NULL-STEERING BEAMSPACE TRANSFORMATION DESIGN FOR ROBUST DATA REDUCTION

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ABSTRACT

In this paper, we present a robust solution for data reduction in array processing. The purpose is to reduce the computation and improve the performance of applied signal processing algorithms by mapping the data into a lower-dimension beamspace through a transformation. Nulls steering to interference are incorporated into a transformation using the subspace projection technique, and the beamspace spatial spectrum estimation accuracy is evaluated and maximized with a measure. The derived transformation tries to preserve the full dimension Cramer-Rao bounds for the parameters of interest while rejecting undesired signals effectively. When compared with an optimal method and an adaptive approach, simulation results show that significant improvements are obtained in terms of beamspace direction-of-arrival estimation root-mean-squared error, bias and resolution probability.

1. INTRODUCTION

The desire for high gain beamforming and accurate direction-of-arrival (DOA) estimation can result in consideration of antenna arrays composed of a large number of sensor elements. However, the computational burden associated with processing data from large arrays is often prohibitively extensive. A common approach to solve this problem is to map the array data from the full dimension element-space (ES) into a lower dimension beamspace (BS), through a linear transformation, which is termed a matrix beamformer, before applying signal processing algorithms.

A key issue in beamspace processing is design of the transformation. Several criteria can be employed to judge the performance of a matrix beamformer, for instance, coverage of a spatial sector [1], the output interference power [2], the closeness between the ES Cramer-Rao bound (CRB) and the associated BS CRB [3], and etc. Various design methods have been proposed based on the above-mentioned guidelines. In addition to the major advantage of reducing computation, a well-designed preprocessor also improves the DOA estimation performance at other aspects, such as a lower resolution threshold and a reduced bias. Further more, Anderson shows that the full-dimension CRB, which is the best performance one could hope for, can also be asymptotically attained in BS [3]. The corresponding design is referred to as the maximum estimation accuracy (MEA) technique in this paper.

MEA is an optimal technique in terms of the DOA estimation accuracy; however, it is sensitive to out-of-sector sources (interferers). For MEA, attenuation of interference and BS DOA estimation accuracy are conflicting requirements, and the trade-off is made by selecting a suitable dimension. The same problem exists with other approaches presented in [1], [2], and etc. How to design a transformation, which minimizes the effect of interference and achieves as much as possible the ES optimal performance, is a topic of practical interest.

Recently, Eriksson and Viberg [4] propose an adaptive transformation design approach, which achieves better performance than MEA when there are out-of-sector sources. It rejects interference based on the information in the data covariance matrix $\mathbf{R}$. However, when the number of snapshots is small, $\mathbf{R}$ cannot be estimated accurately, thus degradation in performance is inevitable. And the technique encounters difficulties while dealing with correlated sources. When some sources are coherent, it malfunctions completely. The problems are common for most adaptive techniques.

In this paper, we present a flexible, robust and conceptually intuitive approach for dimension reduction in the scenarios involving out-of-sector sources. Nulls steering to the undesired signals are incorporated into an optimal transformation by the so-called subspace projection technique. The position, width and depth of the nulls can be designed easily, and degradation in BS DOA estimation accuracy caused by incorporating nulls is slight, which can be evaluated and minimized by a measure. The null-incorporating algorithm is data-independent, which makes the technique robust and perform well in the scenarios involving small number of snapshots and highly correlated, even coherent sources.

2. DATA MODEL

Consider a wave field due to L narrow-band sources and additive noise, sampled spatially by an M ($M > L$) sensor array of arbitrary geometry. The received M-dimensional data vector $\mathbf{x}(t)$ can be written as follows:

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{n}(t); \quad t = 1, \cdots, N. \tag{1}$$

N is the number of snapshots. The $M \times L$ matrix $\mathbf{A}(\boldsymbol{\theta})$ has the following special structure $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_L)]$, where $\mathbf{a}(\theta_k) = [a_1(\theta_k), \cdots, a_M(\theta_k)]^T$, $k = 1, \cdots, L$, is the steering vector, $a_i(\theta_k)$ is the complex response of the i^{th} sensor relative to its response at the reference sensor when a single wavefront impinges at an angle θ_k, $(\bullet)^T$ denotes the transpose; and $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_L]^T$ is a parameter vector corresponding to the true source DOAs. It is assumed that the sensor array is accurately calibrated. We also regard the parameterization of $\mathbf{a}(\theta)$ as being known. $\mathbf{s}(t)$, the L-dimensional vector of source time-series as observed at the reference sensor, and the additive noise $\mathbf{n}(t)$ are assumed to be independent, stationary, zero-mean, Gaussian processes with covariance $E[\mathbf{s}(t)\mathbf{s}^H(t)] = \mathbf{P}_s$ and $E[\mathbf{n}(t)\mathbf{n}^H(t)] = \mathbf{Q}$, where $E[\bullet]$ stands for the expectation operator, and $(\bullet)^H$ denotes the conjugate transpose. The $L \times L$ matrix $\mathbf{P}_s$ and $M \times M$ matrix $\mathbf{Q}$ are Hermitian and positive definite, but otherwise arbitrary. The $M \times M$ covariance matrix of $\mathbf{x}(t)$ is

$$\mathbf{R}_{\mathbf{x}} = E\left[\mathbf{x}(t)\mathbf{x}^H(t)\right] = \mathbf{A}(\boldsymbol{\theta})\mathbf{P}_{\mathbf{s}}\mathbf{A}^H(\boldsymbol{\theta}) + \mathbf{Q} \cdot \quad (2)$$

Assume L_{in}, the number of sources within the location sector Θ, to be known (given or estimated). Θ can be one location sector or a union of multiple separate sectors of interest. If we introduce the $M\times J$ transformation **T** covering Θ, $L_{in} < J \le M$, and the mapping $\mathbf{x}(t) \mapsto \mathbf{z}(t) = \mathbf{T}^H\mathbf{x}(t)$ from ES to BS, a new set of J-dimensional observations is obtained, which will be

$$\mathbf{z}(t) = \mathbf{T}^H\mathbf{x}(t) = \mathbf{T}^H\mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{T}^H\mathbf{n}(t), \quad (3)$$

and the $J\times J$ covariance matrix of $\mathbf{z}(t)$ will be

$$\mathbf{R}_{\mathbf{z}} = E\left[\mathbf{z}(t)\mathbf{z}^H(t)\right] = \mathbf{A}_b(\boldsymbol{\theta})\mathbf{P}_{\mathbf{s}}\mathbf{A}_b^H(\boldsymbol{\theta}) + \mathbf{T}^H\mathbf{Q}\mathbf{T}, \quad (4)$$

where the $J\times L$ matrix $\mathbf{A}_b(\boldsymbol{\theta}) = \mathbf{T}^H\mathbf{A}(\boldsymbol{\theta})$ is the counterpart of $\mathbf{A}(\boldsymbol{\theta})$ in BS.

If $\mathbf{n}(t)$ is spatially white and of variance σ_n^2, $\mathbf{Q} = \delta_n^2\mathbf{I}_M$, where $\mathbf{I}_M$ stands for an M-dimensional identity matrix. It will be required that

$$\mathbf{T}^H\mathbf{T} = \mathbf{I}_J, \quad (5)$$

implying that the BS sensor noise is spatially white whenever the ES noise is so. Thus $\mathbf{R}_{\mathbf{z}}$ is reduced to $\mathbf{R}_{\mathbf{z}} = \mathbf{A}_b(\boldsymbol{\theta})\mathbf{P}_{\mathbf{s}}\mathbf{A}_b^H(\boldsymbol{\theta}) + \sigma_n^2\mathbf{I}_J$.

3. PRODUCING NULLS WITH THE SUBSPACE PROJECTION TECHNIQUE

Assume that a matrix beamformer $\mathbf{T}_0$ focusing on a spatial sector Θ_0 is calculated using an optimal technique such as MEA [3], the spheroidal sequence approach [1], or others. It is sensitive to out-of-sector emitters, since accurate BS DOA estimation is obtained at the expense of high sidelobe. Now, we want to incorporate some nulls into $\mathbf{T}_0$, which steer to the null sector Θ_N, to cancel strong interferers in it. Usually, Θ_N is within the complement of Θ_0. This can be accomplished by adjusting the beams in $\mathbf{T}_0$ into the null space of Θ_N.

Let $\boldsymbol{\theta}_N = [\theta_1,\cdots,\theta_n]^T \in \Theta_N$, $\theta_1 < \cdots < \theta_n$ and $n < M$, be a vector of design notch locations, which are, for example, equally spaced. Assume

$$\mathbf{A}_{\Theta_N} = \mathbf{A}(\boldsymbol{\theta}_N) = \left[\mathbf{a}(\theta_1)\ \mathbf{a}(\theta_2)\ \cdots\ \mathbf{a}(\theta_n)\right]. \quad (6)$$

If we project the matrix beamformer $\mathbf{T}_0$ into the orthogonal complement of the column space spanned by $\mathbf{A}_{\Theta_N}$, the projected columns (beams) will be orthogonal to the space spanned by $\mathbf{A}_{\Theta_N}$, and n notches will be placed at locations $\theta_1,\cdots,\theta_n$, thus some null sectors are incorporated into $\mathbf{T}_0$.

The projection matrix can be constructed by

$$\mathbf{P}_{\Theta_N}^{\perp} = \mathbf{I} - \mathbf{A}_{\Theta_N}\left(\mathbf{A}_{\Theta_N}^H\mathbf{A}_{\Theta_N}\right)^{-1}\mathbf{A}_{\Theta_N}^H, \quad (7)$$

and the projection is given by

$$\mathbf{B} = \mathbf{P}_{\Theta_N}^{\perp}\mathbf{T}_0. \quad (8)$$

If the white noise model is employed in ES and BS processing, (5) must be satisfied, which can be guaranteed by applying an orthonormal transformation to **B**

$$\mathbf{T} = \mathbf{B}\left(\mathbf{B}^H\mathbf{B}\right)^{-1/2}. \quad (9)$$

T and **B** span the same column space, thus represent the same transformation, and **T** has orthonormal columns. Besides the steering vectors $\mathbf{a}(\theta_i)$, the derivatives of them can also be included in $\mathbf{A}_{\Theta_N}$.

Further more, a measure $\eta_1(\mathbf{T})$ is introduced to evaluate the location, width and depth of the incorporated nulls. $\eta_1(\mathbf{T})$ represents the difference within Θ_N between the actual beam pattern or gain response of **T** and an ideal response with desired null specification. The gain of a matrix beamformer **T** is defined by

$$g(\theta,\mathbf{T}) = \frac{\mathbf{a}^H(\theta)\,\mathbf{T}\,\mathbf{T}^H\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\,\mathbf{a}(\theta)}, \quad \theta \in [0, 2\pi]. \quad (10)$$

For orthonormal columns of **T**, $0 \le g(\theta,\mathbf{T}) \le 1$. If **T** is designed to cover the location sector Θ_0, the desired gain will be $g_d(\theta,\mathbf{T}) = \begin{cases} 1 & \theta\in\Theta_0 \\ 0 & otherwise \end{cases}$. To evaluate $\eta_1(\mathbf{T})$, at first, a template formed by the shape of the mainlobe of $\mathbf{T}_0$ and the desired nulls is defined, whose sidelobe level is specified somewhat arbitrarily, since we only pay attention to the gain response in Θ_N, where all strong interferers are assumed to be located. And then, the template is cast over the gain response of **T** to compute the cumulative difference within Θ_N as a form of error-in-gain in decibels. $\eta_1(\mathbf{T}) \ge 0$, with the lower bound being desirable to attain.

Another concern is the DOA estimation accuracy associated with the projected beamformer. As for the white noise model, it is shown that the optimal matrix beamformer $\mathbf{T}_0$ satisfies the condition [3]

$$\Re\{\mathbf{T}_0\} \supseteq \Re\left\{\left[\mathbf{a}(\theta_1)\ \cdots\ \mathbf{a}(\theta_{L_{in}})\ \mathbf{d}^{(1)}(\theta_1)\ \cdots\ \mathbf{d}^{(1)}(\theta_{L_{in}})\right]\right\}, \quad (11)$$

where $\mathbf{d}^{(1)}(\theta_i) = \partial\mathbf{a}(\theta_i)/\partial\theta_i$, $\Re\{\mathbf{X}\}$ denotes the range space of **X**, and $\theta_1,\cdots,\theta_{L_{in}}$ are the true DOAs of the L_{in} in-sector sources. The projection (8) makes $\Re\{\mathbf{T}\}$ deviate from $\Re\{\mathbf{T}_0\}$, thus DOA estimation accuracy degradation is inevitable. A measure $\eta_2(\mathbf{T})$, the closeness between the ES and BS CRB over Θ_0 [3],

$$\eta_2(\mathbf{T}) = \left(\frac{\int_{\Theta_0}\mathrm{CRB}_{ES}(\theta)d\theta}{\int_{\Theta_0}\mathrm{CRB}_{BS}(\theta,\mathbf{T})d\theta}\right)^{1/2}, \quad (12)$$

can be used to evaluate and minimize the degradation, where $0 < \eta_2(\mathbf{T}) \le 1$, and $\eta_2(\mathbf{T}) = 1$ is desired. Here, the element-space stochastic CRB is given by [5]

$$\mathrm{CRB}_{ES}(\theta) = \frac{\sigma_n^2}{2N}\left[\mathrm{Re}\left\{\left[\mathbf{D}^H(\theta)\mathbf{P}_A^{\perp}\mathbf{D}(\theta)\right]\odot\left[\mathbf{P}_{\mathbf{s}}\mathbf{A}^H(\theta)\mathbf{R}_{\mathbf{x}}^{-1}\mathbf{A}(\theta)\mathbf{P}_{\mathbf{s}}\right]^T\right\}\right]^{-1} \quad (13)$$

where $\mathbf{X}\odot\mathbf{Y}$ denotes the Hadamard product of the matrices **X** and **Y**, i.e., $(\mathbf{X}\odot\mathbf{Y})_{ij} = \mathbf{X}_{ij}\mathbf{Y}_{ij}$, $\mathrm{Re}\{\bullet\}$ denotes the real part of the complex variable, and

$$\mathbf{P}_A^{\perp} = \mathbf{I} - \mathbf{A}(\theta)\left[\mathbf{A}^H(\theta)\mathbf{A}(\theta)\right]^{-1}\mathbf{A}^H(\theta),$$

$$\mathbf{D}(\theta) = \left[\mathbf{d}^{(1)}(\theta)\right].$$

The expression for $\mathrm{CRB}_{BS}(\theta,\mathbf{T})$ is obtained by substitution of $\mathbf{A}(\theta)$, $\mathbf{D}(\theta)$ and $\mathbf{R}_{\mathbf{x}}$ by $\mathbf{T}^H\mathbf{A}(\theta)$, $\mathbf{T}^H\mathbf{D}(\theta)$ and $\mathbf{R}_{\mathbf{z}}$, respectively [3].

4. OPTIMAL NULL-STEERING BEAMSPACE TRANSFORMATION DESIGN

We suggest the following procedure for null-steering matrix beamformer design and beamspace DOA estimation, which is more computationally efficient and numerically stable:

1. Specify a spatial sector Θ_0, within which sources of interest are assumed to be located. Let
$$\mathbf{A}_f = \left[\mathbf{a}\left(\theta_{f_1}\right) \cdots \mathbf{a}\left(\theta_{f_k}\right) \mathbf{d}^{(1)}\left(\theta_{f_1}\right) \cdots \mathbf{d}^{(1)}\left(\theta_{f_k}\right)\right], \quad (14)$$
where, $\boldsymbol{\theta}_f = \left[\theta_{f_1}, \cdots, \theta_{f_k}\right]^T$ is a sufficiently dense set of fictitious DOAs within Θ_0.
2. Specify the null sector Θ_N. Select an initial notch location vector $\boldsymbol{\theta}_N = \left[\theta_1, \cdots, \theta_n\right]^T$, where $\theta_1 < \cdots < \theta_n \in \Theta_N$, and construct the $M \times n$ matrix $\mathbf{A}_{\Theta_N}$ as in (6). Taking the orthogonal-triangular (QR) decomposition of $\mathbf{A}_{\Theta_N}$
$$\mathbf{A}_{\Theta_N} = \left[\mathbf{Q}_1 \quad \mathbf{Q}_2\right]\begin{bmatrix}\mathbf{R}_1 \\ \mathbf{0}\end{bmatrix}, \quad (15)$$
where the $M \times (M-n)$ matrix $\mathbf{Q}_2$ is an orthonormal basis, which spans the null space of $\mathbf{A}_{\Theta_N}^H$.
3. Let $\mathbf{Y} = \mathbf{Q}_2^H \mathbf{A}_f$. Assume $J = 2k$ to be the initial beamspace dimension, which is large enough to accommodate the maximum number of expected sources within Θ_0. Compute the singular-value decomposition (SVD) of $\mathbf{Y}$ as $\mathbf{U\Sigma V}^H = \mathbf{Y}$, and the $M \times J$ transformation matrix is calculated as $\mathbf{T}_J = \mathbf{Q}_2\left[\mathbf{u}_1 \quad \cdots \quad \mathbf{u}_J\right]$, where $\mathbf{u}_i, i = 1, \cdots, J$, are ordered left singular vectors of $\mathbf{Y}$.
4. Examine the null position, width, depth and the resultant BS DOA estimation accuracy by evaluating the performance measures $\eta_1(\mathbf{T})$ and $\eta_2(\mathbf{T})$. If the measures are not satisfactory, introduce more notches or change the location of $\boldsymbol{\theta}_N$, and repeat step 2 and 3.
5. Optional step. If possible, reduce the dimension of the transformation matrix by evaluating the performance measure $\eta_2(\mathbf{T})$ (12). Estimate the number of sources $\hat{L}_{in}$ within the beamspace spanned by $\mathbf{T}_J$ using, e.g., the minimum description length (MDL) principle [6]. Suppose the new dimension to be *j*, where $\hat{L}_{in} < j \le J$. Compute the SVD of $\mathbf{A}_f$ (14) as $\mathbf{U}_f \mathbf{\Sigma}_f \mathbf{V}_f^H = \mathbf{A}_f$, and take the $M \times j$ transformation matrix (without nulls) as
$$\mathbf{T}'_j = \left[\mathbf{u}_{f1} \quad \cdots \quad \mathbf{u}_{fj}\right], \quad (16)$$
where $\mathbf{u}_{fi}, i = 1, \cdots, j$, are the *j* first columns of $\mathbf{U}_f$. By depicting the values of $\eta_2(\mathbf{T}'_j)$ obtained for different $j's$, it is easy to see for which dimensions the performance measure is sufficiently close to 1. Assume *K* to be the minimum dimension satisfying $\eta_2(\mathbf{T}'_K) \approx 1$, set the beamspace dimension to *K* and repeat step 3 and 4.

Remarks.

1. Step 1 is based on the theory of (11), and $\theta_{f_i}, i = 1, \cdots, k$, can be chosen somewhat arbitrarily within Θ_0. However, to obtain enough sector coverage, the spacing between any two adjacent $\theta_{f_i}'s$ seems necessary to be less than the beamwidth. In step 2, the initial notch locations $\theta_1, \cdots, \theta_n$ can be selected equally spaced with approximate one-beamwidth spacing. To examine if the null location, width and depth produced by $\boldsymbol{\theta}_N$ are acceptable, one can evaluate the measure $\eta_1(\mathbf{T})$.
2. The focusing sector Θ_0 and the null sector Θ_N can be specified beforehand, or calculated from the output of a low-resolution, but robust DOA estimator, for example, the conventional beamformer, by finding the peak locations of the spatial spectrum. Similar or more accurate *a priori* DOA information is required in numerous popular array processing techniques such as constrained MUSIC [7], pseudo-randomly generated estimator bank [8], generalized sidelobe canceller [9], and others.

5. SIMULATION RESULTS

In this section, we present numerical examples to evaluate the proposed technique against MEA [3] and the adaptive approach [4] in scenarios involving out-of-sector sources. We consider a 16 sensor uniform linear array (ULA) with half-wavelength element spacing. The unconditional maximum likelihood (ML) algorithm computed using genetic algorithms (GA), termed GA-ML [10], which is able to asymptotically achieve the CRB, is employed as the DOA estimator. The simulation results are based on 500 independent Monte-Carlo trials.

Two uncorrelated sources with 0dB SNR are located at 105° and 108° within the sector of interest, which is assumed to be $\Theta_0 = [100^\circ, 115^\circ]$. There are 2 equal-power, uncorrelated interfering signals located outside Θ_0, at 32° and 42°, and their SNR varies within the range [–2dB, 20dB]. The emitter at 32° is correlated with the source at 105° with the correlation factor $\gamma = 0.7$, and so are the emitters at 42° and 108° with $\gamma = 0.9$. The number of snapshots is 20.

Following the design steps of Section 4, we compute the array beamwidth to approximately 7.2°, which means we can choose $\boldsymbol{\theta}_f = [100.5^\circ, 107.5^\circ, 114.5^\circ]^T$. The null sector specification is made on-line from the output of a conventional beamformer. We choose $\Theta_N = [23^\circ, 46^\circ]$, corresponding to the left and right –3dB decrease points relative to the peak level, and specify the desired null depth to be –35dB. Take $\boldsymbol{\theta}_N = [24^\circ, 31.2^\circ, 38.4^\circ, 45.6^\circ]^T$. Then we obtain a transformation matrix $\mathbf{T}_6$ of dimension 16×6 as shown in Fig. 1 by the solid line. The performance measures are calculated as $\eta_1(\mathbf{T}_6) = 17.3$, illustrated by the shadow within Θ_N, and $\eta_2(\mathbf{T}_6) = 0.9623$. To see if it is possible to make further dimension reduction, we estimate $\hat{L}_{in} = 2$ with $\mathbf{T}_6$. By depicting the values of $\eta_2(\mathbf{T}'_j)$ (16) obtained for $j = 3 - 6$, it can readily be seen that it may be sufficient to have the dimension of five. Changing the notch locations with smaller spacing at the right end point of Θ_N, we obtain a 16×5 matrix beamformer $\mathbf{T}_5$ whose beam pattern is shown in Fig. 1 by the dotted line. The corresponding measures are $\eta_1(\mathbf{T}_5) = 0$ and $\eta_2(\mathbf{T}_5) = 0.9623$. The template specifying the null parameters for evaluating $\eta_1(\mathbf{T})$ is illustrated in Fig. 1 by the dashed line.

The results of DOA estimation RMSE and the resolution probability (RP) for the sources within Θ_0, versus the out-of-sector emitter SNR are illustrated in Fig. 2 and Fig. 3. The sources

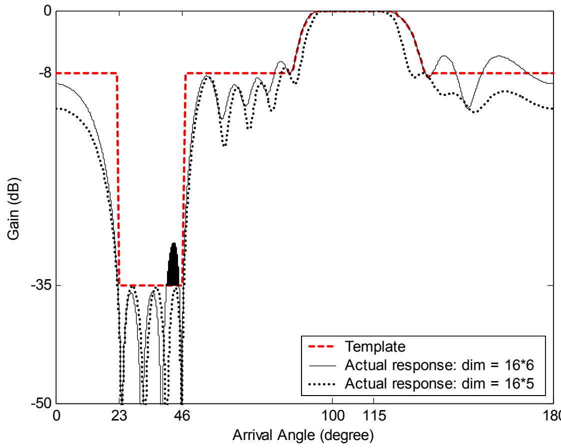

Fig. 1 The gain responses of two null-steering matrix beamformers and a template for evaluating the performance measure $\eta_1(\mathbf{T})$.

are considered to be resolved in a run if DOA estimation error for each of them is less than half of their separation. MEA is sensitive to the power variation of the interferers, and when the interfering signals are 10dB stronger than the sources, it malfunctions completely in terms of RP. The subspace projection technique demonstrates significant advantages over the adaptive approach, in terms of both RMSE and RP. The adaptive approach encounters horrible problems when the interfering signals and the desired sources are highly correlated; however, the proposed technique can deal with them without any difficulties, which yields DOA estimates with accuracy close to the element space CRB. When the out-of-sector signals are 20dB stronger than the desired signals, the proposed technique still performs well.

6. CONCLUSION

This paper presents a robust solution for array processing dimension reduction in scenarios involving out-of-sector sources. Beamspace DOA estimation is sensitive to out-of-sector signals, and the attenuation effect on such interferers and BS estimation accuracy are conflicting requirements. This technique rejects the undesired signals with nulls, and try to minimize the inevitable degradation in BS DOA estimation accuracy, thus retain as much as possible the optimal element space performance. The position, width and depth of the nulls are guaranteed by selecting the so-called design notch locations, and the estimation accuracy is evaluated and maximized with a measure, the closeness between the full dimension CRB and the reduced dimension BS CRB. Simulation results demonstrate that the proposed technique results in much better BS DOA estimation performance in the scenarios involving small number of snapshots and highly correlated sources compared with MEA and the adaptive approach. The subspace projection technique can also be employed to generate nulls for other array processing applications, for instance, array pattern synthesis.

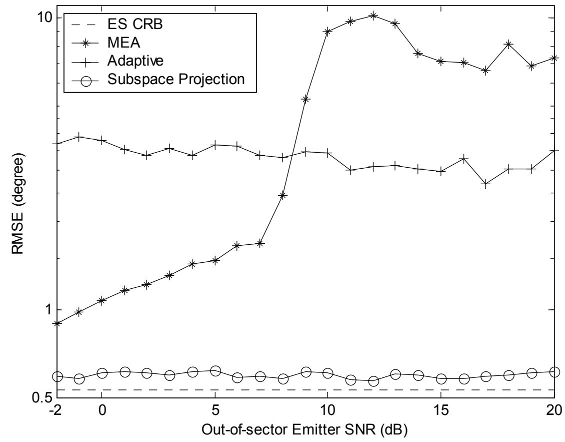

Fig. 2 The beamspace DOA estimation RMSE versus the out-of-sector emitter SNR.

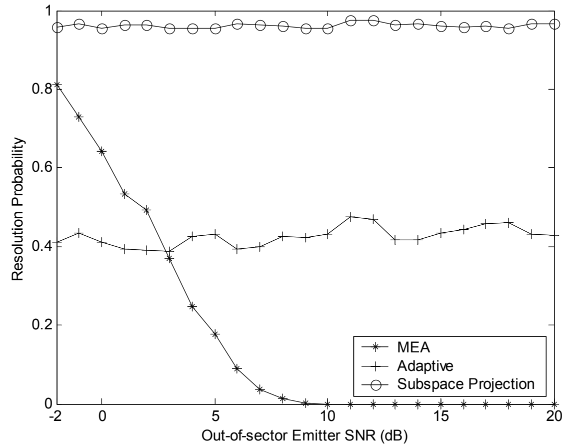

Fig. 3 The beamspace DOA resolution probability versus the out-of-sector emitter SNR.

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