A STOP & GO PRE-WHITENED SIGN ALGORITHM

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ABSTRACT

This paper deals with performance improvement of the sign algorithm, which is known to be suffering from slow rate of convergence, especially when the input signal is highly correlated. To overcome this drawback, decorrelated input signal is used to pilot the adaptive filter. In this paper, we show that using this technique, a punctual degradation can occur when the pre-whitened input direction is the opposite of the right one. To prevent this problem, we introduce the concept of "Stop and Go" to govern the algorithm. It consists on freezing the adaptation when we detect this effect. Experimental results conducted on highly correlated input and long system impulse response indicate clearly the effectiveness of the proposed algorithm.

1. INTRODUCTION

In many fields, such as acoustic echo cancellation [1] and speech coding [2], the family of sign algorithm is retained thanks to its low complexity and robustness against impulsive noise . However, it suffers from slow convergence rate, especially for highly correlated input signal. As an extension to the Least Mean Square (LMS) family, for which filtered X-LMS approach, is introduced to accelerate convergence [3], the family of filtered sign algorithm has also been proposed (see for example [4, 5, 6]). Hence, a variety of algorithms has been investigated. Each version overcomes some limitations of classical sign family. Sometimes, it introduces novel limitations. For example, the filtered sign algorithm using pre-whitened input and filtered error leads to better convergence rate but amplifies additive noise in steady state.

One retained algorithm in term of compromise between low complexity, fast convergence rate and good steady state performances is the Filtered Sign Algorithm (FSA) [6]. Its adaptation term uses the pre-whitened input and the sign of the error signal. In this paper, we show that FSA can suffer from punctual degradation, especially if we adapt the algorithm in the opposite direction of the desired one. Hence, we propose to improve the FSA algorithm by introducing the concept of "Stop & Go" rule: we freeze adaptation when we detect that pre-whitened input signal direction moves away the adaptive filter from the optimal solution. The proposed criterion to decide about adaptation freeze is based on a comparison between the sign of the error and the sign of the filtered error.

The proposed Normalized Filtered Sign Algorithm equipped with "Stop and Go" rule (SGNFSA) offers at least two improvements: better convergence rate, and smaller mean square error in the steady state.

This paper is organized as follows. In section 2, the proposed algorithm is described. Section 3 provides the necessary justifications to the proposed ideas. Section 4 presents some simulation results supporting the proposed algorithm. Finally concluding remarks are provided in section 5.

2. ALGORITHM DESCRIPTION

2.1 Background

Let us consider the identification problem. The input/output equation of the system to be identified is given by

$$y(k) = F^T X(k) + n(k), \tag{1}$$

where $x(k)$ is the input signal, $y(k)$ is the output, F is the unknown system impulse response of length L, $X(k) = [x(k), \cdots, x(k-L+1)]^T$ is the input observation vector and $n(k)$ is a zero mean additive white Gaussian noise. An adaptive FIR filter $H(k)$ is used to identify the system impulse response F.

In this paper, we are interested in sign algorithms using the concept of input pre-whitening [5]. Different solutions are possible. In previous work [6], we investigated three adaptation process: pre-whitening only the input, pre-whitening the input and filtering the error using the same pre-whitener or filtering only the error signal. In this paper, a novel combination between original signals and filtered ones in the adaptation process is proposed. This combination introduces a novelty which consists on adaptation freeze during some iterations. Moreover, we are interested with normalized version of the algorithm.

2.2 Mathematical formulation

The proposed Stop and Go Normalized Filtered Sign Algorithm (SGNFSA) is described as follows:

$$\begin{aligned} H(k+1) &= H(k) + \mu \frac{G\left\{e(k), e^f(k)\right\}}{\mathcal{N}(k)} X^f(k) \\ G\left\{e(k), e^f(k)\right\} &= \frac{sign\left[e(k)\right] + sign\left[e^f(k)\right]}{2} \end{aligned}, \tag{2}$$

where μ is a positive step size, $e(k)$ is the error signal given by:

$$e(k) = y(k) - H(k)^T X(k), \tag{3}$$

$X^f(k) = [x^f(k), \cdots, x^f(k-L+1)]^T$ is the observation vector of the pre-whitened input signal $x^f(k)$, $e^f(k)$ is the filtered error signal, and $\mathcal{N}(k)$ is the normalizing factor.

The pre-whitened input signal $x^f(k)$ is given by:

$$x^f(k) = x(k) - P(k)^T \tilde{X}(k-1), \tag{4}$$

where $P(k)$ is the adaptive predictor of length L_P and $\tilde{X}(k-1) = [x(k-1),...,x(k-L_P)]^T$ is the input observation vector.

In this paper, we apply the same filter, used to de-correlate the input signal, to the error signal to drive the adaptive algorithm. The filtered error $e^f(k)$ is given by:

$$e^f(k) = e(k) - P(k)^T E(k-1), \tag{5}$$

where $E(k-1) = [e(k-1),...,e(k-L_P)]^T$ is the error vector.

The normalizing factor $\mathcal{N}(k)$ is expressed as follows:

$$\mathcal{N}(k) = \sum_{i=0}^{L-1} |x^f(k-i)| + \beta, \tag{6}$$

where β is a constant avoiding division by zero.

The adaptive predictor is a sign-based algorithm. It is described as follows:

$$P(k+1) = P(k) + \frac{\mu_P \operatorname{sign}\left[x^f(k)\right] \tilde{X}(k-1)}{\sum_{i=1}^{L_P} |x(k-i)| + \beta}, \tag{7}$$

where μ_P is the pre-whitener step size.

2.3 Algorithm description

The expression of the proposed algorithm (2) is characterized by three terms : $X^f(k)$, $\mathcal{N}(k)$ and $G\left\{e(k), e^f(k)\right\}$. They are justified as follows:

- The filtered input $x^f(k)$ is used instead of the original input $x(k)$, in order to accelerate convergence. It is obtained using a de-correlation process whose output is the prediction error. We note that, in order to maintain low the computational complexity, we use an adaptive predictor based on sign algorithm.
- The normalization factor $\mathcal{N}(k)$ is introduced in order to control the critical step size and to speed up the convergence.
- The term $G\left\{e(k), e^f(k)\right\}$ describes the "Stop and Go" rule. In fact, two situations are possible:
  - ⋆ if $sign\,[e(k)] = sign\left[e^f(k)\right]$, the algorithm update will be given by:

    $$H(k+1) = H(k) + \mu \frac{sign\,[e(k)]\, X^f(k)}{\mathcal{N}(k)} \tag{8}$$

    It corresponds to the "Go" step in the algorithm, which is equivalent to the Normalized Filtered Sign Algorithm (NFSA).
  - ⋆ if $e(k)$ and $e^f(k)$ have opposite signs, the adaptation is frozen $H(k+1) = H(k)$ and it corresponds to the "Stop" step in the algorithm.

Once the algorithm is presented, it is obvious that the "Stop and Go" rule must be justified. This is the objective of the following section.

3. ALGORITHM JUSTIFICATIONS

3.1 Punctual degradation in NFSA algorithm

As starting point, we propose to use the NFSA algorithm described by (8). In fact, this algorithm was shown to be powerful in term of convergence acceleration [6]. In this subsection, we'll point on one limitation of this algorithm, which consists on punctual degradation when the filtered input direction moves away the adaptive filter from the optimal solution.

First of all, we define the deviation vector between the unknown system impulse response and the adaptive filter:

$$V(k) \stackrel{\triangle}{=} F - H(k). \tag{9}$$

It obeys to the following recursion:

$$V(k+1) = V(k) - \mu \frac{sign\,[e(k)]\, X^f(k)}{\mathcal{N}(k)}. \tag{10}$$

The error signal can be expressed by

$$e(k) = V(k)^T X(k) + n(k). \tag{11}$$

The adaptive algorithm converges without punctual degradation, if the norm of the deviation vector vanishes, which means that:

$$||V(k+1)|| \leq ||V(k)||. \tag{12}$$

The deviation vector can be decomposed as follows:

$$V(k) = U(k) + \frac{V(k)^T X^f(k)}{||X^f(k)||^2} X^f(k), \tag{13}$$

where $U(k)$ is the vector orthogonal to $X^f(k)$.

Using (13), we rephrase the recursion (10) as follows:

$$V(k+1) = U(k) + \left[\frac{V(k)^T X^f(k)}{||X^f(k)||^2} - \mu \frac{sign\,[e(k)]}{\mathcal{N}(k)}\right] X^f(k). \tag{14}$$

From equation (14), it is easy to show that the condition (12) is satisfied if the following inequality is verified

$$\left|\frac{V(k)^T X^f(k)}{||X^f(k)||^2} - \mu \frac{sign\,[e(k)]}{\mathcal{N}(k)}\right| \leq \frac{|V(k)^T X^f(k)|}{||X^f(k)||^2}. \tag{15}$$

From the last equation, it is important to note that if $e(k)$ and $V(k)^T X^f(k)$ have different signs, the condition given in (12) is not satisfied. This leads to a punctual degradation, which may cause noise enhancement and/or convergence delay. To overcome this problem, we propose to stop adaptation if $sign\,[e(k)] \neq sign\left[V(k)^T X^f(k)\right]$, otherwise, adaptation is kept. This is what is called "Stop and Go" rule.

In figure 1, we illustrate this problem of punctual degradation in the NFSA, through the evolution of the Mean Square Deviation $MSD \stackrel{\triangle}{=} E\left\{V(k)^T V(k)\right\}$ and for only one run. The simulation conditions are : the input signal is a first order auto-regressive signal of power $P_x = 5.3$ and whose model is $x(k) = 0.9x(k-1) + g(k)$, where $g(k)$ is a white Gaussian signal. The system impulse response is an acoustic response of a room truncated to $L = 64$ (sampled at 16 Khz), the additive noise $n(k)$ was used with $SNR = 46$ dB.

It's interesting to note, that when the NSFA suffers from punctual degradation, the SGNFSA freezes the adaptation.

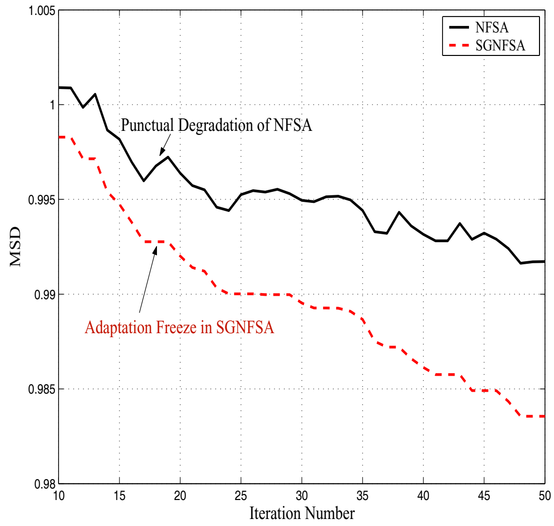

Figure 1: Punctual degradation in NFSA algorithm.

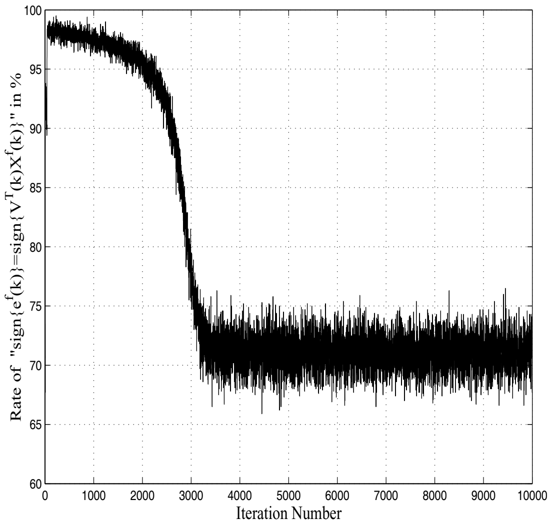

Figure 2: "$sign\left[V^T(k)X^f(k)\right] = sign(\left[e^f(k)\right]$" assumption verification.

3.2 $sign\left[V(k)^T X^f(k)\right]$ approximation

Since $V(k)^T X^f(k)$ is not observable, we must find an approximation to this term in order to propose a practical formulation of the proposed "Stop and Go" rule.

Let us assume the case of fixed optimal predictor $P(k) = P$. This assumption can be argued by the fact that the predictor length is smaller than the filter length, and the predictor converges more rapidly than the adaptive filter. Using this assumption, the pre-whitened input vector and the filtered error are re-written as follows:

$$X^f(k) = X(k) - \sum_{i=1}^{L_P} p_i X(k-i), \qquad (16)$$

$$e^f(k) = e(k) - \sum_{i=1}^{L_P} p_i e(k-i). \qquad (17)$$

Assuming that in the transient state $|n(k)| << |V(k)^T X(k)|$, this means that during that state $e(k) \approx V(k)^T X(k)$, and using equations (11) and (16), the term $V(k)^T X^f(k)$ can be approximated as follows:

$$V(k)^T X^f(k) \approx e(k) - \sum_{i=1}^{L_P} p_i V(k)^T X(k-i). \qquad (18)$$

Denoting

$$\Delta(k,i) = V(k) - V(k-i),$$

and using the same assumption $e(k-i) \approx V(k-i)^T X(k-i)$, the last equation (18) can be re-phrased as follows:

$$V(k)^T X^f(k) \approx e^f(k) - \sum_{i=1}^{L_P} p_i \Delta(k,i)^T X(k-i), \qquad (19)$$

In case of small step size and for slow rate of convergence, $V(k)$ and $V(k-i)$ are too close, and the norm of $\Delta(k,i)$ is very small. So, during the transient phase, one can assume that

$$|e^f(k)| >> \left|\sum_{i=1}^{L_P} p_i \Delta(k,i)^T X(k-i)\right|. \qquad (20)$$

Hence, we may approximate the sign of $V(k)^T X^f(k)$ by the

sign of the filtered error $e^f(k)$.

This approximation will be verified through this simulation. We plot, in figure 2, the rate of occurrence of the statement "$sign\left[V^T(k)X^f(k)\right] = sign(\left[e^f(k)\right]$" for 1000 iterations. From this figure, it is interesting to note that, during transient state, the used assumption is valid for more than 90% of cases, which makes it a good approximation.

3.3 "Stop and Go" rule

The stop and go rule characterized by a comparison between $V(k)^T X^f(k)$ and $e(k)$ signs is modified by a comparison between $e^f(k)$ and $e(k)$ signs. A manner to stop adaptation in the case of opposed signs and to maintain adaptation in case of equal signs is to modify the term $\mu sign\left[e(k)\right]$ in adaptation expression of NFSA (8) by the term $\mu \frac{\left\{sign\left[e(k)\right] + sign\left[e^f(k)\right]\right\}}{2}$. In fact, it is easy to verify that if $e(k)$ and $e^f(k)$ have same sign, the adaptation is carried using $sign\left[e(k)\right] = sign\left[e^f(k)\right]$, otherwise it is stopped.

Finally, thanks to this analysis, we were able to justify the conception of this new algorithm SGNFSA resumed in equation 2.

4. ALGORITHM PERFORMANCES

In order to validate the proposed SGNFSA algorithm, we present simulation results carried in the same simulation conditions as previous, obtained by averaging 1000 independent runs using 10000 samples. The tested algorithms are : Normalized Sign Algorithm (NSA) obtained by replacing

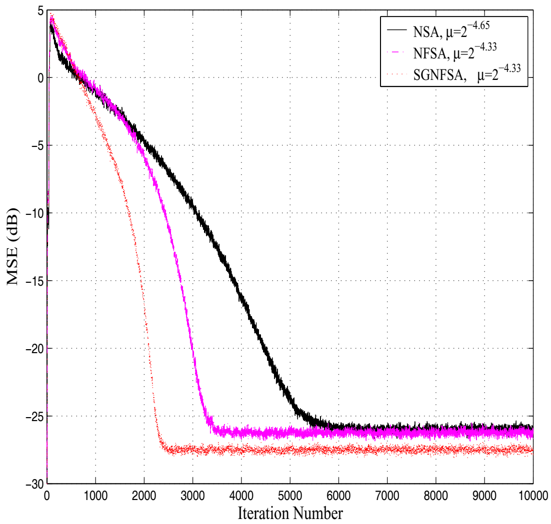

Figure 3: Tested algorithms convergence rate.

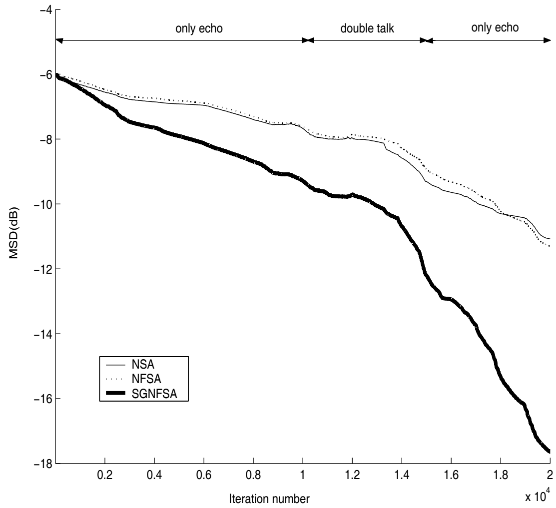

Figure 4: Tested algorithms performance in acoustic echo cancellation.

$x^f(k)$ by $x(k)$ in (8), the Normalized Filtered Sign Algorithm (NFSA) described by (8), and the proposed SGNFSA algorithm described by (2). The pre-whitener parameters are $L_P = 1$ and $\mu_P = 2^{-10}$.

The step sizes for the NSA and NFSA are accorded in order to obtain the same steady state corresponding to a Mean Square Error $MSE \stackrel{\triangle}{=} E\left\{e(k)^2\right\} = -26\,\text{dB}$. They are $\mu_{NSA} = 2^{-4.65}$, $\mu_{NFSA} = 2^{-4.33}$. To show the improvement introduced by the "Stop & Go" rule, we have chosen $\mu_{SGNFSA} = \mu_{NFSA}$.

In figure 3, we report the evolution of the MSE versus iteration number. This figure shows that NFSA has better convergence rate than NSA, and the introduced "Stop & Go" rule increases the convergence rate while reducing the steady state MSE to -27.5 dB. In fact, the SNA converges after 5500 iterations, the NFSA converges after 3400 iterations and the SGNFSA converges after 2400 iterations.

The test algorithm performances are now evaluated in acoustic echo cancellation field. The input signal $x(n)$ is a speech sequence, the system output $y(n)$ is composed of periods of only echo and periods of double talk (near-end speech and echo). In such application, besides convergence rate, steady state performance and complexity issues, an important aspect is the algorithm performance during double talk.

In figure 4, we plot the evolution of the *MSD*. The same step size $\mu = 2^{-6}$ is used for all algorithms and it is adjusted such as bursts during transition from only echo to double talk are avoided. No improvement is observed when passing from NSA to NFSA. However, the proposed SG-NFSA outperforms and gives excellent performances for all operating conditions including double talk.

5. CONCLUSION

In this paper, we proposed a novel sign algorithm using the two concepts of input pre-whitening and "Stop and Go" rule for adaptation. The algorithm is justified by the need of faster convergence for high correlated inputs and by the need of adaptation freeze in case of local degradation. After algorithm presentation, the different steps are analyzed and justified. Some simulation are carried to show that the proposed algorithm presents good performance in term of convergence speed, steady state mean square error and computational complexity.

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