# DESIGN OF CHANNEL-RESILIENT DFT BANK TRANSCEIVERS

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## ABSTRACT

In this paper, we study DFT bank transceivers with filter length longer than the block size. Given a fixed transmit (or receive) prototype filter, it has been shown recently that we are able to design the receive (or correspondingly transmit) prototype filter so that the signal to interference ratio (SIR) is maximized for multipath fading channels. However only the transmit (or receive) filters are guaranteed to have good stopband attenuation. In this paper, we propose an iterative method that allow us to design channel-resilient DFT bank transceivers where both the transmit and receive filters can have good stopband attenuation. Moreover the proposed iterative procedure can increase the SIR significantly.

## 1. INTRODUCTION

In recent years, there has been considerable interest in OFDM (orthogonal frequency division multiplexing) and DMT (discrete multitone) techniques due to their low complexity and their effectiveness in combating intersymbol interference. However the transmit filters and receive filters of these systems are DFT filters. The stopband attenuation of DFT filters is only 13dB and it decays at a rate of $1/f$ only. In many applications, it is often desirable to have filters with better stopband attenuation so that out-of-band spectrum is reduced at the transmitter and narrowband interference is rejected at the receiver.

Many techniques have been proposed for the design of transceivers with good transmit and receive filters. In particular, the filter bank (FB) approach [1] [2] has drawn many attentions recently. Perfect reconstruction FBs have been proposed as the transceiver in [1]. When the channel is a frequency selective channel, the system suffers from severe ISI and costly post processing technique is needed at the receiver. In [2], channel-resilient FB transceivers were first introduced. The proposed AMOUR (a mutually orthogonal usercode receiver) system achieves ISI-free transmission for multipath fading channels. However it is not easy to design AMOUR transceiver with good frequency response due to its short filter length ($\leq$ the block size) and limited number of free parameters. Recently the authors in [3] proposed a technique for the design of channel-resilient DFT bank transceivers. Given a fixed transmitter (or receiver), the receiver (correspondingly transmitter) is optimized so that SIR is maximized for multipath fading. Transceivers with good transmit (or receive) filters are obtained and these transceivers enjoy a moderate SIR. In [4], the authors study channel-resilient DFT bank transceivers with matching transmit and receive filters. It was shown that transceivers with high bit rate can be obtained. However, it is not easy to incorporate the frequency constraint into the design.

Based on the formulation in [3], we introduce an iterative procedure to design FB transceivers with good stopband attenuation. The proposed iterative method can significantly increase the SIR without degrading the stopband attenuation of the filters. Moreover FB transceivers with good transmit and receive filters can be designed using the proposed techniques. Simulation results show that for multipath fading channels, we are able to get FB transceivers with good stopband attenuation and high SIR.

## 2. CHANNEL-RESILIENT FB TRANSCEIVERS

Fig. 1 shows a FB transceiver, where $F_k(z)$ and $H_k(z)$ are respectively the transmit and receive filters. One block of transmit data contains $M$ modulated symbols $x_k(n)$. In this paper, we make the mild assumption that the symbols $x_k(n)$ are zero mean and satisfy:

$$E\{x_i(n)x_j^*(m)\} = \mathscr{E}_x \quad (i-j) \quad (n-m). \qquad (1)$$

The interpolation ratio is $N$. We assume that $N \geq M$. So $(N-M)$ is the number of of redundant samples in one block of data. The transmission channel is assumed to be slowly varying so that it can be modeled as an LTI channel $C(z) = {}_{n=0}^{L_c} c(n)z^{-n}$. The channel taps $c(n)$ are assumed to be complex process satisfying

$$E\{c(l)\} = 0, \quad E\{c(l)c^*(l-k)\} = {}^{2}_{l} \quad (k), \qquad (2)$$

for $0 \leq l \leq L_c$. Such channels are called multipath fading channels. We say that the FB transceiver is channel-resilient if it enjoys a high SIR for multipath fading channels. Note that the exact impulse response $c(n)$ is not known to both the transmitter and receiver.

To reduce cost, we consider only DFT modulated filters[1]:

$$F_k(z) = F_0(zW^k), \quad H_k(z) = H_0(zW^k), \quad \text{where } W = e^{-j\frac{2}{M}},$$

for $1 \leq k \leq M-1$. The transmit and receive prototype filters are FIR filters:

$$F_0(z) = \sum_{n=0}^{n_f} f_0(n)z^{-n}, \quad H_0(z) = \sum_{n=0}^{n_h} h_0(n)z^{n}.$$

The filter orders $n_f$ and $n_h$ can be larger than $N$. Using the efficient polyphase structure, the cost of implementing the transmitter (or receiver) is equal to the cost of one IDFT plus one prototype filter. For convenience, we use non causal receive filters and this non causality can be removed by cascading enough delays.

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[1] It is not difficult to modify the formulation for the more general case of non DFT filters.

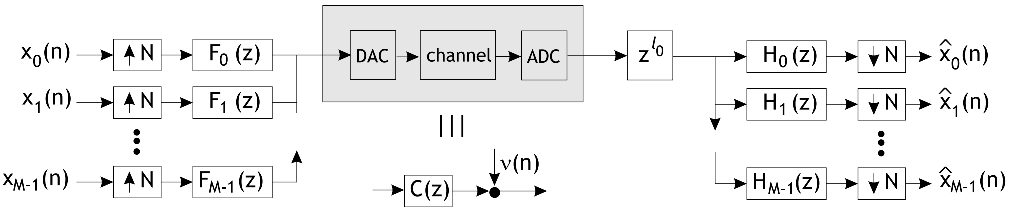


Figure 1: A filter bank transceiver.

Consider the system from $x_i(n)$ to $\hat{x}_j(n)$ in Fig. 1. Using multirate identities, it can be shown that this system is LTI with transfer function:

$$\begin{aligned} T_{ji}(z) &= [F_i(z)C(z)z^{l_0}H_j(z)]_{\downarrow N} \\ &= {}_{l=0}^{L} c(l)[F_i(z)H_j(z)z^{l_0-l}]_{\downarrow N}, \end{aligned}$$

where the notation $[\bullet]_{\downarrow N}$ denotes $N$-fold downsampling. Define the sequences $_{i,l}(n)$ and $_{i,j,l}(n)$ as:

$$\Big[F_i(z)H_j(z)z^{l_0-l}\Big]_{\downarrow N} = \begin{cases} {}_{i,l}(0) + {}_{n,n\neq 0}\; {}_{i,l}(n)z^{-n} & i = j, \\ {}_{n}\; {}_{i,j,l}(n)z^{-n} & i \neq j, \end{cases}$$

for $0 \leq i, j \leq M-1$ and $0 \leq l \leq L$. Then we can write

$$\begin{aligned} \hat{x}_j(n) &= \Big[{}_{l=0}^{L}\; {}_{j,l}(0)c(l)\Big]x_j(n) + {}_{l=0}^{L} c(l)\Big[\; {}_{j,l}(n) - \\ &\quad {}_{j,l}(0)\;\;(n)\Big] * x_j(n) + {}_{i=0}^{M-1}\,{}_{l=0}^{L} c(l)\; {}_{i,j,l}(n) * x_i(n), \end{aligned}$$

where $*$ denotes convolution. For notational simplicity, we have used the definition $_{i,i,l}(n) = 0$, for all $i, l, n$ in the above expression. The three summation terms on the right hand side are respectively the desired signal, the intra-band and cross-band intersymbol interferences. To compute the average signal and interference powers, we can take the statistical expectation with respect to the input symbols $x_k(n)$ and the channel taps $c(n)$. Using (1) and (2) to simplify the results, we can express the signal and interference powers at the output of the $j$th band as

$$P_{sig}(j) = \mathscr{E}_x {}_{l=0}^{L} |\; {}_{j,l}(0)|^2 \;{}_l^2$$

$$P_{isi}(j) = \mathscr{E}_x\Big( {}_{n,n\neq 0}\,{}_{l=0}^{L} |\; {}_{j,l}(n)|^2 \;{}_l^2 + {}_{i,n}\,{}_{l=0}^{L} |\; {}_{i,j,l}(n)|^2 \;{}_l^2\Big).$$

It was shown in [3] that for DFT filters, we have the relation $|\; {}_{j,l}(n)| = |\; {}_{0,l}(n)|$ and $|\; {}_{i,j,l}(n)| = \; {}_{0,((j-i)),l}(n)|$ where $((j-i))$ denotes $j-i \bmod M$. Substituting this result into the above expression, we find that all the outputs $x_j(n)$ have the same signal power equal to $P_{sig}(0)$ and the same interference power equal to $P_{isi}(0)$. Thus the SIR expression is given by

$$SIR = P_{sig}(0)/P_{isi}(0). \tag{3}$$

It was shown in [3] that given a fixed receive prototype (or transmit prototype), the SIR expression can be expressed as a Rayleigh-Ritz ratio. The formulation is given below.

**Matrix Formulation:** Note that $_{0,l}(n)$ and $_{0,j,l}(n)$ are functions of the prototype filters $h_0(n)$ and $f_0(n)$. Using the definitions of and , one can write

$$\begin{bmatrix} {}_{0,0}(n) \\ {}_{0,1}(n) \\ \vdots \\ {}_{0,L}(n) \end{bmatrix} = \mathbf{A}_h(n) \begin{bmatrix} f_0(0) \\ f_0(1) \\ \vdots \\ f_0(n_f) \end{bmatrix} = \mathbf{A}_f(n) \begin{bmatrix} h_0(0) \\ h_0(1) \\ \vdots \\ h_0(n_h) \end{bmatrix}$$

$$\begin{bmatrix} {}_{0,j,0}(n) \\ {}_{0,j,1}(n) \\ \vdots \\ {}_{0,j,L}(n) \end{bmatrix} = \mathbf{B}_{h,j}(n) \begin{bmatrix} f_0(0) \\ f_0(1) \\ \vdots \\ f_0(n_f) \end{bmatrix} = \mathbf{B}_{f,j}(n) \begin{bmatrix} h_0(0) \\ h_0(1) \\ \vdots \\ h_0(n_h) \end{bmatrix},$$

The matrices $\mathbf{A}_h(n)$ and $\mathbf{B}_{h,j}(n)$ are $(L+1)$ by $(n_f+1)$ and their entries depends only on $h_0(n)$, whereas $\mathbf{A}_f(n)$ and $\mathbf{B}_{f,j}(n)$ are $(L+1)$ by $(n_h+1)$ and their entries depends only on $f_0(n)$. Define the vectors

$$\mathbf{f}_0 \triangleq \begin{bmatrix} f_0(0) \\ f_0(1) \\ \vdots \\ f_0(n_f) \end{bmatrix}, \quad \mathbf{h}_0 \triangleq \begin{bmatrix} h_0(0) \\ h_0(1) \\ \vdots \\ h_0(n_h) \end{bmatrix}.$$

Then it was shown [3] that the SIR can be formulated as two different Rayleigh-Ritz ratios:

$$SIR = \frac{\mathbf{f}_0^\dagger \mathbf{Q}_0 \mathbf{f}_0}{\mathbf{f}_0^\dagger \mathbf{Q}_1 \mathbf{f}_0} = \frac{\mathbf{h}_0^\dagger \mathbf{Q}_2 \mathbf{h}_0}{\mathbf{h}_0^\dagger \mathbf{Q}_3 \mathbf{h}_0}, \tag{4}$$

where $^\dagger$ denotes transpose conjugate. The matrices $\mathbf{Q}_0$ and $\mathbf{Q}_1$ are some positive matrices dependent on $h_0(n)$ and ${}_l^2$, and $\mathbf{Q}_2$ and $\mathbf{Q}_3$ are some positive matrices dependent on $f_0(n)$ and ${}_l^2$. One can use the above expressions to design FB transceivers with good transmit (or receive) filters [3]. For example, if receive filters with a large stopband attenuation are needed, one can first design a good lowpass filter $h_0(n)$, then $f_0(n)$ is chosen so that SIR is maximized.

Using (4), one can also design an iterative procedure to optimize the SIR. Starting with a given $h_0(n)$, one can find $f_0(n)$ so that the SIR is maximized. The solution is obtained by solving the first Rayleigh-Ritz ratio in (4). For this optimal $f_0(n)$, we can design $h_0(n)$ to maximize the SIR. The solution is achieved by solving the second Rayleigh-Ritz ratio in (4). Repeating the above process, one can improve the SIR as the iteration number increases [5]. It is not difficult to see that SIR cannot decrease with the iteration. Though we can get transceivers with very high SIR, the resulting transmit and receive filters will have poor stopband attenuation[5]. In the following, we will propose an iterative procedure that can increase the SIR without sacrificing the filter stopband attenuation.

## 3. FB TRANSCEIVERS WITH FREQUENCY CONSTRAINTS

In this section, our goal is to find the transmit and receive prototype filters $f_0(n)$ and $h_0(n)$ so that (i) $F_0(z)$ and $H_0(z)$ have a large stopband attenuation and (ii) the SIR given in (3) is large. We must emphasize that in transceiver design, the filters do not need to have a good passband. This is because the perfect reconstruction property is ensured by the SIR condition. On the other hand, high stopband attenuation is needed at transmitter for limiting out-of-band spectrum and at receiver for reducing narrowband interference. In order to preserve the frequency responses of the filters, we split the prototype filters into two parts:

$$F_0(z) = F_a(z)F_b(z), \quad H_0(z) = H_a(z)H_b(z), \tag{5}$$

where the orders of $F_a$, $F_b$, $H_a$ and $H_b$ are respectively $n_{fa}$, $n_{fb}$, $n_{ha}$ and $n_{hb}$. Thus we have the relations $n_f = n_{fa} + n_{fb}$ and $n_h = n_{ha} + n_{hb}$. In the design of FB transceiver, the filters $F_a(z)$ and $H_a(z)$ are designed as good lowpass filters whereas $F_b(z)$ and $H_b(z)$ are optimized so that SIR is maximized. In the case when only transmit (or receive) filters with good stopband are needed, one can set $n_{ha} = 0$ (correspondingly $n_{fa} = 0$). That means all the coefficients $h_0(n)$ (correspondingly $f_0(n)$) are employed for SIR optimization. Since $F_0$ is a cascade of $F_a$ and $F_b$, the stopband attenuation of $F_0$ is the product of those of $F_a$ and $F_b$. If $F_a$ has a large stopband attenuation and $\| F_b \| = 1$ (constraint applied in the optimization), then $F_0$ will have a large stopband attenuation. The same is true for the receive prototype filter $H_0(z)$. Define the vectors

$$\mathbf{f}_b \triangleq \begin{bmatrix} f_b(0) \\ f_b(1) \\ \vdots \\ f_b(n_{fb}) \end{bmatrix}, \quad \mathbf{h}_b \triangleq \begin{bmatrix} h_b(0) \\ h_b(1) \\ \vdots \\ h_b(n_{hb}) \end{bmatrix}.$$

Then we can write the expression (5) in time domain as

$$\mathbf{f}_0 = \mathbf{F}_a\mathbf{f}_b, \ \ \mathbf{h}_0 = \mathbf{H}_a\mathbf{h}_b, \tag{6}$$

where $\mathbf{F}_a$ and $\mathbf{H}_a$ are the convolution matrices with dimensions $(n_f + 1) \times (n_{fb} + 1)$ and $(n_h + 1) \times (n_{hb} + 1)$ respectively. These convolution matrices are lower triangular Toeplitz matrices with their first columns given by $[f_a(0) \ \ f_a(1) \ \ \dots \ \ f_a(n_{fa}) \ \ 0 \ \ \dots \ \ 0]^T$ and $[h_a(0) \ h_a(1) \ \dots \ h_a(n_{ha}) \ 0 \ \dots \ 0]^T$ respectively. Using (6), we can rewrite the SIR expression in (4) as:

$$SIR = \frac{\mathbf{f}_b^\dagger \mathbf{F}_a^\dagger \mathbf{Q}_0 \mathbf{F}_a \mathbf{f}_b}{\mathbf{f}_b^\dagger \mathbf{F}_a^\dagger \mathbf{Q}_1 \mathbf{F}_a \mathbf{f}_b} = \frac{\mathbf{h}_b^\dagger \mathbf{H}_a^\dagger \mathbf{Q}_2 \mathbf{H}_a \mathbf{h}_b}{\mathbf{h}_b^\dagger \mathbf{H}_a^\dagger \mathbf{Q}_3 \mathbf{H}_a \mathbf{h}_b}.$$

The SIR can also be expressed as Rayleigh-Ritz ratios in terms of the vectors $\mathbf{f}_b$ and $\mathbf{h}_b$. Using this formulation, we can design channel-resilient FB transceivers with good stopband attenuation by the following iterative algorithm:

- Design $F_a(z)$ and $H_a(z)$ so that they have large stopband attenuation.
- Select $F_b^0(z)$ as any lowpass filter.
- For $i \geq 1$, do the following
  - Given $F_0^{i-1}(z) = F_a(z)F_b^{i-1}(z)$, design $H_b^i(z)$ so that the SIR is maximized.
  - Given $H_0^i(z) = H_a(z)H_b^i(z)$, design $F_b^i(z)$ so that the SIR is maximized.
- Stop if $i$ reaches the maximum number of iterations or SIR reaches the desired value or the increase in SIR is less than a predetermined value. Else $i = i + 1$, repeat the above process.

## 4. SIMULATIONS

Two examples will be given below. In the first example, we do not put any frequency constraint on the transmit filter, that is, $n_{fa} = 0$. In the second example, both the transmit and receive filters are designed to have good stopband attenuation. The fading channels satisfy the channel model given in (2). The channel order is $L_c = 4$. The five channel taps are independent identically distributed circularly-symmetric complex Gaussian random variables with zero mean and variances $\ _l^2 = 0.2$ for $0 \leq l \leq 4$. The number of subbands is $M = 64$ and the interpolation ratio is $N = 80$.

**Example 1.** In this example, we design prototype filters with order $n_f = n_h = 160$. The receive prototype filter $H_0(z)$ is split into $H_a(z)$ and $H_b(z)$. Initially, $H_0(z)$ are selected as a lowpass filter with $\ _s = 0.033$ . Two different values of $n_{ha} = 40$ and $n_{ha} = 80$ are considered. The transmit prototype $F_0(z)$ are not split and hence all of its parameters are utilized for SIR maximization. Fig. 2 shows the SIR versus the number of iterations. From the plot, we see that the SIR values can be increased significantly by the proposed iterative algorithm. Fig. 3 shows the magnitude responses of receive prototype filter $H_0(z)$ for different cases of $n_{ha}$ after carrying out 15 iterations of SIR optimization. For comparison, we have also plotted the magnitude response when we set $n_{ha} = 0$ (that is, no frequency constraint is applied on $H_0(z)$ and all of its parameters are utilized for SIR maximization). As we can see, if we do not split the prototype filter $H_0(z)$ into $H_a(z)$ and $H_b(z)$, the magnitude response of $H_0(z)$ after 15 iterations is not much better than the DFT filters even though it is initially selected as a lowpass filter with more than 60 dB attenuation. By splitting the receive prototype filter into $H_a$ and $H_b$, we are able to preserve the stopband attenuation of the receive prototype filter in the iterative process and at the same time increase the SIR value by around 10 dB and 5 dB respectively for $n_{ha} = 40$ and $n_{ha} = 80$.

**Example 2.** In this example, we design prototype filters with order $n_f = n_h = 320$. Both the transmit and receive prototype filters are split into two parts. We consider the two cases $n_{ha} = n_{fa} = 40$ and $n_{ha} = n_{fa} = 80$. The filters $H_a(z)$ and $F_a(z)$ are taken to be the same lowpass filter. The parameters in $H_b(z)$ and $F_b(z)$ are optimized for SIR maximization. The SIR results are shown in Fig. 4. We see that the iterative process can increase the SIR by around 8 dB and 5 dB respectively for $n_{ha} = 40$ and $n_{ha} = 80$. The magnitude responses of the resulting transmit filter $F_0(z)$ and receive filter $H_0(z)$ (after 15 iterations of SIR optimization) are shown in Fig. 5 and Fig. 6 respectively. From these plots, we see that the filters continue to have good stopband attenuation.

## 5. CONCLUSIONS

In this paper, we propose a simple iterative method for designing FB transceivers. Frequency constraint is included in the iteration so that the stopband attenuation is preserved.

Experiments show that for multipath fading channels, the iterative procedure can significantly increase the SIR without destroying the stopband attenuation.

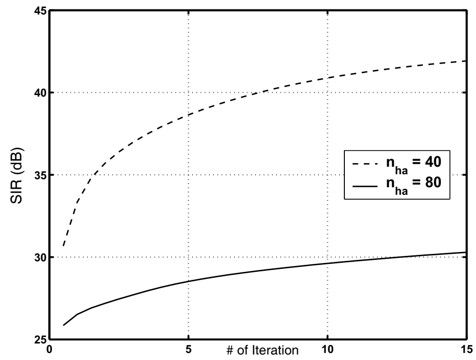


Figure 2: SIR versus number of iterations.

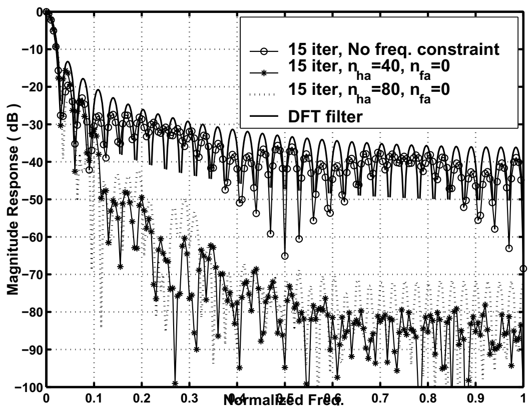


Figure 3: Magnitude response of $H_0(z)$.

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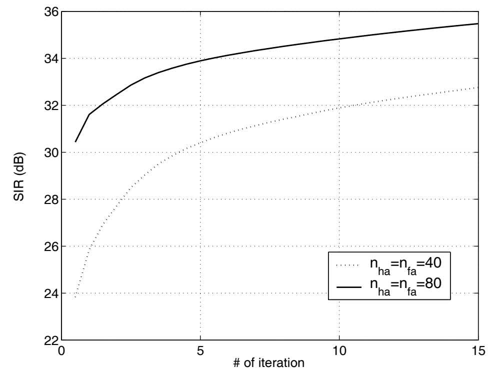


Figure 4: SIR versus number of iterations.

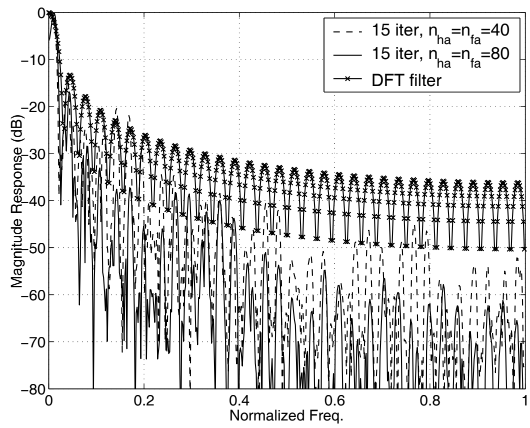


Figure 5: Magnitude response of $F_0(z)$.

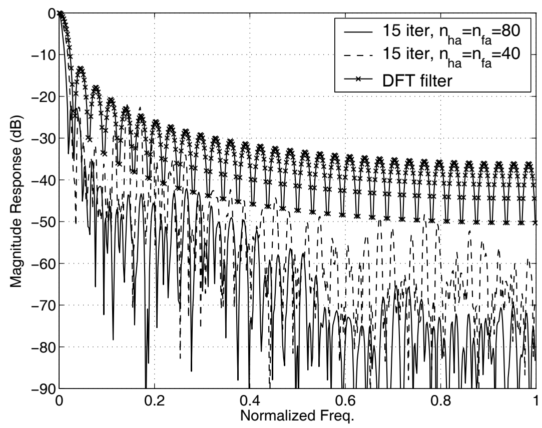


Figure 6: Magnitude response of $H_0(z)$.