NON-PARAMETRIC ML CHANNEL ESTIMATOR AND DETECTOR FOR OFDM

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ABSTRACT

A maximum-likelihood channel estimator for the orthogonal frequency division multiplexing (OFDM) communication environments, in presence of interference is discussed here. We study a training based scenario, where the channel is estimated based on pilots that precede the transmission of the information. To reduce the number of estimation parameters, we estimate the channel iteratively in time-domain. Since interference from other users provides no useful information we do not estimate parameters of the interference and neither we neglect the affect of the interference instead interference along with Gaussian noise is perceived as non-Gaussian noise. The algorithm assumes no *apriori* knowledge about the interfering channel and signal at the receiver, further no-assumption on the statistical properties of the interferer is assumed which makes this algorithm robust. The estimated channel information along with the estimated distribution are then utilized to equalize the subsequent data blocks.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a promising multi-carrier digital communication technique for transmitting data at high bit-rates over wireless or wire-line channels. The high-speed serial data is converted into many low bit rate streams that are transmitted in parallel, thereby increasing the symbol duration and reducing the intersymbol interference (ISI). These features have led to an increase in the use of OFDM or related techniques in many high bit rate communication systems. Discrete multi-tone modulation which is quite similar to OFDM is extensively used in digital subscriber line (xDSL) communication systems. OFDM has been chosen for digital audio broadcasting (DAB) and digital video broadcasting (DVB). It is also used for the 2.4 GHz wireless local area networks (IEEE 802.11g).

Coherent OFDM transmission invariable requires estimation of the channel frequency response (i.e. the gains of the OFDM tones). Currently there can be three possible solutions: 1) blind, 2) semi-blind, and 3) pilot aided. In blind channel estimation techniques, the channel is estimated without the knowledge of the transmitted sequence. It is attractive as the throughput is higher as no bits are lost in training. However it requires large amount of data to be stored before channel estimation can begin, which invariably introduces delays. The pilot based technique estimates the channel by transmitting a known (at the receiver) training sequence along with the unknown data at the receiver. The receiver estimates the channel using some criterion based on comparing the change in these pilots due to channel. The semi-blind techniques try to reduce the size of the training sequence by exploiting both the known and the unknown (blind) portions of the data.

Channel estimation in OFDM is critical to the overall performance of the communication system. Insertion of pilots in OFDM symbols provides a base for reliable channel estimates. There has been considerable increase in channel estimation research over the years [1], [2] etc. However most of the current work is based on channel estimation for Gaussian channels or assuming that the interference is very low. This assumption is usually based on two reasons: first the interference to have tractable mathematical models and by central limit theorem. This assumption is however not always valid in scenarios where there are a small number of interferers (e.g. Bluetooth device or microwave oven operating in presence of a WLAN). With the co-existence of various wireless equipments in home or office environments the interference from neighboring devices has become a major concern [3]. In interference affected channels we can be sure that algorithms designed for Gaussian assumption are not optimal [4]. From here on we refer to the traditional Gaussian assumption estimator (which assumes zero or negligible interference) as least squares (LS) estimator.

Here we estimate the fading channel in presence of interference directly in time domain using maximum likelihood (ML) technique. The channel is assumed to be deterministic for a given block. The algorithm discussed in [2] specifically deals with the synchronous interference, however it was noted that interference was modelled as Gaussian, which may not be the case if only a few (or in fact one major interferer as in [5]) are present. In this paper we make no such *apriori* assumption on the interfering received signal distribution. Moreover no parameter of the interferer is estimated specifically. In fact, the presence of interference along with Gaussian noise is jointly considered as a Gaussian mixture noise [4] and [6]. It is noted that traditional zero forcing equalization technique fall short of performance in presence of interference. Simulation results confirm the non-optimal estimates when LS is used and improved bit error rate (BER) performance by using the presented algorithm. Throughout the paper capitalized variables represents frequency domain values while the bold variables represents vectors. Also $\Re$ and $\Im$ represents real and imaginary part.

The paper is organized as follows. In section-2 the problem statement is formulated for a general OFDM communication system followed by brief discussion on den-

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sity estimation. The iterative non-parametric maximum-likelihood (NPML) channel estimator is described in section-3. Section-4 discusses the modified non-parametric symbol-by-symbol equalizer. To test the robustness of the algorithm, in section-5, the simulation results are presented. Conclusions based on analysis and simulation are drawn at the end.

2. FORMULATION OF THE PROBLEM

2.1 OFDM System Model

The baseband equivalent representation of a typical OFDM system as in Fig-1 is considered here. We focus our discussion on estimation of one OFDM symbols instead of a sequence of symbols for the reasons justified below. At the transmitter side, the serial input data is converted into M parallel streams, and each data stream is modulated by a linear modulation scheme, such as QPSK, 16QAM or 64QAM. If QPSK is used, for instance, the binary input data of $2M$ bits will be converted into M QPSK symbols by the serial-to-parallel converter (S/P) and the modulator. The modulated data symbols, which are denoted by complex-valued variables $X(0),...,X(m),...,X(M-1)$, are then transformed by the IFFT, and the complex-valued outputs $x(0),...,x(k),...,x(M-1)$ are converted back to serial data for transmission. A guard interval is inserted between symbols to avoid inter-symbol interference (ISI). If the guard interval is longer than the channel delay spread, and if we discard the samples of the guard at the receiving end, the ISI will not affect the actual OFDM symbol. Therefore, the system can be analyzed on a symbol-by-symbol basis. At the receiver side, after converting the serial data to M parallel streams, the received samples $y(0),...,y(k),...,y(M-1)$ are transformed by the FFT into $Y(0),...,Y(m),...,Y(M-1)$ [1]. Using the notations for the OFDM symbols, the output of the channel can be written as

$$y(k) = \sum_{l=0}^{L-1} h^*(l)x(k-l) + \sum_{p=0}^{P-1}\sum_{l=0}^{L-1} g_p^*(l)u_p(k-l) + n(k), \quad 0 \le k \le M-1 \quad (1)$$

where h and x represents desired user's channel and data respectively. Without loss of generality we choose complex conjugate h^* instead of h in above equation [7]. L represents the channel length and $n(k)$ is the additive white Gaussian noise. P represents the number of interferers where g_p and u_p is the interfering channel and signal respectively. Note that $y(k)$, $x(k)$, $n(k)$, $h(l)$, $u_p(k)$ and $g_p(l)$ are all complex valued. It is assumed that the channel and interference doesn't change during the block transfer and interference is synchronous which makes the above representation possible.

If cyclic prefix is used for the guard interval, intercarrier interference (ICI) in multipath channel can also be avoided. Then it can be shown that the following simple relation between $Y(m)$ and $X(m)$ holds:

$$\begin{aligned} Y(m) &= \left(\sum_{l=0}^{L-1} h^*(l)\exp^{(-j2\pi\frac{ml}{M})}\right)X(m) \\ &\quad + \left(\sum_{p=0}^{P-1}\sum_{l=0}^{L-1} g_p^*(l)\exp^{(-j2\pi\frac{ml}{M})}U_p(m)\right) + N(m) \quad (2) \\ &= H(m)X(m) + I(m) + N(m), 0 \le m \le M-1 \quad (3) \\ &= H(m)X(m) + N'(m), 0 \le m \le M-1 \quad (4) \end{aligned}$$

where $H(m)$ is the complex frequency response of the channel at the subchannel m, $I(m)$ be the complex interference at that subchannel m and $N(0),...,N(M-1)$ are the DFT of $n(0),...,n(M-1)$. If $n(0),...,n(M-1)$ are i.i.d. Gaussian random variables, so are the transformed variables $N(0),...,N(M-1)$. It is assumed that the interfering signals $U_p(0),...,U_p(M-1)$ are also OFDM signals, with same block and cyclic pre-fix lengths, and they are block synchronous with the desired signal. Eq. (4) shows that the received signal is the transmitted signal attenuated and phase shifted by the frequency response of the channel at the sub-channel frequencies due to fading in presence of interference and noise [1]. It is assumed to be that noise is represented as complex independent identically distributed (i.i.d.) with vector $\mathbf{n} = [n(0), n(1), ..., n(M-1)]^T$ with each component of $\mathbf{n}$ distributed as $\mathbb{C}\mathscr{N}(\mu_i, \sigma_i^2)$ and are also independent. The multivariate complex Gaussian pdf is just the product of the marginal pdf or

$$f(\mathbf{n}) = \prod_{i=0}^{M-1} f(n(i)) \quad (5)$$

which follows from the usual property of the pdf for real independent random variables, this can be written as

$$f(\mathbf{n}) = \frac{1}{\pi^M \prod_{i=0}^{M-1}\sigma_i^2}\exp\left[-\sum_{i=1}^{M-1}\frac{1}{\sigma_i^2}\mid n(i)\mid^2\right] \quad (6)$$

Since the joint pdf depends on $\Re$ and $\Im$ only through $\mathbf{n}$, we can view the pdf to be that of the 'scalar random variable n'. This pdf eq. (6) is called a 'complex Gaussian pdf' for a scalar complex random variable and is denoted by $\mathbb{C}\mathscr{N}(0, \sigma_i^2)$ [8].

3. KERNEL DENSITY ESTIMATION

Since we have complex noise and interference we can model it as a 'complex Gaussian mixture' pdf, where the real and complex are assumed independent as discussed earlier. Parzen window or kernel density estimation assumes that the probability density is a smoothed version of the empirical sample. Its estimate $\hat{f}(y)$ of a complex random variable $y = \Re\{y\} + i\Im\{y\}$ is simply the average of radial kernel function centered on the points in a sample M of the instance of y:

$$\hat{f}(y) = \frac{1}{M}\sum_{j=1}^{M}\phi(y - y(j)) \quad (7)$$

We here assume ϕ to be Gaussian kernel (Parzen kernel) [6]:

$$\phi(y) = \mathscr{N}(0, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{-\mid y\mid^2}{2\sigma^2}\right) \quad (8)$$

variance defined as σ^2. The joint pdf $\hat{f}(y)$ depends on the real and complex components through y, we can view the pdf to be that of the scalar random variable y, as the notation suggest [8]. Other choices of kernel like *Epanechnikov kernel* are also possible. It can be shown that under the right conditions $\hat{f}(y)$ will converge to the true density $f(y)$ as $\mid M \mid \rightarrow \infty$.

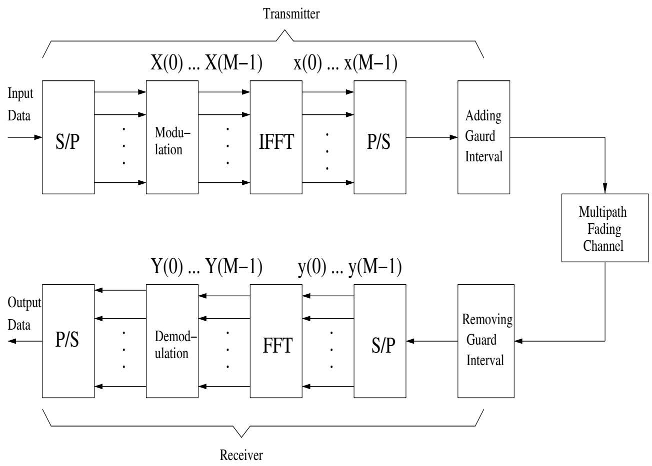

Figure 1: A typical OFDM communication system

4. NON-PARAMETRIC ML CHANNEL ESTIMATION

The channel impulse response $\mathbf{h} = [h(0),...,h(L-1)]$ are independent complex-valued Gaussian random variables (which represents a frequency-selective Rayleigh fading channel). In regular OFDM system, the channel delay spread L is much smaller than the number of subcarriers. This leads to a high correlation between the channel frequency responses $H(m), 0 \le m \le M-1$, even when $h_l, 0 \le l \le L-1$,are independent [1]. We estimate the channel impulse response $\mathbf{h} = [h(0),...,h(L-1)]$ directly, as the channel frequency response $H(0),...,H(M-1)$ are generally correlated among each other (as discussed above) and the impulse response may be independently specified, thus the number of parameters in the time domain is smaller than that in the frequency domain.

The combined interference and AWGN $N'(m)$ in eq. (4) is together taken as a noise that is non-Gaussian because of the presence of interference [6]. As also discussed in [6] the LS estimator does not find the optimal solution in the case of non-Gaussian noise. If the noise was Gaussian then the solution to the ML leads to the LS estimate. However, in communication systems where the noise is non-Gaussian (or Gaussian mixture) i.e. Gaussian in presence of interference, no closed form ML solution exists for such non-Gaussian distributions. Thus we rely on the iterative algorithm to find the ML estimate of the channel. In this algorithm we first initialize channel update algorithm with LS estimate, then we estimate the likelihood on the pilots. After estimating likelihood we find the ML solution iteratively on the pilot symbol. The classical stochastic gradient algorithm is used with a log-likelihood being the cost function i.e. the gradient here is the first derivative of the log-likelihood function with a constant multiplier (similar to well known gradient ascent algorithm) [9]. The update equation is:

$$\hat{\mathbf{h}}_k = \hat{\mathbf{h}}_{k-1} + \mu(k) \nabla_{\mathbf{h}} \mathscr{L}(\mathbf{h} \mid \mathbf{Y})\big|_{\mathbf{h}=\hat{\mathbf{h}}_{k-1}} \tag{9}$$

where $\mu(k)$ is the adaptation constant and $\nabla_{\mathbf{h}}$ represents the gradient of the cost function. Referring to eq. (4) and eq. (9) the likelihood function can be written as:

$$L(\mathbf{h} \mid \mathbf{Y})\big|_{\mathbf{h}=\hat{\mathbf{h}}_{k-1}} = f(\mathbf{Y} \mid \mathbf{h}) = \prod_{i=1}^{M} f_{N'}(E(i))$$

$f_{N'}(.)$ is scalar pdf of 'complex Gaussian mixture' of data length from $i = 1,...,M$ and the previous estimation error is defined as:

$$E(i) = Y(i) - \left(\sum_{l=0}^{L-1} h_k^*(l) \exp^{(-j2\pi \frac{il}{M})} \right) X(i) \tag{10}$$

Kernel density estimators are known to be effective in estimating the pdf over short data record and also provide a differentiable smooth estimated pdf. Using kernel density estimator we obtain:

$$\hat{f}_{N'}(E) = \frac{1}{M} \sum_{j=1}^{M} \phi(E - E(j)) \tag{11}$$

where M is the number of subcarriers.

$$\hat{\mathscr{L}}(\mathbf{h} \mid \mathbf{Y})\big|_{\mathbf{h}=\hat{\mathbf{h}}_{k-1}} = \sum_{i=1}^{M} \log \left(f_{N'}(E(i)) \right)$$
$$= \sum_{i=1}^{M} \log \sum_{j=1}^{M} \phi(E(i) - E(j)) - \log \mid M \mid \tag{12}$$

Maximizing the log-likelihood function w.r.t to channel weight vector. By definition of complex vector differentiation [7] we obtain,

$$\nabla_{\mathbf{h}} \hat{\mathscr{L}}(\mathbf{h} \mid \mathbf{Y})\big|_{\mathbf{h}=\hat{\mathbf{h}}_{k-1}} = \sum_{i=1}^{M} \frac{\sum_{j=1}^{M} \frac{\partial \phi(E(i)-E(j))}{\partial \mathbf{h}}}{\sum_{j=1}^{M} \phi(E(i) - E(j))} \tag{13}$$

Thereby substituting this gradient in eq. (9) gives an iterative solution. As with any stochastic gradient algorithm the choice of optimal $\mu(k)$ varies with application and requirements. As discussed in [9] we choose $\mu(k) = \frac{\sigma^2}{M}$ in eq. (9) (where σ is chosen as in [6]) and witnessed convergence in a few iterations.

5. NON-PARAMETRIC SYMBOL-BY-SYMBOL EQUALIZER

Similar to the channel estimator discussed before, the conventional detector (equalizer [1]) is based on the Gaussian assumption that is again not optimal for the interference affected channels. The performance of this zero-forcing equalizer [1] is highly sensitive to the quality of estimated channel and the ratio of interfering received signal with estimated channel. Thus for the said equalizer structure the decision boundary is clearly non-linear. Thereby we use a probabilistic equalizer whose decision is based on the estimated likelihood. For the estimated channel impulse response $\hat{\mathbf{h}}_{\mathbf{k}}$ (after convergence) from eq. (9) the ML estimate of the transmitted signal can be obtained by

$$\hat{X}(m) = argmax_{X=\hat{X}} \left(\hat{f}_E(Y(m) \mid \hat{H}(m)) \right)\big|_{\mathbf{h}=\hat{\mathbf{h}}_k} \tag{14}$$

where $\hat{H}(m)$ is the frequency response of the estimated channel and without loss of generality it is assumed that X is equi-probable. It should be noted that estimated pdf $\hat{f}_E$ for detection is generated by using eq. (11). Based on the higher probability of occurrence the hard-decision is taken on $\hat{X}(0),...,\hat{X}(M-1)$ to generate the output data as shown in Fig-1. From simulation results we observe that significant BER improvement is achieved by using this probabilistic equalizer.

6. SIMULATION RESULTS AND DISCUSSION

We assume a packet based OFDM system (similar to WLAN), we assume that the first symbol is known at the receiver hence used for channel estimation, while the remaining payload is the useful information. A multi-path fading channel model is considered with synchronous multipath OFDM interference as in eq. (2). The model is considered slowly fading, i.e. the channel is constant for the OFDM packet (of size 64-subcarriers and 8-symbols). To verify the robustness of the algorithm, simulations were carried out on Matlab for ensemble of 1000-runs. The performance measure is average BER for fixed signal to noise ratio (SNR) and for various values of signal to interference ratio (SIR). The SIR is defined as $SIR = \frac{\mathbb{E}\{(HX)(HX)^*\}}{\mathbb{E}\{II^*\}}$. The channel is defined as two-path Rayleigh fading channel with transfer function [1]:

$$h(z) = 0.8\alpha_0 \exp(j\theta_0) + 0.6\alpha_1 \exp(j\theta_1) z^{-1} \qquad (15)$$

The interfering channel is defined as:

$$g(z) = 0.5\alpha_2 \exp(j\theta_2) + 0.1\alpha_3 \exp(j\theta_3) z^{-1} \qquad (16)$$

where α_0, α_1, α_2, α_3 are the i.i.d random variables with Rayleigh distribution, and θ_0, θ_1, θ_2, θ_3 are i.i.d. random variables with uniform distribution.

The average BER plot is shown in Fig-2. The legends 'LS' and 'NPML' represents least squares and non-parametric maximum likelihood channel estimator respectively, when the detection for both algorithms is based on Gaussian assumption. 'Exact' represents the BER when it is assumed that the receiver has exact knowledge of the channel and the detection is based on Gaussian assumption. For lower two legends 'NPML-Det' and 'Exact-Det', suffix 'Det' stands for when the detection is based on estimated density, whereas the channel is estimated by NPML in 'NPML-Det' and channel is assumed known at the receiver for the 'Exact-Det'. The SNR is kept fixed at 17.63 dBs while SIR is varied over a large range. From the simulations we find, there is negligible BER improvement when detection is based on Gaussian assumption, moreover exact knowledge of channel also does not improve the BER performance because of interference and channel ratio effect at detector. However significant BER improvement is obtained by using the non-parametric symbol-by-symbol equalizer. It is also worth noting that there is a little difference when this equalizer is used with 'NPML' and exact channel knowledge, this also confirms that the estimated channel using 'NPML' is closer to the exact channel. It is also interesting to see that the BER curves follow the pattern as noted in [6].

7. CONCLUSION

It is shown that the channel estimator based on Gaussian noise assumption are inferior in interference affected channels. This non-Gaussian noise was estimated using kernel density estimator to estimate the likelihood function. A new channel estimation and symbol detection scheme was presented using the estimated density. Significant performance gains were achieved for multipath fading scenarios. It was also highlighted that major performance gain is achieved by using the proposed non-parametric symbol-by-symbol equalizer in interference limited channels.

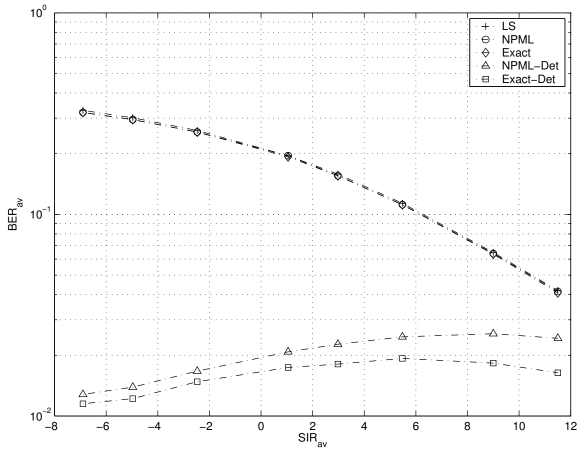

Figure 2: Average BER performance in multipath fading channel

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