# NEW BLIND SOURCE SEPARATION ALGORITHM FOR CYCLOSTATIONARY SIGNAL ESTIMATION BASED ON SECOND ORDER STATISTICS


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## ABSTRACT

Blind Source Separation (BSS) is a general signal processing method, which consists in recovering, from a set of observations recorded by sensors, the contributions of different physical sources independently from the propagation medium and without any a priori knowledge of the sources [4].
All BSS algorithms are based on the assumption that the sources are statistically independent and generally stationary processes. But, real sources are not necessarily stationary processes. In this paper, we are considering the mixture of two sources. The first one is cyclostationary and the second is a stationary process. Our aim is to elaborate a new BSS algorithm able to restore the cyclostationary process by using only the knowledge of its fundamental cyclic frequency $\alpha_o$ and the second order statistical properties of the sources.


## 1. INTRODUCTION

Blind Source Separation (BSS) is a promising technique for signal processing and data analysis that allows the recovery of unknown signals (called sources) from observed signals mixed by an unknown propagation medium.
Figure 1 represents a BSS general scheme. For instantaneous mixtures, the general model becomes :

$$X(k) = A\, S(k) \quad (1)$$

where both the mixing matrix A and the sources S are unknown.
In this work, we elaborate a new Blind Source Separation (BSS) algorithm for a linear instantaneous model de-mixing signals from two unknown sources received by two sensors. First, we apply the widely used whitening step of the mixed sources (matrix W) [2]. The non-standard part is the second step which uses the knowledge of the fundamental cyclic frequency of the cyclostationary process and the second order statistical properties of the sources in order to find the optimal rotation of the de-mixing matrix V.
In our simulations, the first source corresponds to a simple cyclostationary process, $S_1(t) = a(t).\cos(2\pi f_s t)$, a sinusoid modulated by a random amplitude. The second source is a random process (stationary process). We estimate the second order statistical coefficients involved in the criterion used in our algorithm. After separation, the two 'unknown' sources are successfully restored.

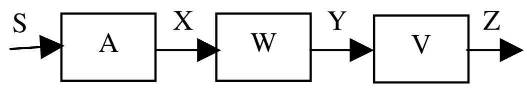


***Figure 1 : BSS general scheme***

## 2. ASYMMETRIC CRITERION

The idea behind restoring our source of interest, i.e., the cyclostationary process, is to maximize the cyclostationary contribution of the first estimated source $Z_1$ and the stationary contribution of the second estimated source $Z_2$, using just the second order properties of the sources and a combined criterion.

## 3. CHARATERIZATION OF THE CYCLOSTATIONARY PROCESS

Let us denote by s(t) a cyclostationary process and by $\alpha_o$ its fundamental cyclic frequency. Its autocorrelation $R_s(t,\tau)$ is a periodic function in t and $T_o = 1/\alpha_o$ is the fundamental cyclic period.

We compute the n[th] coefficient of the Fourier Series Decomposition :

$$R_s^{n\alpha_o}(\tau) = \frac{1}{T_o}\int_0^{T_0} R_s(t,\tau).e^{-j2\pi n\alpha_o t}\,dt \qquad (2)$$

If n is different from zero, then :

$$R_s^{n\alpha_o}(\tau) \neq 0 \ ,\ n \in N^* \qquad (3)$$

One can deduce that if n is equal to zero, then :

$$R_s^{o}(\tau) = \frac{1}{T_o}\int_0^{T_0} R_s(t,\tau)\,dt = K(\tau) \qquad (4)$$

$K(0) = K_o \in R$ , the mean square value.

## 4. CHARATERIZATION OF THE STATIONARY PROCESS

Note that $s'(t)$ is a stationary process.

We deduce that for different non zero values of $n$ :

$$R_{s'}^{n\alpha_o}(\tau) = \frac{1}{T_o}\int_0^{T_0} R_{s'}(t,\tau).e^{-j2\pi n\alpha_o t}\,dt = 0 \qquad (5)$$

and if $n$ is equal to zero :

$$R_{s'}^{o}(\tau) = R_{s'}(\tau) \neq 0 \qquad (6)$$

## 5. COMBINED CRITERION

For an instantaneous linear model the estimate source vector $Z(t)$ is a linear combination of the unknown sources :

$$Z(t) = a\,S_1(t) + b\,S_2(t) \qquad (7)$$

where $S_1(t)$ is our source of interest : the cyclostationary process and $S_2(t)$ the stationary process.

Computing $\left|R_z^{n\alpha_o}(\tau)\right|$ for different non zero values of $n$ leads to :

$$\left|R_z^{n\alpha_o}(\tau)\right| = \left|a^2.R_{s_1}^{n\alpha_o}(\tau) + b^2.R_{s_2}^{n\alpha_o}(\tau) + 2ab.R_{s_1.s_2}^{n\alpha_o}(\tau)\right| \qquad (8)$$

Since $S_2$ is a stationary process, one obtains :

$$R_{s_2}^{n\alpha_o}(\tau) = 0 \qquad (9)$$

As the sources are assumed to be statistically independent and the second source to be centred, then :

$$R_{s_1.s_2}^{n\alpha_o}(\tau) = 0 \qquad (10)$$

We conclude that :

$$\left|R_z^{n\alpha_o}(\tau)\right| = a^2.\left|R_{s_1}^{n\alpha_o}(\tau)\right| \ ,\ n \neq 0 \qquad (11)$$

We denote by $C_1$ the first criterion,

$$C_1 = \left|R_z^{n\alpha_o}(\tau)\right| \ ,\ n \neq 0 \qquad (12)$$

which depends only on the contribution of the cyclostationary source.

If $n=0$ , then $\left|R_z^{o}(\tau)\right|$ equals :

$$\left|R_z^{o}(\tau)\right| = \left|a^2.R_{s_1}^{o}(\tau) + b^2.R_{s_2}^{o}(\tau) + 2ab.R_{s_1.s_2}^{o}(\tau)\right| \qquad (13)$$

As $S_1$ is a cyclostationary process :

$$R_{s_1}^{o}(\tau) = K(\tau) \qquad (14)$$

and for the same hypotheses about the two sources,

$$R_{s_1 s_2}^{o}(\tau) = 0 \qquad (15)$$

This gives :

$$\left|R_z^{o}(\tau)\right| = \left|a^2.K(\tau) + b^2.R_{s_2}^{o}(\tau)\right| \qquad (16)$$

Let us now denote by $C_2$ the second criterion,

$$C_2 = \left|R_z^{o}(\tau)\right| \qquad (17)$$

which includes the (constant) contribution of the stationary source.

The aim of our algorithm is to restore in the first channel $Z_1$ the contribution of our source of interest $S_1$. Thus we need to maximize $\left|R_{z_1}^{n\alpha_o}(\tau)\right|$ and to minimize $\left|R_{z_1}^{o}(\tau)\right|$ in the first channel.

Minimizing of the contribution $\left|R_{z_1}^{o}(\tau)\right|$ leads to maximizing its inverse function : "$-\left|R_{z_1}^{o}(\tau)\right|$".

Finally, we need to maximize the following combined criterion :

$$C = \alpha^2.\left|R_{z_1}^{n\alpha_o}(\tau)\right|^2 - \beta^2.\left|R_{z_1}^{o}(\tau)\right|^2 \qquad (18)$$

where $\alpha$ and $\beta$ are the weighting coefficient of $C_1$ and $C_2$, respectively, for the first channel $Z_1$ , where :

$$\alpha + \beta = 1 \qquad (19)$$

As the contributions of the two sub-criteria $C_1$ and $C_2$ are both important in restoring our source of interest in the first channel, we opted for equal weighting factor :

$$\alpha = \beta = 0.5 \qquad (20)$$

## 6. CRITERION ESTIMATION

The computation of the first criterion gives :

$$R_{z_1}^{n\alpha_o}(\tau) = v_{11}^2.R_{y_1}^{n\alpha_o}(\tau) + v_{12}^2.R_{y_2}^{n\alpha_o}(\tau) + 2.v_{11}.v_{12}.R_{y_1.y_2}^{n\alpha_o}(\tau) \quad (21)$$

and for the second one :

$$R_{z_1}^{o}(\tau) = v_{11}^2.R_{y_1}^{o}(\tau) + v_{12}^2.R_{y_2}^{o}(\tau) + 2.v_{11}.v_{12}.R_{y_1.y_2}^{o}(\tau) \quad (22)$$

where $v_{11}$ and $v_{12}$ represent the coefficients of the first row of the de-mixing rotation matrix V.
After whitening of the observations, one has WA=U that defines a unitary matrix, i.e. $U(U)^H = I$. This leads to $V = (U)^H$.
One can restrict oneself to matrices satisfying det(U)=1 without loss of generality. U is parameterised by a (2X2) Givens rotation matrix [2], where the angle of the rotation plane of the whitening observations Y is bounded by :

$$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \quad (23)$$

Then :

$$V = \hat{U}^H = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad (24)$$

where $v_{11} = \cos\theta$ *and* $v_{12} = -\sin\theta$. Our criterion depending only on one rotation variable $\theta$, we are looking for the optimal rotation angle $\theta_{opt}$ which maximizes the criterion *C(θ)*.

## 7. SIMULATION RESULTS

To validate our new algorithm we tried to apply it to a mixture composed of two sources. The first one is :

$$S_1(t) = a(t).\cos(2\pi f_s t) \quad (25)$$

where, *a(t)* is a white noise (see Figure 2). Simple calculation of the first and second order moments proves this signal to be a cyclostationary process with a fundamental cyclic frequency $\alpha_o = 2f_s$ [1].
The second source is a white random process.
We mix these two sources with the following matrix:

$$A = \begin{bmatrix} 0.8 & 0.5 \\ 2.8 & 2.5 \end{bmatrix} \quad (26)$$

In the obtained mixtures, our source of interest is completely undistinguishable as shown in Figure 3.

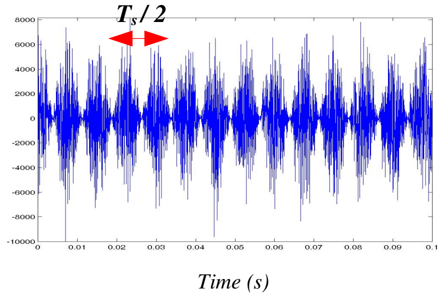


***Figure 2 : The cyclostationary source***

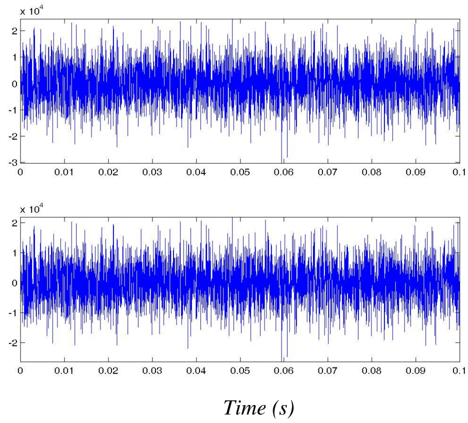


***Figure 3 : The observed signals***

### 7.1 FIRST STEP

After estimation of the different autocorrelations and cross-correlations of the whitened observations Y for $\tau = 0$, by using ten realizations of the two processes, one obtains the function *C(θ)* illustrated in Figure 4.
We remark that our criterion exhibits a maximum, from which one can deduce $\theta_{opt}$ *=0.6692 rd*.

Applying the rotation matrix V for the whitened observations gives the estimated sources of Figure 5.
Our algorithm provides a very good result since it can restore the cyclostationary process in the first channel.

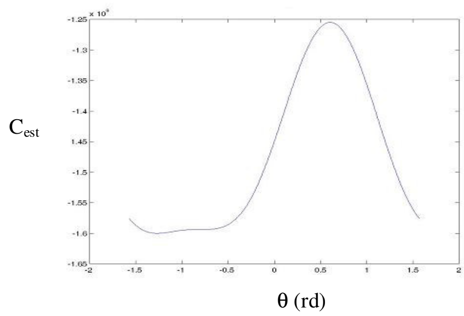


***Figure 4 : $C_{est}$ versus θ***

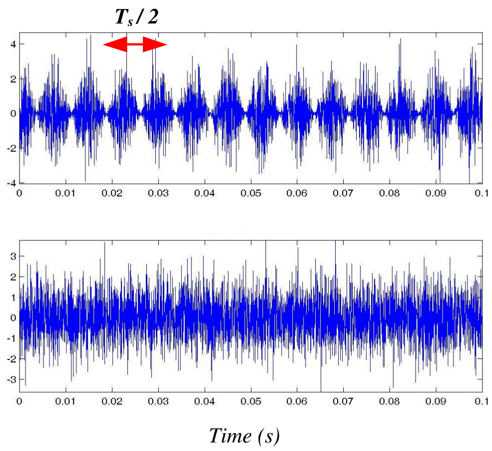


***Figure 5 : The estimated sources***

### 7.2 SECOND STEP

In order to quantify our algorithm separation quality, we estimate our sources using the different values of $\theta$ : $\hat{S}(\theta)$. We then compute the *M*ean *N*ormalized *Q*uadratic *E*rror (*MNQE*) between the real source vector and its estimates at different angle values $\theta$ :

$$MNQE(\theta) = \left\| S - \hat{S}(\theta) \right\|^2 / \| S \|^2 \qquad (27)$$

Figure 6 illustrates the obtained result. Hence, the theoretical optimal rotation $\theta_{th\_opt}$ *=0.6592 rd.*

$\theta_{opt}$ and $\theta_{th\_opt}$ values being close one to each other confirms the efficiency of our algorithm.

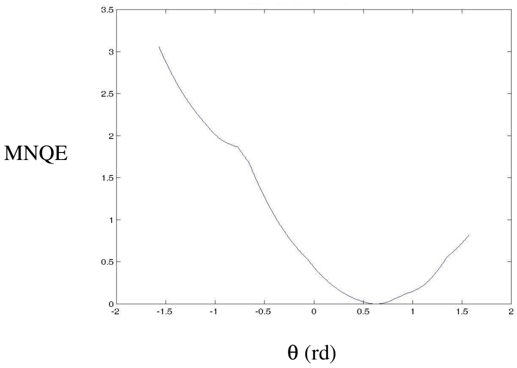


***Figure 6 : MNQE versus*** $\boldsymbol{\theta}$

### 8. CONCLUSION

In this paper, a new BSS technique has been used in order to separate a cyclostationary and a stationary process. The later uses only the knowledge of the cyclostationary process fundamental cyclic frequency and the sources second order statistical properties.

Application of this new algorithm leads to very promising results, since the signature of the cyclostationary source is perfectly restored.

The possibility of extracting a cyclostationary source from a mixture of sources can be applied for rotating machines diagnosis in order, for example, to predict a defect in a bearing. Recent work has shown that the signal emitted by this kind of defect is a cyclostationary process [5].

The use of the proposed algorithm may be expanded into other signal processing areas, where the observations contain one or more cyclostationary processes (telecommunications, etc…).

### 9. REFERENCES


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