (or $\theta_{eff}\leq 30°$). Actually, it is quite simple and intuitive to see how the complex PDF of the exact formula (8) should equal a 1-dimensional PDF for small angle spreads. At these small angles, the lines bounding different small 'slices' of the 2-dimensional PDF are nearly parallel, and so it is as if we are calculating the marginal PDF of the 2-dimensional spatial PDF along the x axis. Since the marginal PDF of a 2-dimensional Gaussian distribution is a 1-dimensional Gaussian distribution, our approximate result (9) is intuitively of the correct form.

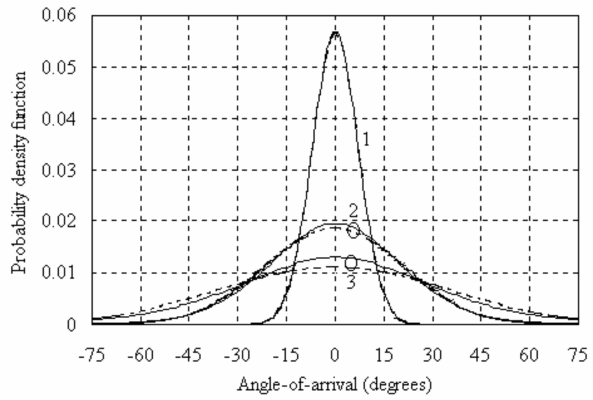

Figure 2 PDF of the AoA of the multipaths at the BS. The angle spread Δ (where Δ=2θ_{eff}) is equal to 20, 60 and 100 degrees (curves 1,2,3 respectively). The solid and dashed curves correspond to the exact formula (8) and its Gaussian approximation (9) respectively.

The comparison of the theoretical PDF against real measurement data is of course of interest, in order both to validate and to parameterise the GCM. Histograms of the estimated azimuthal Power Angle Density and scatterer occurrence probabilities are presented by the authors of [**8**]. This measurement data was obtained in Aarhus with a BS antenna located 12m above the rooftop level. We wish to take this measured data and compare it to the three proposed theoretical channel models: 1) our GCM of equation (8), 2) the Geometrical Based Single Bounce Model (GBSBM) developed in [**7**] (in which the scatterers are assumed to be uniformly randomly distributed within the area of a circle), and 3) Clarke's model [**5**], [13] (in which the scatterers are assumed to lie on the circumference of a circle).

It was derived in [**7**] that the PDF of the AOA of the multipath components for GBSBM is given by

$$p(\theta) = \begin{cases} \dfrac{2\cos(\theta)\sqrt{\sin^2\theta_{\max}-\sin^2\theta}}{\pi\sin^2\theta_{\max}}, & -\theta_{\max}\leq\theta\leq\theta_{\max} \\ 0, & otherwise \end{cases} \quad (10)$$

[1] N.B. There is a minor typographical error (a missing factor of -1) in the version of this equation printed in [12], which is corrected within the addenda of the original Russian version

where $\theta_{\max} = \sin^{-1}(r_0/D)$ and r_0 is the radius of the circle within which all the scatterers are uniformly distributed.

Whilst we omit the derivation here, for reasons of brevity, it can be shown that the PDF of the AOA of the multipath components for Clarke's model is equal to

$$p(\theta) = \frac{1}{\pi}\frac{1}{\cos^2\theta}\frac{1}{\sqrt{\tan^2\theta_{\max} - \tan^2\theta}} \qquad (11)$$

where in this case, when calculating θ_{max}, r_0 has the meaning of the radius of the circle periphery on which the scatterers are uniformly distributed.

Figure 3 shows the PDFs for the AoA of the multipath components at the BS for GCM, GBSBM and Clarke's models, and the measured scatterer occurrence probability histograms taken from [**8**]. We have chosen the model parameters (θ_{max}, θ_{eff}) so that the best agreement was obtained for each model. For both the GBSBM and Clarke models the value chosen was θ_{max}=10°, and for GCM θ_{eff}=8.8°. It can be seen that the GCM ensures the best agreement with real-life results for the whole angular region and especially for the tails of the histogram. Clarke's model produces the worst match to the real-life data.

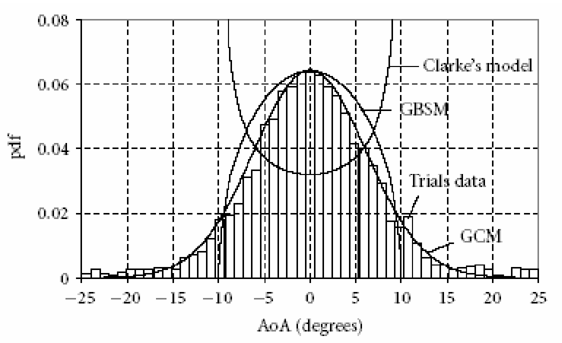

Figure 3 The PDFs for the AoA of the multipath components at the BS for GCM, GBSBM and Clarke's models and for the measured histograms.

3. CONCLUSIONS

In this paper we have developed a model for an angle spread source which we term the Gaussian Channel Model (GCM). This model is suitable for representing the signal seen at the BS antenna, and assumes that the probability of the scatterer occurrence decreases in accordance with a Gaussian law when its distance from the UE antenna increases. Such an assumption about the scatterer location is closer to the real-life environment than some of the other known models. An analytical expression for the PDF of the multipath Angle-of-Arrival at the BS has been derived for the general case of an arbitrary angle spread. It is shown that this PDF can be approximated by a Gaussian curve for sources with a small spread. The comparison of the obtained PDF of AoA of the multipath for the GCM with the published experimental results gives a better agreement than for some other known angle scattering models. In a related paper [14] the authors demonstrate the application of this GCM model to the performance analysis of a bearing estimation algorithm, and demonstrate the importance in this application of any assumptions made about the mean number of scatterers within the model.

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