PEAK POWER REDUCTION IN OFDM SYSTEMS USING DYNAMIC CONSTELLATION SHAPING

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ABSTRACT

In this paper, we propose a new peak-to-average power ratio (PAPR) reduction algorithm for OFDM transmission. The algorithm is based on dynamically shaping the signal constellation using a decision metric, and it does not require transmitting any side information to the receiver. Compared to other recently introduced PAPR reduction techniques based on constellation shaping, the proposed algorithm is very simple and does not involve any complex optimization procedures. Its performance is investigated using OFDM signaling with a QPSK signal constellation.

1. INTRODUCTION

The attractive features of Orthogonal Frequency Division Multiplexing (OFDM) have made this technique very popular for future wireless communications systems. But the attractiveness of OFDM may be outweighed by its handicap of having a large peak-to-average power ratio (PAPR) at the transmitter output. The PAPR problem of OFDM has been studied considerably and a number of techniques have been developed to reduce it.

Possible peak power reduction techniques include coding, phase optimization and multiple signal representation [1]. Coding leads to satisfactory results, but it reduces the useful data rate, which is undesirable [2]. As an alternative, PAPR reduction can be achieved using phase optimization [3] or schemes relying on multiple signal representation, namely, Selective Mapping (SLM) and Partial Transmit Sequences (PTS) algorithms introduced in [4] and [5]. The main problem of these techniques is that they require the transmission of side information to the receiver. More recent attempts reduce the peak power by changing the signal constellation, introducing new constellations, or inserting pilot signals either in unused subcarriers or over some or all of the used subcarriers [1], [6] and [7]. These techniques are very complex, however, as they use an iterative optimization procedure based on gradient search.

In this paper, we propose a very simple scheme based on constellation shaping. Peak power reduction in the proposed scheme is based on a simple metric calculation for the input symbols and does not need any optimization or iterative search. Its performance was investigated by means of computer simulations and the discrete time-domain samples were considered for PAPR calculation.

The paper is organized as follows: In Section 2, we give a brief review of OFDM together with the PAPR problem. Section 3 describes the proposed PAPR reduction technique based on constellation shaping. In Section 4, we present our preliminary simulation results using QPSK. Finally, we give our conclusions in Section 5.

2. THE PAPR PROBLEM IN OFDM

In OFDM transmission, the complex data symbol block $\mathbf{a} = (a_0, a_1, \ldots, a_{N-1})$ is passed through an N-point inverse fast Fourier transform (IFFT) to obtain the discrete time-domain samples to be transmitted. The transmitted signal samples can be written as

$$b_n^i = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} a_m^i e^{j2\pi nm/N}, \tag{1}$$

where i is the OFDM symbol index and a_m^i is the data symbol transmitted over the mth subcarrier. For convenience, the symbol index will be omitted in the sequel.

The data symbols (the a_m 's) are i.i.d. random variables, and from the central limit theorem, with a large number of subcarriers the time-domain samples at the IFFT output can be modeled as truncated Gaussian random variables with zero mean. Thus, most of the magnitudes will be small (close to zero), but a very small percentage of them will have a very large magnitude. This results in the problem of PAPR from which multicarrier systems suffer considerably.

The PAPR of the time-domain sample sequence $\mathbf{b} = (b_0, b_1, \ldots, b_{N-1})$ is defined as

$$PAPR(\mathbf{b}) = \frac{\max_{0 \le n < N} |b_n|^2}{E\{\|\mathbf{b}\|^2\}/N}, \tag{2}$$

where $\|\cdot\|$ denotes the norm of the enclosed vector.

In general, it is more relevant to consider the approximated continuous-time PAPR with oversampled signal values. The continuous-time PAPR can be approximated by employing the IFFT of the zero-padded input data sequence of length *LN*, where *L* denotes the oversampling rate. In this paper, we only investigated the case $L = 1$.

3. SIMPLE CONSTELLATION SHAPING

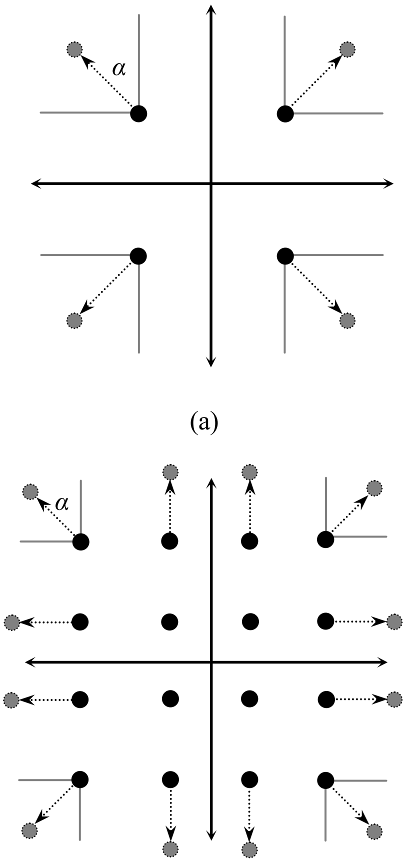
With the increasing popularity of OFDM, there have been numerous attempts to reduce the PAPR of this type of signals. Recently, the attention for PAPR reduction has been turned to schemes which tend to play with the constellation intelligently (see, e.g. [6], [7]). These techniques lead to large improvements compared to the previous ones without having to transmit any side information to the receiver. Constellation shaping consists of modifying the transmitted data symbol values without affecting the minimum distance and consequently the system bit error rate (BER). Constellation shaping actually increases the transmitted average signal power, but this increase can be controlled and what is truly important is the peak power rather than the average power.

In our proposed scheme, a metric is computed for each input data symbol which measures how this symbol contributes to the IFFT output samples with large values. More specifically, the metric indicates how much the peak values of the time-domain signal can be reduced by predistorting the symbol at hand without reducing minimum distance. The symbol metrics involve the magnitudes of the output signal samples $w(n) = |b_n|$ and an appropriate measure of the angle between the output sample b_n and the contribution of symbol a_m to it, which is $a_m e^{j2\pi nm/N}$. The angle measure used in this work is chosen as

$$f(n,m) = -Cos(\varphi_{nm}), \quad (3)$$

where φ_{nm} is the angle between $a_m e^{j2\pi nm/N}$ and b_n. The idea here is to predistort a data symbol if this operation is likely to reduce the peak values of the output block. Note that the $f(n,m)$ function has its maximum for $\varphi_{nm} = \pi$ and decreases monotonically around this phase value. Consequently, a large value of this function indicates that $a_m e^{j2\pi nm/N}$ and b_n are almost in opposite phase and symbol a_m can be predistorted to reduce the magnitude of b_n without reducing minimum distance and degrading performance. Once the metric is computed for all input symbols of the block, the symbols are sequentially predistorted in the decreasing order of their metrics. The procedure stops when the peak power at the IFFT output stops decreasing.

The symbol predistortion technique adopted here involves a simple expansion, as shown in Fig. 1(a) for the QPSK signal constellation. In this modulation, predistortion of a symbol a_m consists of transmitting αa_m, where α is a real number greater than 1. This type of constellation shaping only scales upward the magnitude of the transmitted symbols leaving their phase unchanged. Fig. 1(b) shows the extension of this procedure to 16-QAM. In this case, the corner points of the signal constellation are expanded as in QPSK, while only the real or the imaginary part of the side symbols is expanded in order not to reduce the minimum distance in the signal space. Note that the inner points of the constellation cannot be predistorted without degrading BER performance.



(b)

Fig. 1. Dynamic constellation shaping with scaling factor α : a) QPSK, b) 16-QAM.

More generally, the same procedure can be applied to *M*-QAM signal formats by appropriately predistorting the outermost constellation points. The choice of the scaling factor has a significant impact on the PAPR reduction performance of the proposed technique and may be optimized. In this paper, we determined a positive constant leading to the highest reduction of the average peak power, the averaging being made on OFDM symbols.

The proposed algorithm involves five steps and can be summarized as follows:

1. Obtain the output sequence **b** via IFFT of the input data symbol block $\mathbf{a} = (a_0, a_1, \ldots\ldots, a_{N-1})$.
2. For each sample b_n of the output sequence define a weighting function $w(n)$, which is an increasing monotone function of its power.
3. For each input data symbol a_m compute the decision metric:

$$\mu_m = \sum_{n=0}^{N-1} f^p(n,m) w^q(n) , \quad (4)$$

where p and q are design parameters (with p odd).

4. Determine the K symbols with largest decision metrics with the scaling factor α .
5. Finally, for $n = 0, \ldots, N-1$ update the IFFT output as

$$\hat{b}_n = b_n + \frac{\alpha - 1}{\sqrt{N}} \sum_{k \in S_K} a_k e^{j2\pi nk/N} , \quad (5)$$

where S_K is a set of size K whose elements are the indices of the expanded symbols in the input sequence. The number K is determined by observing the output peak power reduction. The peak output power is indeed reduced using the described procedure up to some value of K, but reduction stops at that point and increasing K beyond that value actually increases peak output power. The K parameter is accordingly selected such that peak power reduction is maximized, i.e., expanding any additional symbols increases peak power.

Although determining the K parameter looks like an iterative procedure, no iterations are actually needed and the algorithm is a one-shot process. Indeed, for a given IFFT block size and signal constellation, the K parameter can be determined before hand by computer simulations and no iterations are needed in a particular implementation.

Further improvements of this technique are possible through an optimization of the decision metric. Indeed, the $w(n)$ and $f(n,m)$ functions in this work were selected arbitrarily, and better performance can be expected if they are optimized. Also, the number of subcarriers in this technique is a critical parameter which gives an increased flexibility and results in a larger reductions of the PAPR as it increases.

4. SIMULATION RESULTS

In this section, we investigate the performance of the proposed scheme for QPSK signaling. In our simulations, we considered the complex baseband representation of the OFDM signal and we used $N = 256$ subcarriers. The results are obtained by averaging over 10^6 randomly generated OFDM symbols. The proposed scheme was applied when the PAPR is greater than 6 dB and the averaging is performed over all trials. The design parameters p and q were taken as 1 and 6, respectively. Note that in PAPR calculation, the ratio of the peak power to the initial average power (before the application of the reduction algorithm) was taken into consideration.

First, the scaling values and the number of symbols to be modified were determined. Then, the results were presented as the Complementary Cumulative Distribution Function (CCDF) defined as

$$CCDF(PAPR(\mathbf{b})) = \Pr(PAPR(\mathbf{b}) > \gamma^2) , \quad (6)$$

which indicates the probability that the PAPR of a symbol block exceeds the threshold level γ^2 .

Fig. 2 shows the change in the peak power as a function of the number of expanded data symbols with the expansion factor α as parameter (solid-line curves). It also shows (dashed curves) the increase of the average signal power. We can see that the number of symbol predistortions which reduce the peak power is a function of the expansion factor. As can be seen on this figure, the optimum number of predistorted symbols is on the order of 20 for $\alpha = 2$ and on the order of 50 for $\alpha = 1.3$. The figure also shows that the largest peak power reduction is achieved for $\alpha = 1.6$ and 25 data symbols predistorted. For all values of α , the results indicate a reduction of the peak power by approx. 1.5 dB.

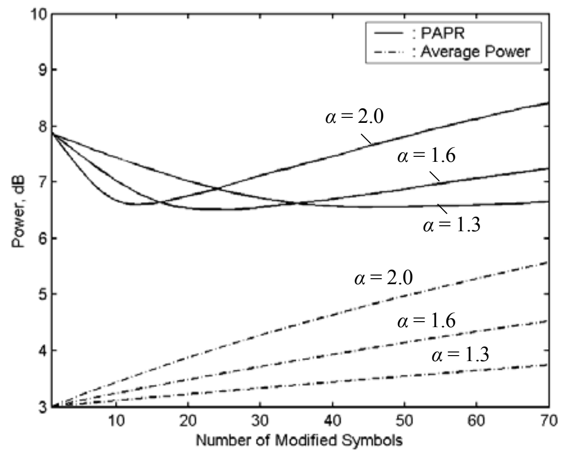


Fig. 2. PAPR and average power vs. number of modified symbols for different scaling factors.

Next, Fig. 3 shows the CCDF of the proposed scheme using the optimum α and K parameters. The solid-line curve corresponds to conventional OFDM without any compensation and the dotted curve corresponds to the PAPR reduction technique proposed. These curves indicate that the improvement is on the order of 2.2 dB at the probability of 10^{-3} and of 2.7 dB at the probability of 10^{-5} .

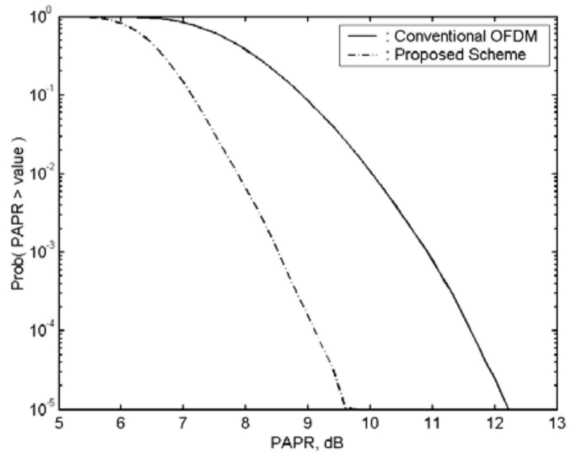


Fig. 3. CCDF of PAPR for conventional OFDM. and the proposed scheme.

5. CONCLUSIONS AND PERSPECTIVES

We have introduced a simple PAPR reduction scheme for OFDM systems, which relies on constellation shaping. Specifically, the algorithm employs a simple decision metric for each input symbol that measures its contribution to the output signal samples of large magnitude and indicates how much these samples can be reduced by upscaling the value of that symbol. The computational complexity of the proposed algorithm is very low compared to other PAPR reduction techniques based on constellation shaping, because it is one-shot process and it does not need any complex optimization procedures. The algorithm was described for QPSK, but it equally applies to higher-level QAM signal constellations using an expansion of the outermost signal points. Finally, additional improvements can be expected by introducing other cost functions in order to better select the symbols to be expanded and also by extending this symbol predistortion technique to the phase dimension.

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